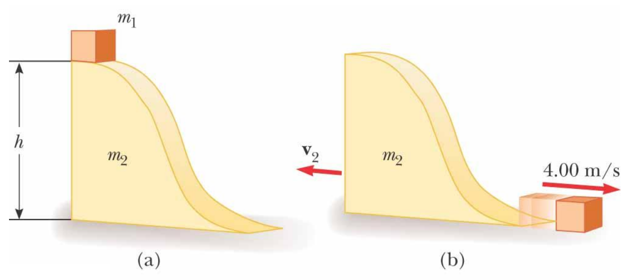
1. (**20 pts**) A small block of mass $m_1 = 0.5kg$ is released from rest at the top of a curved-shaped frictionless wedge of mass $m_2 = 3.0kg$, which sits on a frictionless horizontal surface as in the figure below. When the block leaves the wedge, its velocity is measured to be $4.0m/s$ to the right, as in (b).

   (a) What is the velocity of the wedge after the block reaches the horizontal surface?

   (b) What is the height h of the wedge?



A POSSIBLE SOLUTION

(a) Since there are not external forces (system = both masses) acting in the x direction of the system, $\Delta p_x = 0$.

$$p_{fx} = p_{ix} \tag{1}$$
$$m_1\, v_1 - m_2\, v_2 = 0 \tag{2}$$
$$\Rightarrow v_2 = \frac{m_1\, v_1}{m_2} \tag{3}$$
$$\Rightarrow v_2 = \frac{0.5\, kg\, (4.0\, m/s)}{3.0\, kg} = \boxed{0.67\,\mathrm{m/s}}\checkmark \tag{4}$$

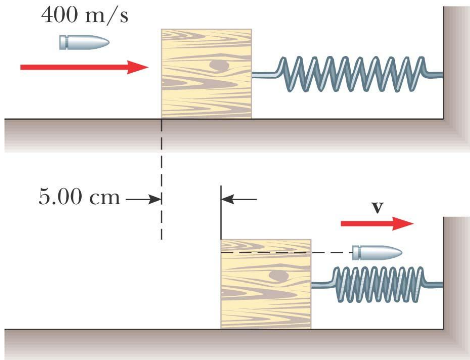
(b) Total mechanical energy is conserved since there is no friction ($\Delta E = 0$).

$$\Delta E = 0 = \Delta K_T + \Delta U_T. \tag{5}$$
$$0 = \frac{1}{2} m_1\, ({v_{1f}}^2 - {v_{1i}}^2) + \frac{1}{2} m_2\, ({v_{2f}}^2 - {v_{2i}}^2) + m_1\, g\, (y_f - y_0) \tag{6}$$
$$h = \frac{m_1\, {v_{1f}}^2 + m_2\, {v_{2f}}^2}{2\, m_1\, g} \tag{7}$$
$$\Rightarrow h = \boxed{0.95\,\mathrm{m}}\checkmark \tag{8}$$

2. (**20 pts**) A 5.0 g bullet moving with an initial speed of 400 m/s is fired into and passes through a 1.0 kg block of wood, as in the figure below. The block, initially at rest on a frictionless, horizontal surface, is connected to a spring with force constant 900 N/m. If the block moves 0.05 m to the right after impact, find:

(a) the speed at which the bullet emerges from the block.

(b) the mechanical energy lost in the collision.



A POSSIBLE SOLUTION

(a) Since there are no external forces acting in the x direction on the system (bullet and block), momentum is conserved.

$$\begin{aligned} p_{fx} &= p_{ix} && (9)\\ M_B\,V_{Bf} + m_b\,v_{bf} &= m_b\,v_{bi} && (10)\\ \Rightarrow v_{bf} &= \frac{m_b\,v_{bi} - M_B\,V_{Bf}}{m_b} && (11)\end{aligned}$$

We need to find V_{Bf} in order to solve the problem. We can find V_{Bf} by considering $\Delta E = 0$. Immediately following the impact:

$$\begin{aligned} \Delta E = 0 &= \Delta K_T + \Delta U_T. && (12)\\ 0 &= \frac{1}{2}M_B\,(V^2_{Bfinal} - V^2_{Binitial}) + \frac{1}{2}k\,(\Delta x_f^2 - \Delta x_i^2) && (13)\end{aligned}$$

but $V_{Bfinal} = 0$ and $V_{Binitial} = V_{Bf}$ from Eq. 11, and $\Delta x_i = 0$.

$$\begin{aligned} \Rightarrow V_{Bf} &= \sqrt{\frac{k}{M_B}}\,\Delta x_f && (14)\\ \Rightarrow V_{Bf} &= \boxed{1.5\,\mathrm{m/s}} && (15)\end{aligned}$$

