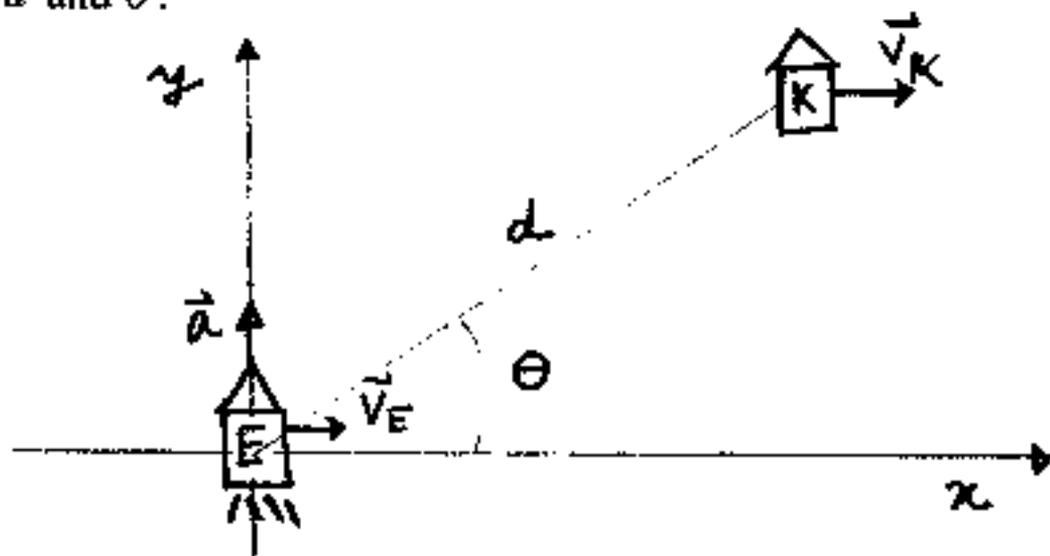


- (1) Print your LAST and FIRST names on the front of your blue book, on this question sheet and the multiple-choice question sheet.
- (2) The test consists of two free-response questions plus ten multiple-choice questions.
- (3) The test is graded on a scale of 100 points; each free-response question accounts for 35 points and each multiple-choice question accounts for 3 points.
- (4) Answer the two free-response questions in the blue book; answer the 10 multiple-choice questions by circling the single most nearly correct answer on the multiple-choice-question sheet directly.
- (5) Consult no books or notes of any kind. You may use a hand calculator in non-programmed and non-graphing mode.
- (6) Do NOT take any of the test materials out of the room at any time. Return questions sheets with your blue book and the multiple-choice-question answer sheet.
- (7) Write and sign the pledge on the front of your blue book.

LAST NAME: _____ FIRST NAME: _____
 (PLEASE PRINT)

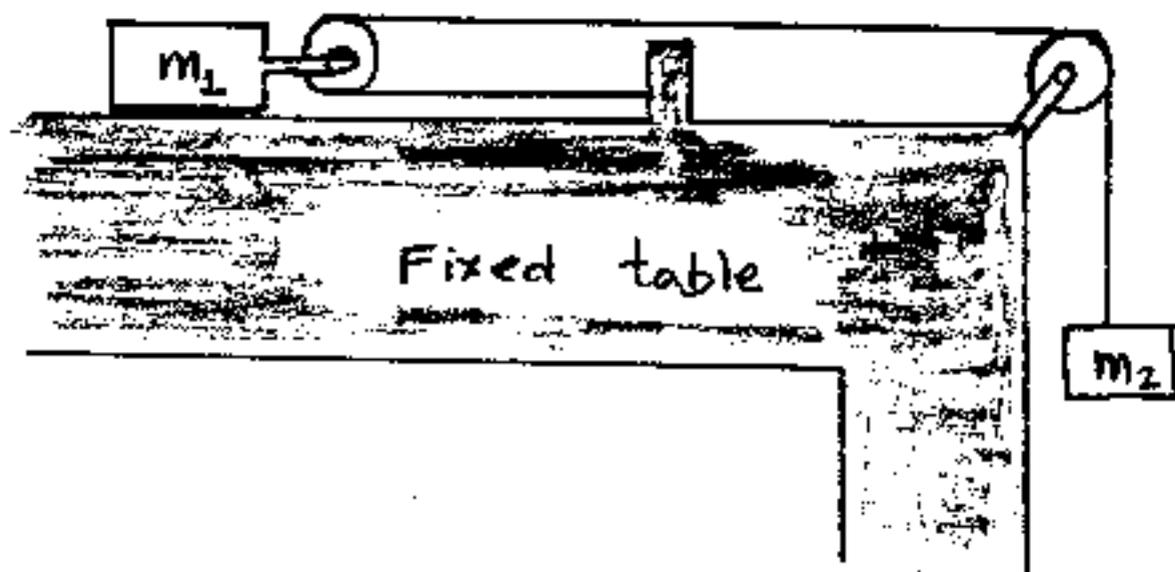
Show your work, including neat and clearly labeled figures on the free-response questions. Even correct answers without explanation may be denied credit.

