15 I. A rectangular loop of conducting wire of length $b$, width $a$, and total resistance $R$ is pulled through a region of uniform magnetic field as shown. The magnetic field has magnitude $B$ and points into the page. The right end of the loop is within the region of uniform field, but the left end is out of the field. The loop is pulled to the right at a constant velocity $v_0$.

3 (a) Determine the current induced in the loop in terms of quantities given. Indicate both direction and magnitude.
4 (b) Determine the constant force which must be exerted by an external agent to keep the loop moving at constant velocity.
4 (c) Determine the rate at which the external agent does work on the loop.
4 (d) Show that the rate at which the external agent does work is equal to the power dissipated by the resistance of the loop.

II. The figure below shows a uniform magnetic field $\vec{B}$ confined to a cylindrical volume of radius $R$, with $\vec{B}$ pointing into the page. The magnitude of $\vec{B}$ is decreasing at the constant rate $\alpha$; that is the rate of change of $B$ is negative: $\frac{dB}{dt} = -\alpha$. Determine the acceleration of an electron immediately after being released from rest at the points $a$, $b$, and $c$, in terms of $\alpha$, $R$, $r_1$, $r_2$, the mass of the electron $m_e$, and the charge of the electron, $-e$. Point $a$ is on the axis of the cylinder; point $b$ is a distance $r_1 < R$ from the axis; point $c$ is a distance $r_2 > R$ from the axis. Be sure to give both magnitude and direction.

5 points for each location
(a) First determine the emf $\varepsilon = - \frac{d\phi}{dt}$

$$\phi_B = \oint \vec{E} \cdot d\vec{A} = B a x$$

$$\frac{d\phi}{dt} = B a \frac{dx}{dt} = B a v_0 = 1 E_1$$

$$I = \frac{\varepsilon}{R} = \frac{B a v_0}{R} = I$$

Direction is counter-clockwise

(b)

Each segment of the loop will experience a force

$$F = I L \times \vec{B} \quad \text{top and bottom cancel,}$$

$$\vec{F}_3 \text{ (force on right segment) is to the left}$$

$$\vec{F}_3 = - I L \hat{B} \times \vec{A} = - \frac{B a v_0}{R} \hat{A}$$
For the loop to continue to move at constant velocity $\nu_0$, an external agent must exert a constant force to the right.

$$F_{\text{ext}} = \frac{B^2a^2\nu_0}{R} \hat{i} \quad \text{(to the right)}$$

(c) $dW = \overrightarrow{F} \cdot d\overrightarrow{L}$

Power $= \frac{du}{dt} = \overrightarrow{F} \cdot \frac{d\overrightarrow{L}}{dt} = \overrightarrow{F} \cdot \overrightarrow{v}$

$$\frac{du}{dt} = \frac{B^2a^2\nu_0^2}{R}$$

(d) Power dissipated in resistor is $P = I^2R$

$P = I^2R = \frac{B^2a^2\nu_0^2}{R} \cdot \frac{1}{R}$

$$P = \frac{B^2a^2\nu_0^2}{R} = \frac{du}{dt} \text{ above!}$$

So the work done by an external agent to keep the loop moving at constant $\nu_0$ appears as Joule heating ($I^2R$) in the wire!
II.

First we need to determine the electric field at the points a, b, c.

From Faraday's Law

\[ \oint E \cdot dr = -\frac{d\phi_b}{dt} = 2\pi n E. \]

For \( r < R \), \( \phi_b = B\pi r^2 \)

\[ \frac{d\phi_b}{dt} = \frac{dB}{dt} \pi r^2 = -2\pi n^2 \]

The direction of the field, by Lenz's Law, will oppose the change. Since \( |B| \) is decreasing, \( E \) will form clockwise loops, as shown.

\[ 2\pi n |E| = \pi r^2 \quad (r < R) \]

\[ |E| = \frac{xR}{2}, \text{ direction as indicated.} \]

An electron will experience a force due to \( E \), remember the electron has (-) charge, \( \vec{E} = q\vec{E} \)

Point a: \( \vec{E} = 0 \) since \( n = 0 \); \( \vec{E} = 0 \)
Point b, \( E_1 = \frac{\hbar v}{2} \), direction is downward.

\[
\vec{E}(a+b) = \frac{\lambda n e}{r^2} \hat{r} = m \frac{\vec{E}}{r^2} \text{ (upward)}
\]

\[
\vec{a} = \frac{\lambda n e}{2 m_e} \hat{r}
\]

Point b

For point c, we need to find \( E \) for \( r > R \):

\[
\phi_B = \pi R^2 B \quad \text{since \( B \) ends at \( R \)}
\]

\[
\frac{d\phi_B}{dt} = \pi R^2 \omega = \oint E \cdot \hat{n} = 2\pi mE_1
\]

\[
E_1 = \frac{\pi R^2 \omega}{2\pi n^2}
\]

direction at c is upward.

\[
\vec{E} = q \vec{E'} = -\frac{e\lambda R^2}{2\pi n^2} \hat{r} = m \vec{a}
\]

\[
\vec{a} = -\frac{e\lambda R^2}{2\pi m_e n^2} \hat{r}
\]

force on electron will be downward.

\[
\vec{F} = q \vec{E} = -\frac{e\lambda R^2}{2\pi n^2} \hat{r} = m \vec{a}
\]