Using the expression obtained for V_{Bf} in Eq. 14, leads to the following expression for v_{bf}:

$$\begin{aligned} v_{bf} &= v_{bi} - \frac{\sqrt{M_B\,k}\,\Delta x_f}{m_b} && (16)\\ \Rightarrow v_{bf} &= \boxed{100\,\mathrm{m/s}}\checkmark && (17)\end{aligned}$$

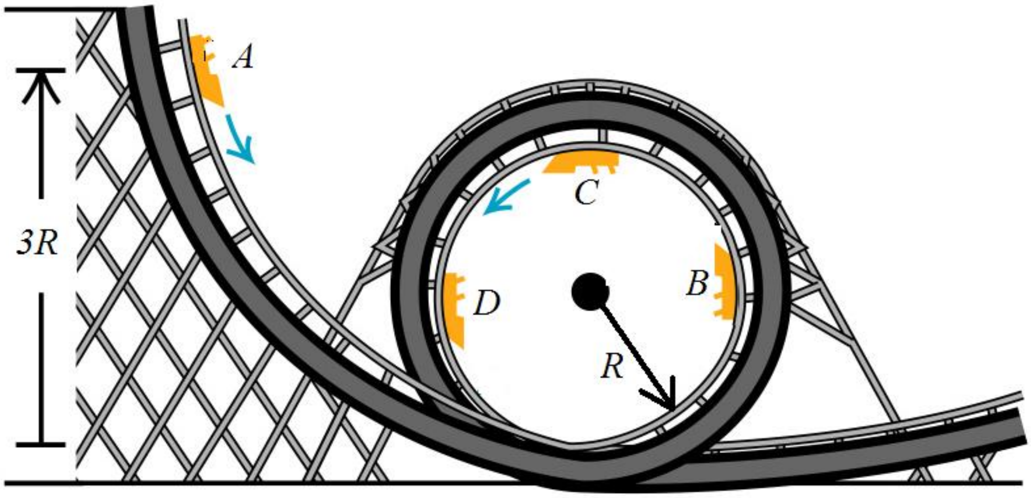
(b) The mechanical energy lost in the collision is determined by considering ΔE.

$$\begin{aligned} \Delta E &= \Delta K_T + \Delta U_T && (18)\\ \Delta E &= \frac{1}{2}m_b\,(v_{bf}^2 - v_{bi}^2) + \frac{1}{2}M_B\,(V_{Bf}^2 - V_{Bi}^2) + \frac{1}{2}k\,(\Delta x_f^2 - \Delta x_i^2) && (19)\end{aligned}$$

Using the following values: $v_{bf} = 100\ m/s$, $v_{bi} = 400\ m/s$, $V_{Bf} = 0 = V_{Bi}$, $\Delta x_f = 0.05\ m$, and $\Delta x_i = 0$ leads to the following answer:

$$\Delta E = \boxed{\text{-374 J}}\checkmark \qquad (20)$$

3. (**20 pts**) A car in an amusement park ride (roller-coaster) runs without friction around the track shown in the figure below. The car is initially at rest at point A at a height $3R$ above the bottom of the loop which has radius R. Treat the car as a particle.



(a) Compute the acceleration (*vector*) at position B.

(b) Compute the acceleration (*vector*) at position C.

A POSSIBLE SOLUTION

(a) At position B, the roller coaster will experience an inward acceleration due to its speed at B, and the downward acceleration of gravity. The inward acceleration (centripetal acceleration) is $a_c = \frac{v_B{}^2}{R}$. One needs to find the speed of the roller coaster at B. (Consider total mechanical energy!)

$$\Delta E = \Delta K_T + \Delta U_T \quad (21)$$
$$0 = \frac{1}{2}m(v_f{}^2 - v_i{}^2) + m\,g\,(y_f - y_0) \quad (22)$$
$$\Rightarrow v_f = \sqrt{2\,g(y_0 - y_f)} \quad (23)$$

To find the speed at B, set $y_0 = 3R$ and $y_f = R$ in Eq. 23:

$$v_B = \sqrt{2\,g(2\,R)} \quad (24)$$
$$\Rightarrow a_c = \frac{v_B{}^2}{R} = 4\,g \quad (25)$$
$$\Rightarrow \vec{a}_B = \boxed{-4\,g\,\hat{\imath} - g\,\hat{\jmath}}\,\checkmark \quad (26)$$

