Math 102 Practice #1 Solutions for Exam 2

Spring 2008

1. Find the third degree Taylor polynomial for \( f(x) = \tan x \) at \( a = 0 \).

\[
x + \frac{1}{3}x^3
\]

2. Determine whether the infinite series \( \sum_{n=2}^{\infty} \frac{5 \cdot 3^n}{4n+1} \) converges or diverges. If it converges, find its sum.

It converges to \( \frac{45}{16} \).

3. Determine whether the following infinite series converge or diverge.

(a) \( \sum_{n=1}^{\infty} \frac{5n + 19}{n^2 + 7n + 10} \).

Diverges (try comparison test)

(b) \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \).

Diverges (try integral test)

(c) \( \sum_{n=1}^{\infty} \frac{\cos^2 \left( \frac{1}{n} \right)}{\sqrt{n^3 + 1}} \).

Converges (try comparison test)
4. Use the Taylor series for $\sin x$ to find a power series representation for $\frac{\sin(x^2)}{x}$.

$$x - \frac{x^5}{3!} + \frac{x^9}{5!} - \frac{x^{13}}{7!} + \ldots$$

5. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4 + 2}$. Does it converge? Does it converge absolutely?

Converges absolutely (try AST and comparison)

6. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-3)^n(x - 1)^n}{\sqrt[n]{n + 3}}$.

$$\left[\frac{2}{3}, \frac{4}{3}\right]$$