- 1). Out in deep space (far from any gravitational forces), the starship Enterprise (E) is travelling at a constant velocity \vec{v}_E . The crew spots a Klingon starship (K) drifting at a constant velocity \vec{v}_K parallel to \vec{v}_E (call this the x direction) with $|\vec{v}_K| < |\vec{v}_E|$. Failing to get any response from the Klingon ship, they decide to intercept it. The Enterprise's engines are fired giving it ~~requires~~ a constant acceleration along the y direction. At the time $t = 0$ when the engines are fired the Klingon starship is at a distance d from the Enterprise at an angle θ to the x axis as shown. Express all your answers below in terms of some or all of the quantities \vec{v}_E , \vec{v}_K , d and θ .



- a). Determine the time t_1 at which the interception occurs.
- b). Determine the acceleration \vec{a} of the Enterprise that results in a successful interception.

- c). Determine the position vector \vec{r}_i of the point at which the interception occurs. Assume that the Enterprise is at the origin of your coordinate system at $t = 0$.
- d). Determine the velocity vector \vec{v}_i of the Enterprise at the time of interception t_i .
- e). Sketch a position-time plot showing x versus time for both the Enterprise and the Klingon starship. Repeat this for y versus time.

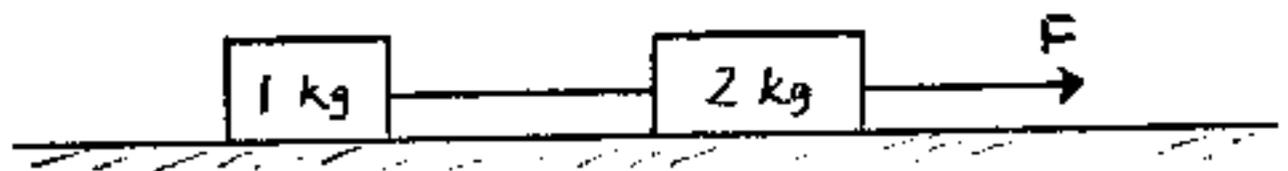


2). A mass m_1 with a pulley attached as shown slides along a frictionless surface. It is connected by a string to a mass m_2 via the arrangement shown in the figure. Let g be the value of the acceleration due to gravity close to the earth's surface and let both pulleys and the string have negligible mass. Express all your answers below in terms of some or all of the quantities g , m_1 and m_2 .

- a). Determine the acceleration of m_1 .
- b). Determine the acceleration of m_2 .
- c). Determine the tension T in the string.