(b) To find the speed of the roller coaster at position C, set $y_0 = 3R$ and $y_f = 2R$ in Eq. 23:

$$v_C = \sqrt{2\,g\,(R)} \quad (27)$$
$$\Rightarrow a_c = \frac{v_C{}^2}{R} = 2\,g \quad (28)$$
$$\Rightarrow \vec{a}_C = \boxed{-2\,g\,\hat{\jmath}}\,\checkmark \quad (29)$$

Physics 101 Fall 2005: Test 2—Multiple-Choice Questions

1. At time $t = 0$ a particle initially at rest is subject to a resultant force in the $+x$ direction that increases linearly with t. If the value of the force at $t = 0$ is zero, the kinetic energy of the particle subsequently increases at a rate proportional to:

   (a) $\sqrt{t}$
   (b) t
   (c) t^2
   (d) t^3
   (e) t^4

2. A box sliding on a frictionless flat surface runs into a fixed spring, which compresses a distance x to stop the box. If the initial speed of the box were doubled, how much would the spring compress in this case?

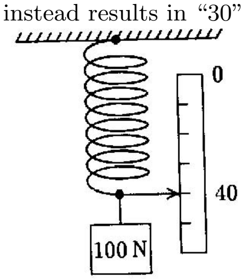
   (a) half as much.
   (b) the same amount.
   (c) $\sqrt{2}$ times as much.
   (d) twice as much.
   (e) four times as much.

3. A golfer making a putt gives the ball an initial speed of v_0, but he has badly misjudged the putt, and the ball only travels one-quarter of the distance to the hole. What initial speed should he have given the ball in order to make it into the hole?

   (a) $2\,v_0$.
   (b) $3\,v_0$.
   (c) $4\,v_0$.
   (d) $8\,v_0$.
   (e) $16\,v_0$.

4. A spring, with a pointer attached to its end, hangs next to a ruler. With a 100 N weight attached, the pointer indicates "40" on the ruler as shown in the figure. Using a 200 N weight instead results in "60" on the ruler. Using an unknown weight X instead results in "30" on the ruler. The weight of X is

   (a) 10 N
   (b) 20 N
   (c) 30 N
   (d) 40 N
   (e) 50 N



5. The graph below represents the potential energy $U(x)$ for a particle of mass m that moves along the x axis. The dashed horizontal line represents the total mechanical energy of the particle. Which statement(s) most accurately describe(s) the particles behavior?

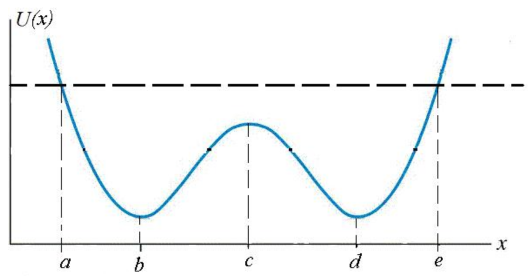
I. The particle experiences the largest magnitude of force at $x = b$, $x = c$, and $x = d$.

II. The particle experiences no force at $x = b$, $x = c$, and $x = d$.

III. The particle has the largest speed at $x = a$ and $x = e$.

IV. The particle has the largest speed at $x = b$ and $x = d$.

V. The particle oscillates back and forth between $x = a$ and $x = e$.



(a) only I and IV are true

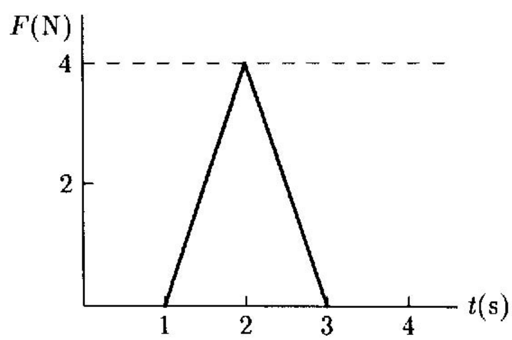
(b) only II and IV are true

(c) only II, IV and V are true

(d) only III and IV are true

(e) only I, III, and V are true

6. A 5 kg object can only move along the x axis. It is subjected to force F in the positive x direction. A graph of F as a function of time t is shown below. Over the time the force is applied, the change in velocity of the object is



(a) $0.8 m/s$.

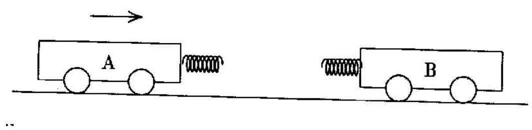
(b) $1.2 m/s$.

(c) $1.44 m/s$.

(d) $2.0 m/s$.

(e) $4.0 m/s$.

