I. A very long wire, carrying a current $I$, enters a region of space from the left. It is bent upward to form a circle, then downward to continue along the original line to the right. The circular part has radius $R$ with the center $C$ a distance $a$ from the straight section.

(a) Determine the magnetic field $\vec{B}$ at $C$ due to the circular part of the wire.
(b) Determine the magnetic field at $C$ due to the straight, horizontal sections of the wire.
(c) Determine the magnetic field at $C$ due to the straight, vertical sections of the wire.

II. Three long, parallel wires are located at the corners of an equilateral triangle, as shown below. Each side of the triangle has length $a$, and each wire carries current $I$. For the two wires at the top of the triangle, the current is out of the page; for the wire at the bottom, the current is into the page.

(a) Determine the force per unit length experienced by the bottom wire due to the presence of the two wires at the top of the triangle.
(b) Determine the magnetic field $\vec{B}$ at the point $P$ shown in the figure. Taking the position of the lower wire to be the origin, $P = (a/2, 0)$
(c) Determine the line integrals $\int \vec{B} \cdot d\ell$ for the three contours shown in the sketch on the right, $C_1, C_2,$ and $C_3$. The direction of the contours is indicated by the arrows.
(a) $\overrightarrow{B}$ due to the circular part:

$$d\overrightarrow{B} = \frac{\mu_0 I}{4\pi} \frac{\overrightarrow{dl} \times \hat{z}}{r^2}$$

(Biot-Savart Law)

Note that $\overrightarrow{dl} \parallel \hat{z}$, and that the direction $\overrightarrow{B}$ $\overrightarrow{dl} \times \hat{z}$ is the same for all points on the circle. $r^2 = R^2$

$$d\overrightarrow{B} = \frac{\mu_0 I}{4\pi} \frac{\overrightarrow{dl}}{R^2} \left[ -\hat{z} \right] \text{ (negative z direction)}$$

$$|\overrightarrow{B}| = \int d\overrightarrow{B} = \frac{\mu_0 I}{4\pi} \int 2\pi R = \frac{2\pi R}{2\pi} = \frac{\mu_0 I}{2R}$$

$$\overrightarrow{B} = \frac{\mu_0 I}{2\pi} \left[ -\hat{z} \right] \quad \text{into page}$$

(due to circle)
(b) $\vec{B}$ due to wire

\[ \vec{B} = \frac{\mu_0 I}{2\pi a} \hat{r} \]

The field due to a long wire forms concentric loops. At the center of the circle, $\vec{B}$ due to the wire will be out of the page ($+z$ direction).

From Ampere's law:

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} = \mu_0 I \]

\[ 2\pi R \vec{B} = \mu_0 I \]

\[ \vec{B} = \frac{\mu_0 I}{2\pi R} \]

with $n = a$

$\vec{B}$ at $C$ due to the wire is

Total $\vec{B}$ is then $\vec{B} = \mu_0 I \left[ \frac{1}{2\pi a} - \frac{1}{2R} \right] \hat{r}$

(c) Field due to vertical straight sections

Note that $d\vec{l}$ is $\hat{n} \cdot \hat{n} = 0$.

Also, since the currents are in the opposite direction, they would cancel.

$\vec{B} = 0$ due to the vertical segments.
(a) First find \( \vec{B} \) at the location of the bottom wire.

\[ |\vec{B}_2| = |\vec{B}_1| = \frac{\mu_0 I}{2\pi a} \quad \text{from Ampere Law} \]

\[ B_{2x} = B_{1x} = \frac{\mu_0 I}{2\pi a} \quad \cos 30^\circ \]

\[ \vec{B}_{\text{tot}} \text{ at bottom wire} = \frac{\mu_0 I}{2\pi a} \cdot \frac{\vec{B}}{2} \quad \hat{\imath} \]

\[ \vec{B}_{\text{tot}} = \frac{\sqrt{3} \mu_0 I}{2\pi a} \quad \hat{\jmath} \]

The \( y \)-components of \( \vec{B}_1, \vec{B}_2 \) cancel, \( x \)-components add.

4 meters length on the bottom wire:

\[ \vec{F} = I \vec{L} \times \vec{B} = I L \vec{B} \text{ downward} \]

\[ \frac{\vec{F}}{L} = \frac{\sqrt{3} \mu_0 I}{2\pi a} \quad (-\hat{\jmath}) \]
There will be three contributions to the field, \( \vec{B}_1, \vec{B}_2, \) and \( \vec{B}_3 \).

Find each and add.

\[
\vec{B}_1 = \frac{\mu_0 I}{2\pi a} \frac{\hat{\lambda}}{\sqrt{3}}
\]

\[
\vec{B}_3 = \frac{\mu_0 I}{2\pi (a r)} (-\hat{\delta})
\]

\[
\vec{B}_3 = -\frac{\mu_0 I}{\pi a} \hat{\delta}
\]

\[
|B_3| = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{\sqrt{7} \pi a}
\]

\[
\vec{B}_2 = \frac{\mu_0 I}{\sqrt{7} \pi a} \cos \theta \hat{i} + \frac{\mu_0 I}{\sqrt{7} \pi a} \sin \theta \hat{\delta}
\]

\[
\vec{B}_{\text{tot}} = \frac{\mu_0 I}{\pi a} \left( \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{7} \right) \hat{i} + \frac{\mu_0 I}{\pi a} \left( -1 + \frac{2}{7} \right) \hat{\delta}
\]

\[
\vec{B}_{\text{tot}} = \frac{\mu_0 I}{\pi a} \left[ \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{7} \right] \hat{i} - \frac{5 \mu_0 I}{7 \pi a} \hat{\delta}
\]

\[
\theta = \frac{3a^2}{4} + \frac{3a^2}{4} = \frac{7a^2}{4}
\]

\[
r = \frac{\sqrt{7}a}{2}
\]

\[
\cos \theta = \frac{\sqrt{3} a \hat{\delta}}{2 \sqrt{7} a} = \frac{\sqrt{3}}{7}
\]

\[
\sin \theta = \frac{\hat{i}}{\sqrt{7} a} = \frac{2}{\sqrt{7}}
\]
\[ \oint \mathbf{B} \cdot d\mathbf{A} = \mu_0 I \text{ and } \mu_0 (I - I) = 0 \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \text{ on cl.} \]

Note that the direction of the contour is such that \( \oint \mathbf{B} \cdot d\mathbf{A} = -\mu_0 I \text{ for the current out of the page, hence the (-) sign.} \]