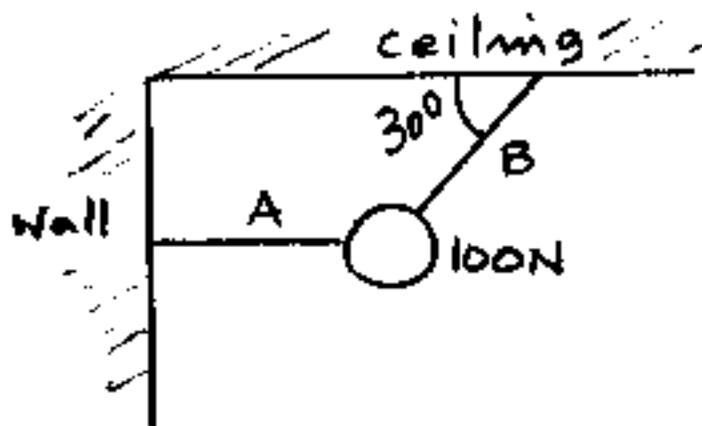
5. A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is fired at an angle 45° from the horizontal, what maximum height can be reached by the projectile?

- a) $\frac{h}{4}$ b) $\frac{h}{2\sqrt{2}}$ c) $\frac{h}{2}$
 d) $\frac{h}{\sqrt{2}}$ e) h



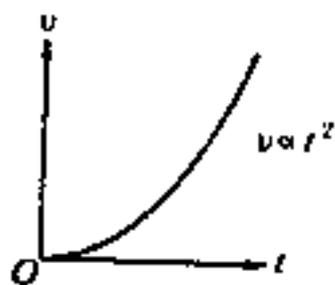
6. When the system of two masses shown above is accelerated along a flat frictionless surface by an applied force of magnitude F , the tension in the string between the blocks is

- a) $2F$ b) F c) $2F/3$
 d) $F/2$ e) $F/3$

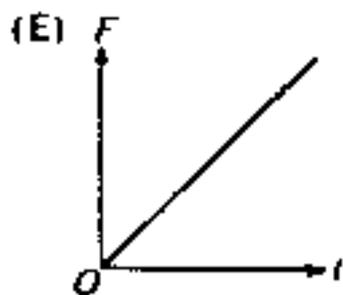
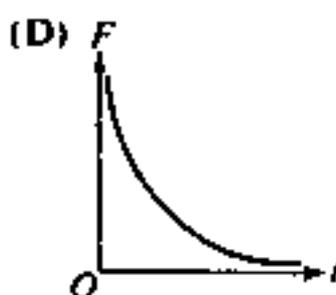
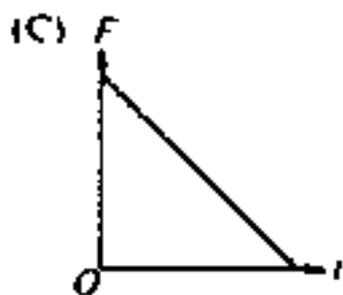
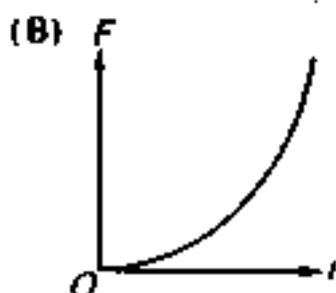
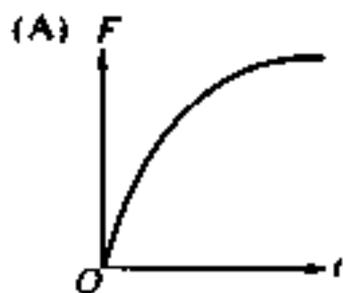


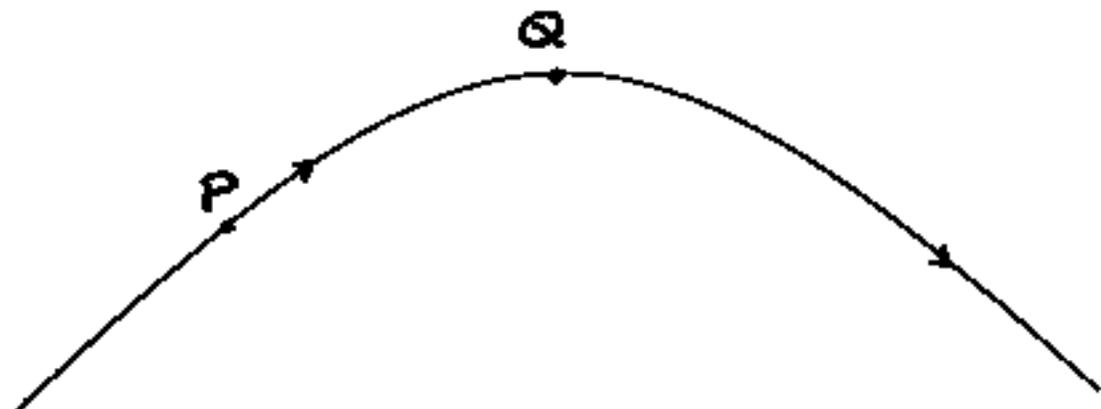
7. A 100-Newton weight is suspended in a corner of a room by two cords A and B as shown in the figure above. The tension in the slanted cord B is

