1) 30 points total

(A) +10 pts

- +4 pts Energy conservation $\Delta K + \Delta U = 0$
- 4 pts $U_f = mg(R + \Delta h)$
  - +2 pts $\Delta h = R \sin 30° = R/2$
  - +2 pts $U_f = \frac{3}{2} mgR$
- +2 pts correct answer $v_f = \sqrt{2gR}$.

B) +5 pts

$a_N = \frac{v^2}{R} = 2g$ "No vector notation"

(−1) if in vector notation

(c) +6 pts

- +4 pts correct geometry (setup) i.e., $g_{tan} = g \cos 30°$
  (Not $\sin 30°$)
- +2 pts – correct answer.

(d) +9 pts

$\overset{2\,pts}{N} + \overset{2\,pts}{\tfrac{1}{2}mg} = \overset{2\,pts}{ma_N}$

- 6 pts Newton's 2nd law (2 for each part)
- 2 pts correct a_N from (B)
- 1 pt correct answer $\boxed{N = \frac{3}{2} mg}$

1. (**30 pts**) Consider the friction-free loop-the-loop apparatus shown in the figure below. The radius of the circular part is R. A mass m is released from rest as shown at a height $h = 5R/2$. When the mass reaches point A derive expressions for

SOLUTION

A). the velocity (vector) of the mass.

First find the magnitude of the velocity vector using energy considerations.

$$\Delta K + \Delta U = 0. \tag{1}$$

$$\frac{1}{2} m\, v_f^2 + m\, g\, (y_f - y_0) = 0. \tag{2}$$

$$y_0 = \frac{5\,R}{2} \quad \text{and } y_f = R + \Delta h. \tag{3}$$

$$\Delta h = R\, \sin 30 = \frac{1}{2}\, R. \tag{4}$$

$$\Rightarrow \frac{1}{2} m\, v_f^2 = m\, g\, \left(\frac{5}{2} - \frac{3}{2}\right)\, R. \tag{5}$$

$$v_f = \sqrt{2\, g\, R}. \tag{6}$$

Since a vector is needed we need the components of the vector. Using the conventional coordinate system (x-axis is horizontal and y-axis is vertical), we obtain the following vector.

$$\boxed{\vec{\mathbf{v}}_f = \sqrt{2\, g\, R}\, \left(-\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}\right).} \tag{7}$$

B). the normal component of its acceleration.

The normal component of its acceleration is the centripetal acceleration given by the following equation.

$$\boxed{a_N = \frac{v_f^2}{R} = 2\, g.} \tag{8}$$

C). the tangential component of its acceleration.

Since gravity acts vertically down, it will have a tangential component which has to be resolved.

$$g_T \text{ (Tangent) } = g\, \cos 30. \text{ (Obtained with the geometry of the problem).} \tag{9}$$

$$\boxed{a_T = -\frac{\sqrt{3}}{2}\, g} \tag{10}$$

D). the magnitude of the normal force exerted by the track on the mass.

To obtain the magnitude of the normal force, we need to consider Newton's 2^{nd}-Law.

$$\sum_N F : -N - m\,g\,\frac{1}{2} = -m\,a_N \tag{11}$$

$$N = 2\,m\,g - \frac{1}{2}\,m\,g \tag{12}$$

$$\boxed{N = \frac{3}{2}\,m\,g} \tag{13}$$

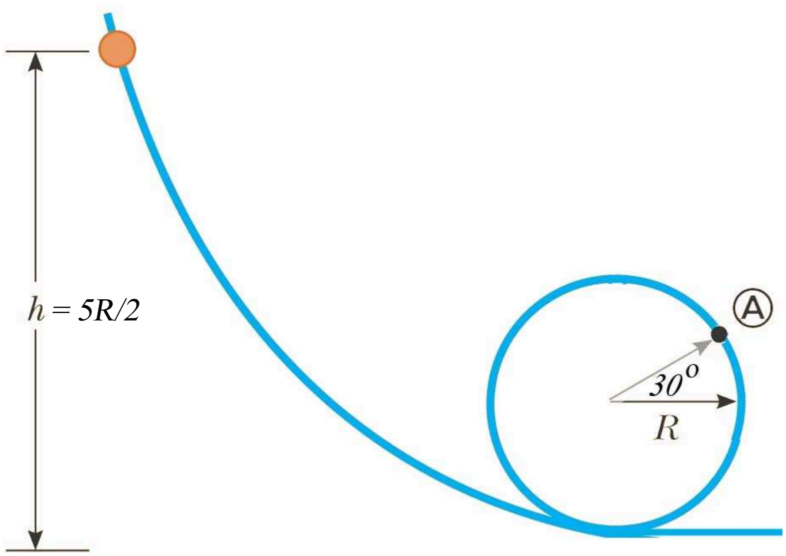

Figure 1: Problem 1

2 20pts total

(A) +10 pts

- +2 pts $\overline{F}_{superman} = -\overline{F}_{Bullet}$
- +2 pts $v_{f,Bullet} = v_{o,Bullet} = 400\ m/s$ "elastically"
- +3 pts $\overline{F}_{superman} = \frac{m_{bullet}\ 400 m/s}{\Delta t} (\hat{i} - \hat{j})$
- +1 pt $\Delta t = 1/200\ s$

