1. The carts-pulley assembly shown below contains a pulley attached to a cart of mass $m_3$ which slides along a friction-less, horizontal table. A light string connects mass $m_1$ to mass $m_2$ via the pulley. There is friction between $m_1$ and $m_3$ (the coefficient of static friction between the two surfaces is $\mu_s$), but no friction between $m_2$ and $m_3$. If $m_3$ is held at rest with respect to the earth, $m_2$ is sufficient to cause $m_1$ to begin sliding across $m_3$. A horizontal force $F$ is now applied to $m_3$. For what range of values of $F$ will $m_1$, $m_2$ and $m_3$ not move relative to each other.

Since we are asked to find the range of values, we need to consider two cases which will define the upper and lower bounds of force $F$. First draw a Free-Body-Diagram for the three bodies. Don’t forget the action/reaction pairs.

From the Free-Body-Diagrams, write down Newton’s 2nd law for each mass.
CASE I (Max Force):

\[
\sum_{x,m_1} F_{Max} : T + f_s = m_1 a_1 \tag{1}
\]

\[
\sum_{y,m_1} F_{Max} : N_1 - m_1 g = 0 \tag{2}
\]

\[
\sum_{x,m_2} F_{Max} : N_2 = m_2 a_2 \tag{3}
\]

\[
\sum_{y,m_2} F_{Max} : T - m_2 g = 0 \tag{4}
\]

\[
\sum_{x,m_3} F_{Max} : F - f_s - N_2 = m_3 a_3 \tag{5}
\]

\[
\sum_{y,m_3} F_{Max} : N_G - N_1 - m_3 g = 0 \tag{6}
\]

Since we are finding the maximum force needed to keep all masses from moving relative to each other, the horizontal accelerations of the individual masses must be the same (called \( a \) from this point forward). Using \( f_s = \mu_s N_1 \) and Eq. 3, we can rewrite Eq. 5 as:

\[
F = \mu_s m_1 g + (m_2 + m_3) a \tag{7}
\]

Knowledge of \( a \) is still needed since we need it to solve for \( F \). Using Eq. 1 and Eq. 4 we obtain

\[
a = \frac{(m_2 + \mu_s m_1) g}{m_1} \tag{8}
\]

\[
F_{Max} = \left[ \frac{(m_2 + m_3)(m_2 + \mu_s m_1)}{m_1} + \mu_s m_1 \right] g \tag{9}
\]

CASE II (Minimum Force - frictional force reverses and points opposite to Tension on \( m_1 \))
Since we are finding the minimum force needed to keep all masses from moving relative to each other, the horizontal accelerations of the individual masses must be the same (called \( a \) from this point forward). Using \( f_s = \mu_s N_1 \) and Eq. 12, we can rewrite Eq. 14 as:

\[
F = -\mu_s m_1 g + (m_2 + m_3) a \tag{16}
\]

Knowledge of \( a \) is still needed since we need it to solve for \( F \). Using Eq. 10 and Eq. 13 we obtain

\[
a = \frac{(m_2 - \mu_s m_1) g}{m_1} \tag{17}
\]

\[
F_{\text{Min}} = \left[ \frac{(m_2 + m_1)(m_2 - \mu_s m_1)}{m_1} - \mu_s m_1 \right] g \tag{18}
\]

The range of forces which will cause all three masses to not move relative to each other is given by the following relationship:

\[
F_{\text{Min}} < F < F_{\text{Max}} \tag{19}
\]
2. A 1-kg physics book is connected by a string to a 0.5-kg coffee cup as illustrated in the figure below. The book is given a push up the slope and released with a speed of 3-m/s. The coefficient of static friction is \( \mu_s = 0.5 \), and the coefficient of kinetic friction is \( \mu_k = 0.2 \).

(a) How far does the book slide up the inclined table? The speed at the book’s peak is zero.

\[ v_f^2 = v_0^2 - 2|a| \Delta x \] (20)

\[ \Delta x = \frac{v_0^2}{2|a|} \] (21)

To solve for \( \Delta x \), one needs to find the deceleration (\(|a|\)). Draw the Free-Body-Diagrams for the book and the cup.
Apply Newton’s 2nd law.

\[ T - m_C g = -m_C |a| \]  \hspace{1cm} (22)

\[ n_B - m_B g \cos \theta - 0 \]  \hspace{1cm} (23)

\[ -m_B g \sin \theta - \mu_k n_B - T = -m_B |a| \]  \hspace{1cm} (24)

\[ \Rightarrow |a| = \frac{m_B g (\sin \theta + \mu_k \cos \theta) + m_C g}{(m_B + m_C)} \]  \hspace{1cm} (25)

\[ \Rightarrow |a| = 6.73 m/s^2 \]  \hspace{1cm} (26)

\[ \Delta x = 0.668 m \]  \hspace{1cm} (27)

(a) At the highest point on the table, does the book stick to the table, or does it slide back down? (Support your answer with a calculation.)

To test whether the book will stick to the table or slide down, one needs to compare the maximum amount of static friction (which will point up the incline) to the forces (tension and gravity) pointing down the incline.

\[ f(s, max) = \mu_s n_B \]  \hspace{1cm} (28)

\[ n_B = m_B g \cos \theta \]  \hspace{1cm} (29)

\[ f(s, max) = 4.61 N. \]  \hspace{1cm} (30)

\[ T = m_C g = 4.91 N \]  \hspace{1cm} (31)

\[ m_B g \sin \theta = 3.34 N \]  \hspace{1cm} (32)

Since the forces pulling the book down the incline sum to a total that exceeds the maximum force provided by static friction, **the book will slide back down the incline.**