- a) 50 N b) 100 N c) 115 N
 d) 200 N e) 866 N



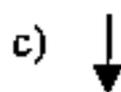
8. Consider the parabola above, showing that the speed v is a quadratic function of the time t for an object. Which of the graphs shown best represents the magnitude F of the net (resultant) force acting on that object as a function of the time t ?





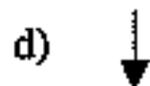
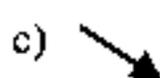
[For Question 9 and 10]. A ball is thrown and follows a parabolic path as shown above. Air friction is negligible. Point Q is the highest point on the path.

9. Which of the following best indicates the direction of the acceleration, if any, of the ball at point Q?



e) There is no acceleration of the ball at point Q.

10. Which of the following best describes the direction of the net force on the ball at the point P?



2) $x_E(t) = v_E t$ (1) $y_E(t) = \frac{1}{2} a t^2$ (3)

$x_K(t) = v_K t + d \cos \theta$ (2) $y_K(t) = d \sin \theta$ (4)

Interception $\Rightarrow x_E(t_1) = x_K(t_1)$ (5) and $y_K(t_1) = y_E(t_1)$ (6)
(at t_1)

(1), (2), (5) $\Rightarrow v_E \cdot t_1 = v_K t_1 + d \cos \theta$

$\Rightarrow t_1 = \frac{d \cos \theta}{v_E - v_K}$ (a) \checkmark

3, 4, 6, a

$y_E(t_1) = y_K(t_1)$

Now substitute t_1 from above
and for $y_E(t)$, $y_K(t)$ from (3) & (4)

$\Rightarrow \frac{1}{2} a t_1^2 = d \sin \theta$

$\Rightarrow a = \frac{2 d \sin \theta}{t_1^2} = \frac{2 d \sin \theta \cdot (v_E - v_K)^2}{d^2 \cos^2 \theta}$