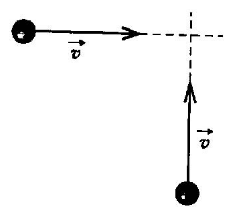
7. Two carts (labeled A and B), each have springs for bumpers, collide as shown below. Cart A has a mass of 2 kg and is initially moving to the right. Cart B has a mass of 3 kg and is initially stationary. When the separation between the carts is a minimum:



(a) cart B is still at rest.
(b) cart A has come to rest.
(c) both carts have the same momentum.
(d) both carts have the same initial kinetic energy.
(e) the kinetic energy of the system is at a minimum.

8. Two identical bodies of mass M move with equal speeds v. The direction of their velocities is illustrated in the figure below. The magnitude of the total linear momentum of the system is

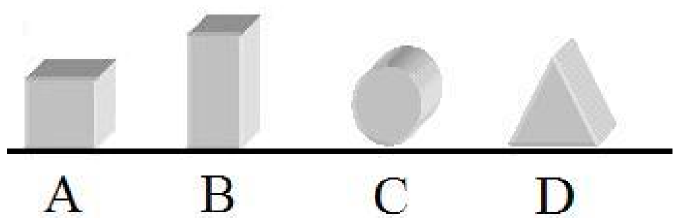
(a) $2\,M\,v$.
(b) $M\,v$.
(c) $4\,M\,v$.
(d) $\sqrt{2}\,M\,v$.
(e) $4\,\sqrt{2}\,M\,v$.



9. Consider the objects depicted below all made of the same uniform material:

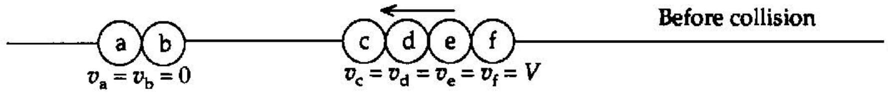
A. A square block.
B. A rectangular block.
C. A cylinder.
D. An triangular prism.

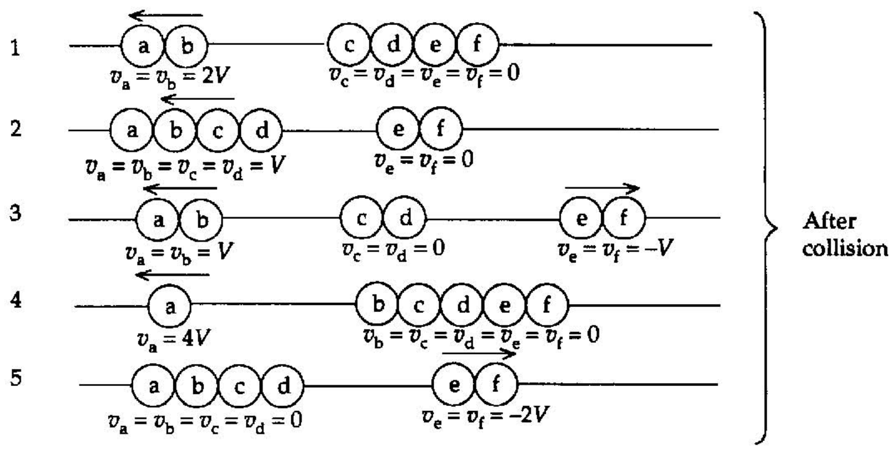
Which of these objects shown in the figure below has its center of mass furthest from the ground (indicated by the solid line) in its depicted orientation?



(a) A
(b) B
(c) C
(d) D

10. The balls shown below in the figure are strung on a very tight wire and slide without friction. If the balls are of equal mass, the diagram that best represents a totally elastic collision is




1
a
b
$v_a = v_b = 2V$
c
d
e
f
$v_c = v_d = v_e = v_f = 0$
2
a
b
c
d
$v_a = v_b = v_c = v_d = V$
e
f
$v_e = v_f = 0$
3
a
b
$v_a = v_b = V$
c
d
$v_c = v_d = 0$
e
f
$v_e = v_f = -V$
4
a
$v_a = 4V$
b
c
d
e
f
$v_b = v_c = v_d = v_e = v_f = 0$
5
a
b
c
d
$v_a = v_b = v_c = v_d = 0$
e
f
$v_e = v_f = -2V$
After collision

(a) 1.
(b) 2.
(c) 3.
(d) 4.
(e) 5.

Physics 101 Fall 2005: Test 2—Multiple-Choice Answers

	A	B	C	D	E
1					X
2				X	
3	X				
4					X
5			X		
6	X				
7					X
8				X	
9		X			
10		X			