+1 pt $\overline{F}_{superman} = 800N (\hat{i} - \hat{j})$

+1 pt $a_{x,superman} = 11.42\ m/s^2$ ("x-direction")

(B) +10 pts

- 6 pts Newton's 2nd Law

-+2 pts for each component → Force's acting on Superman (y direction)
(2pts N ; 2pts Mg ; 2pts $F_{impulse}$)

- +2 pts $F_{impulse} = 800N$
- +2 pts correct answer $N = 1487\ N$

2. (**20 pts**) After a long day of once again saving the planet earth from the evil Lex Luthor, Superman decides to relax on a frozen pond which also serves to cool him down. He slumps down on the frozen pond such that his back makes a 45° angle with the vertical as shown in the figure below. Lex Luthor is aware of the tired Superman's plan to relax, and wants to disturb Superman's quest for peace. Lex Luthor fires bullets horizontally at Superman with his machine gun at a rate of 200 bullets/second, and notices that the bullets bounce elastically off Superman's back and travel vertically. To answer the following questions, assume the mass of Superman is 70 kg, the mass of a bullet is 10 g, the speed of a bullet exiting the machine gun is 400 m/s, and the bullets travel horizontally before they hit Superman's back.

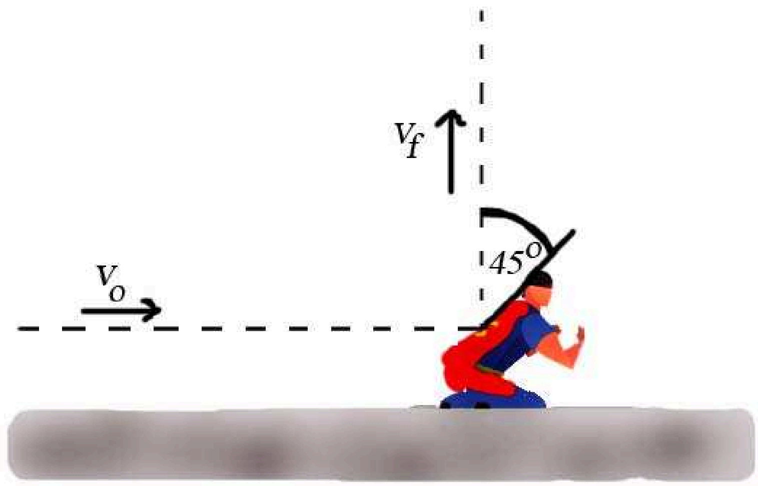

Figure 2: Problem 2

SOLUTION

A). What is Superman's initial average acceleration as the bullet's begin to bounce off his back?

Since Superman is resting (or trying to) on a frozen pond we can neglect friction. The initial average acceleration Superman acquires is a direct consequence of the bullets bouncing off his back. Let's calculate the average force exerted on a bullet that is deflected off Superman's back.

$$\overline{\vec{\mathbf{F}}_{bullet}} = \frac{\Delta p}{\Delta t} \tag{14}$$

$$\overline{\vec{\mathbf{F}}_{bullet}} = \frac{m_{bullet}\,(v_f\,\hat{\jmath} - v_0\,\hat{\imath})}{\Delta t} \tag{15}$$

$$v_f = v_0 = 400m/s. \tag{16}$$

$$\Delta t = \frac{1}{200}s \quad \text{(One bullet)} \tag{17}$$

$$\overline{\vec{\mathbf{F}}_{bullet}} = \frac{m_{bullet}\,(v_f\,\hat{\jmath} - v_0\,\hat{\imath})}{\Delta t} \tag{18}$$

$$\overline{\vec{\mathbf{F}}_{bullet}} = 800N\ (\hat{\jmath} - \hat{\imath})\,. \tag{19}$$

$$\overline{\vec{\mathbf{F}}_{bullet}} = -\overline{\vec{\mathbf{F}}_{Superman}} \tag{20}$$

$$\overline{\vec{\mathbf{F}}_{Superman}} = 800N\ (\hat{\imath} - \hat{\jmath})\,. \tag{21}$$

$$\Rightarrow \boxed{a_{x,Superman} = \frac{800N}{70kg} = 11.42m/s^2.} \tag{22}$$

B). What is the initial average normal force exerted by the ground on Superman?