$a = \frac{2 \sin \theta (v_E - v_K)^2}{d \cos^2 \theta}$ (b) \checkmark

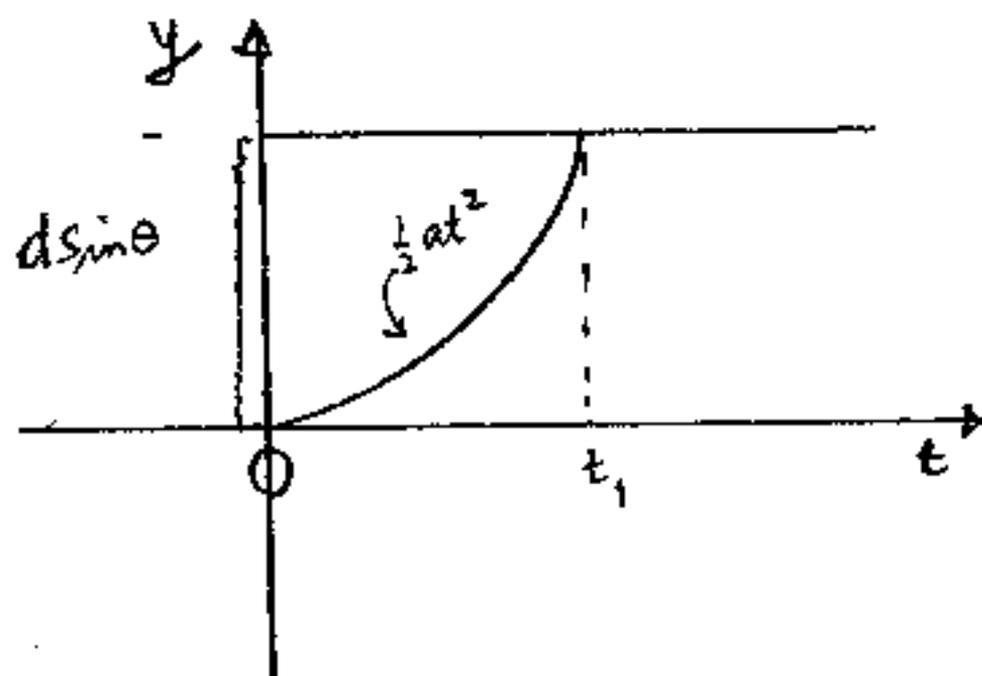
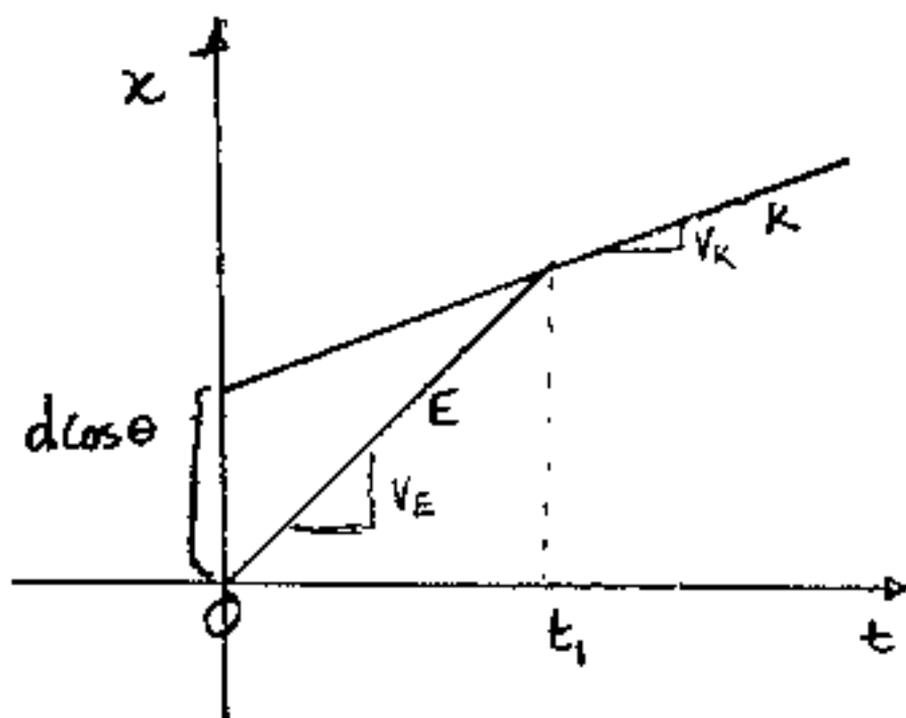
(c) $\vec{r}_i(t_1) = v_E t_1 \hat{i} + d \sin \theta \hat{j}$

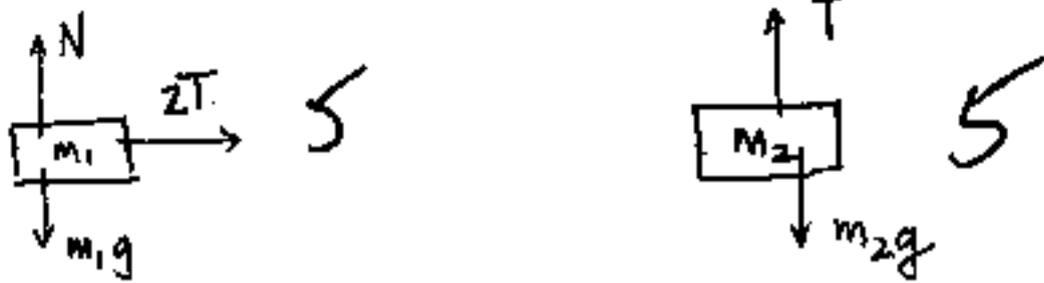
$= \frac{v_E d \cos \theta}{v_E - v_K} \hat{i} + d \sin \theta \hat{j}$

