Math 102 Solutions to Practice #2 for Exam 2

Spring 2008

1. Find the Taylor polynomial of degree 4 for \( f(x) = \cos x \) at \( a = \frac{\pi}{4} \).

\[
\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left( x - \frac{\pi}{4} \right) - \frac{\sqrt{2}}{2!} \left( x - \frac{\pi}{4} \right)^2 + \frac{\sqrt{2}}{3!} \left( x - \frac{\pi}{4} \right)^3 + \frac{\sqrt{2}}{4!} \left( x - \frac{\pi}{4} \right)^4.
\]

2. Determine whether the infinite series \( \sum_{n=0}^{\infty} \frac{5^{2-n}}{2^n} \) converges or diverges. If it converges, find its sum.

It converges to \( \frac{250}{9} \).

3. Determine whether the following infinite series converge or diverge.

(a) \( \sum_{n=1}^{\infty} \frac{n}{10n + 17} \).

Diverges (nth term test)

(b) \( \sum_{n=1}^{\infty} \frac{\ln n}{n} \).

Diverges (integral test)

(c) \( \sum_{n=1}^{\infty} \frac{4 + 3 \sin 2n}{n} \).

Converges (try comparison test)
4. Use the Taylor series for $\cos x$ to find a power series representation for $x^2 \cos(2x)$.

$$x^2 - \frac{4x^4}{2!} + \frac{16x^6}{4!} - \frac{64x^8}{6!} + \ldots$$

5. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n + 4}$. Does it converge? Does it converge absolutely?

It converges (AST), but it does not converge absolutely (try comparison on $\sum \frac{1}{3n+4}$).

6. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(3n)!} x^n$.

$(-\infty, \infty)$