Since there is a vertical component of the impulsive force acting down on Superman, the Normal force will be larger than Superman's Weight.

$$N - M\,g - F_{Impulse,Y} = 0. \tag{23}$$

$$N = M\,g + F_{Impulse,Y} = M\,g + 800N. \tag{24}$$

$$\boxed{N = 1487N.} \tag{25}$$

3) 20 pts – Total

A + B + 15 pts

+ 2 pts $\Delta p_x = 0$

+ 2 pts $\Delta p_y = 0$

+ 3 pts $\Delta p_y = 0 \Rightarrow v_2 = \frac{\sqrt{5}}{2} \frac{\sin\theta_1}{\sin\theta_2} v_0$ "Applied correctly"

+ 3 pts $\Delta p_x = 0 \Rightarrow 3v_0 = \sqrt{5}\, v_0 \cos\theta_1 + 2 v_2 \cos\theta_2$ "Applied correctly"

+ 2 pts $\theta_2 = 45°$

+ 3 pts $v_2 = \sqrt{2}\, v_0$ or $1.414\, v_0$

c) + 5 pts

+ 1 pts KE Before Collision

+ 1 pt KE After Collision

+ 2 pts $KE_{Before} = KE_{After}$

+ 1 pt "elastic"

3. (**20 pts**) Two unequal masses of mass m and $2\,m$ collide on a horizontal, friction-less table as shown in the figure below. Mass $2\,m$ is initially at rest and the initial speed of mass m is $3\,v_0$. After the collision, the masses follow the paths indicated in the figure below. Mass m has the speed and direction indicated.

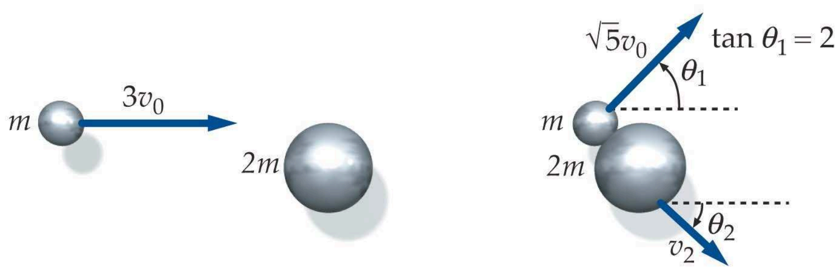

Figure 3: Problem 3

A). Find the speed v_2 of the $2\,m$ mass after the collision.

Considering the system of two masses, total momentum is conserved *since NO* external forces (to the two masses) act on the system.

$$\vec{\mathbf{p}}_i = 3\,m\,v_0\,\hat{\imath} \tag{26}$$

$$\vec{\mathbf{p}}_f = \left(\sqrt{5}\,v_0\,m\,\cos\theta_1 + 2\,m\,v_2\,\cos\theta_2\right)\,\hat{\imath} + \left(\sqrt{5}\,m\,v_0\,\sin\theta_1 - 2\,m\,v_2\,\sin\theta_2\right)\,\hat{\jmath} \tag{27}$$

$$\Delta p_x = 0 \quad \text{AND} \quad \Delta p_y = 0. \tag{28}$$

$$\boxed{v_2 = \frac{\sqrt{5}}{2}\,v_0\,\frac{\sin\theta_1}{\sin\theta_2}} \tag{29}$$

$$3 = \sqrt{5}\,\cos\theta_1 + \frac{\sqrt{5}\sin\theta_1}{\tan\theta_2} \tag{30}$$

$$\tan\theta_2 = \frac{\sqrt{5}\,\sin\theta_1}{\left(3 - \sqrt{5}\,\cos\theta_1\right)} = 1 \tag{31}$$

$$\Rightarrow \boxed{\theta_2 = 45^\circ} \tag{32}$$

Using Eq. 32 in Eq. 29 yields

$$\boxed{v_2 = \sqrt{2}\,v_0} \tag{33}$$

B). Find the angle θ_2 after the collision.

From the previous work and Eq. 32

$$\boxed{\theta_2 = 45^\circ} \tag{34}$$

C). Is the collision elastic or inelastic? (*Support your answer with a calculation.*)

To determine whether the collision is elastic or inelastic, we need to look at ΔK.

$$\Delta K = K_{\text{after}} - K_{\text{before}} \tag{35}$$

$$K_{\text{after}} = \frac{1}{2}\,m\,(\sqrt{5}\,v_0)^2 + \frac{1}{2}\,(2\,m)\,(\sqrt{2}\,v_0)^2 \tag{36}$$

$$K_{\text{after}} = 9\left(\frac{1}{2}\,m\,v_0^2\right) \tag{37}$$

$$K_{\text{before}} = \frac{1}{2}\,m\,(3\,v_0)^2 = 9\left(\frac{1}{2}\,m\,v_0^2\right) \tag{38}$$

$$\Delta K = 0 \quad \text{(Collision was elastic.)} \tag{39}$$

Last Name: First Name:

Physics 101 Fall 2007: Test 2 —Multiple-Choice Answers

	A	B	C	D	E
1				X	
2					X
3	X				
4					X
5	X				
6			X		
7				X	
8			X		
9			X		
10				X	