(d) $\vec{v}_i(t_1) = v_E \hat{i} + a t_1 \hat{j}$

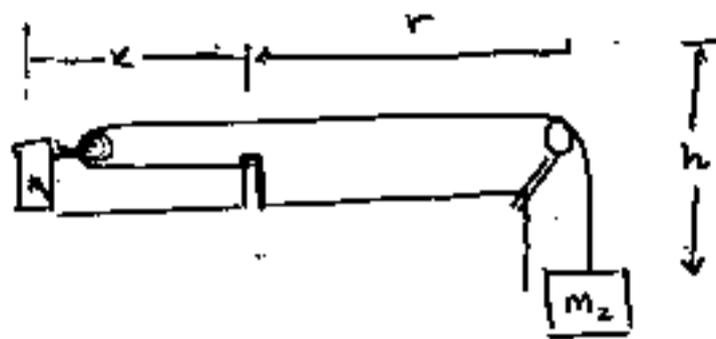
$= v_E \hat{i} + \frac{2 \sin \theta (v_E - v_K)^2}{d \cos^2 \theta} \cdot \frac{d \cos \theta}{v_E - v_K} \hat{j}$

$$\vec{V}_1(t_1) = V_E \hat{i} + 2 \tan \theta (V_E - V_K) \hat{j}$$





Constraint?



Conservation of string. Let string have length L .

$$\Rightarrow 2x + r + h = L$$

$$\Rightarrow 2 \frac{dx}{dt} + 0 + \frac{dh}{dt} = 0 \quad \text{since } \frac{dL}{dt} = 0, \frac{dr}{dt} = 0$$

$$\text{or } 2 \frac{d^2x}{dt^2} + 0 + \frac{d^2h}{dt^2} = 0$$

$$\text{or } \boxed{2|a_1| = |a_2|} \quad \checkmark$$

Newton's laws: $2T = m_1 a_1$

$$m_2 g - T = m_2 a_2$$

$$a_1 = \frac{a_2}{2}$$

$$\text{So } 2T = m_1 \frac{a_2}{2} \rightarrow \textcircled{1} \quad \checkmark$$

$$: (m_2 g - T = m_2 a_2) \rightarrow \textcircled{2} \quad \checkmark$$

$$\text{Add } \textcircled{1} + 2\textcircled{2} \Rightarrow 2m_2 g = \left(\frac{m_1}{2} + 2m_2\right) a_2$$

$$\Rightarrow a_2 = g \left(\frac{4m_2}{m_1 + 4m_2} \right) \quad \checkmark$$

$$a_1 = a_2 = \frac{g}{2} \left(\frac{4m_2}{m_1 + 4m_2} \right) = \frac{2gm_2}{m_1 + 4m_2} \quad 3$$

$$2T = m_1 a_2 = \frac{gm_1 m_2}{m_1 + 4m_2} \quad 3$$

Grading criteria # 1

4 kinematic equations	→ 10 pts	(3 each for y equations, 2 each for x eqn)
2 equalities	5 pts	(3 for y, 2 for x)
2 scalar solutions	5 pts	(2 for t_1 , 3 for a)
2 vector solutions	5 pts	(2 for \vec{r}_1 , 3 for \vec{v}_1)
2 sketches/graphs	10 pts	(5 for x: 2st lines, slopes, intercept, 5 for y: const + curve, intercept, t_1 1 2 pts)
	<hr/>	
	35	

Grading criteria # 2

5 pts	for FBD 1	(y forces optional)
5 pts	for FBD 2	
5 pts	for NII for 1	
5 pts	for NII for 2	
5 pts	for constraint	
4 pts	for a_2	
3 pts	for a_1	
3 pts	for T	

1 c

~~2~~

3 d

4 c

5 c

6 e

7 d (look for θ)

8 e

9 c

10 d