4.5 Externalities and Public Goods

Ref: MWG Chapter 11.

FFWT any competitive equil. is Pareto optimal

SFWT (given suitable convexity assumptions) any Pareto optimal allocation can be supported as a competitive equil.

Tends to suggest possibilities for welfare-enhancing intervention in marketplace can be strictly limited to carrying out wealth transfers for purposes of achieving distributional aims.

Market Failures

1) externalities → production side
   - chemical plant’s discharges
   - reducing fishery’s catch

2) public goods → non-rivalry in consumption
   - national defence
   - flood control
   - non-excludable
   - lighthouse
**DEFN.** An *externality* is present whenever the well-being of a consumer or the production possibilities of a firm are *directly* affected by the actions of another agent in the economy.

c.f. “*Pecuniary*” externality.

√ Fishery’s productivity affected by emissions from oil refinery.

× Fishery’s profitability affected by price of oil.

Latter is *mediated* by *prices* through mkts and outcome in competitive mkts is Pareto optimal/efficient.

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**Example of Bilateral Externality**

Consumer 1 chooses consumption bundle $x^1$ and some action $h \in \mathbb{R}_+$

**UMP**s $v_1 (p, w^1, h) = \max \{x^1 \geq 0, h \geq 0\} \ u^1 (x^1, h) \ \text{s.t.} \ p.x^1 \leq w^1.

$v_2 (p, w^2, h) = \max \{x^2 \geq 0\} \ u^2 (x^2, h) \ \text{s.t.} \ p.x^2 \leq w^2.$

Simplify exposition by supposing preferences are additively separable between consumption and $h$,

$v_1 (p, w^1, h) = \phi_1 (h) \ (+ \ \hat{v}_1 (p, w^1))$

$v_2 (p, w^2, h) = \phi_2 (h) \ (+ \ \hat{v}_2 (p, w^2))$

*Efficient outcome:* $\max_{h \geq 0} \phi_1 (h) + \phi_2 (h)$

FONC: $\phi_1' (h^0) \leq -\phi_2' (h^0)$ (with equality if $h^0 > 0$)

*Equilibrium outcome:* Consumer 1: $\max_{h \geq 0} \phi_1 (h)$

FONC: $\phi_1' (h^*) \leq 0$ (with equality if $h^*> 0$)
Pigouvian Taxation

For negative externality set tax \( t_h = -\phi'_2 (h^0) \)

Consumer 1's problem becomes: \( \max_{h \geq 0} \phi_1 (h) - t_h h \)

FONC: \( \phi'_1 (\hat{h}) \leq t_h \) (with equality if \( \hat{h} > 0 \))

Notice \( \hat{h} = h^0 \), optimality-restoring tax is exactly equal to marginal externality at the optimal solution.

Positive Externality

Govt sets subsidy \( s_h = \phi'_2 (h^0) > 0 \)

Consumer 1's problem becomes: \( \max_{h \geq 0} \phi_1 (h) + s_h h \)

FONC: \( \phi'_1 (\hat{h}) + s_h \leq 0 \) (with equality if \( \hat{h} > 0 \))

Coasean Critique

- Coase argued:
  1. If there are no transactions costs of bargaining, then the Pigouvian solution is wrong.
  2. If there are transactions costs of bargaining, then the Pigouvian solution is incomplete.

- Bargaining between two parties \( \rightarrow \) Pareto efficient outcome (irrespective of who has property rights).

  a) Assign right to "externality-free" environment to consumer 2.

Consumer 2 can make "take-it-or-leave-it" offer, \((h, T)\) to consumer 1.

Consumer 2's problem \( \max_{h \geq 0, T} \phi_2 (h) + T \) s.t. \( \phi_1 (h) - T \geq \phi_1 (0) \)

\[ \Rightarrow T = \phi_1 (h) - \phi_1 (0) \]
So 2’s pblm may be expressed
\[
\max_{h \geq 0} \phi_2(h) + \phi_1(h) - \phi_1(0)
\]

FONC: \( \phi'_1(h^0) \leq -\phi'_2(h^0) \) (with equality if \( h^0 > 0 \))

I.e. socially optimal outcome.

b) Assign right “to pollute” to consumer 1.

Consumer 1 can make “take-it-or-leave-it” offer, \((h, T)\), to consumer 2.

Consumer 1’s pblm
\[
\max_{h \geq 0, T} \phi_1(h) + T \quad \text{s.t.} \quad \phi_2(h) - T \geq \phi_2(h^*)
\]
\[\Rightarrow T = \phi_2(h) - \phi_2(h^*)\]

So 1’s pblm may be expressed
\[
\max_{h \geq 0} \phi_1(h) + \phi_2(h) - \phi_2(h^*)
\]

And once again,
\[
\text{FONC: } \phi'_1(h^0) \leq -\phi'_2(h^0) \quad \text{(with equality if } h^0 > 0)\]

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Pigouvian Tax when 2 assigned right to "externality-free" environment.

\[
\max_{h \geq 0, T} \phi_2(h) + T
\]
\[\text{s.t. } \phi_1(h) - t_h h - T \geq \phi_1(0)\]
\[\text{i.e. } T = \phi_1(h) - \phi_1(0) - t_h h\]

So 2’ pblm may be expressed
\[
\max_{h \geq 0} \phi_2(h) + \phi_1(h) - t_h h - \phi_1(0)
\]

FONC: \( \phi'_1(\hat{h}) + \phi'_2(\hat{h}) - t_h \leq 0 \) (with equality if \( h^0 > 0 \))
\[\Rightarrow \phi'_1(\hat{h}) = -\phi'_2(\hat{h}) - \phi'_2(h^0)\]
Public Goods

**DEFN:** A *public good* is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.  
[i.e. non-depletable, non-rivalrous in consumption].

**Distinction**

*Excludable* — e.g. patent

*Non-excludable* — e.g. national defense, flood control

**Conditions for Pareto Optimality**

Quasi-linear preferences:-

\[
\max_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q) \rightarrow \text{FONC} \sum_{i=1}^{I} \phi'_i(q^0) \leq c'(q^0) \quad (\Rightarrow \text{ if } q^0 > 0)
\]

More generally, \((x_1^1, \ldots, x_L^1; \ldots; x_1^I, \ldots, x_L^I; q)\) satisfies

\[
\sum_{i=1}^{I} \frac{\partial u^i(x^i, q^0)}{\partial q} \frac{\partial x^i}{\partial q} = \text{MRT}_{q^0}.
\]

**Inefficiency of Private Provision of Public Goods**

\[
\max_{x_{i \geq 0}} \phi_i(x_i + \sum_{k \neq i} x_k^*) - p^*x_i
\]

\[
\text{FONC } \phi'_i(x_i^* + \sum_{k \neq i} x_k^*) \leq p^* \quad (\text{with equality if } x_i^* > 0)
\]

Letting \(x^* = \sum_{i=1}^{I} x_i^*\), we have

\[
\phi'_i(x^*) \leq p^* \quad (\text{with equality if } x_i^* > 0)
\]

Firm’s supply:- \(q^*\) solves

\[
\max_{q \geq 0} p^*q - c(q)
\]

\[
\text{FONC } p^* \leq c'(q^*) \quad \text{with equality if } q^* > 0
\]
In equilibrium $q^* = x^*$.

Letting $\delta_i = \begin{cases} 1 & \text{if } x_i^* > 0 \\ 0 & \text{if } x_i^* = 0 \end{cases}$

FONCs of consumers’ UMPs and Firm’s PMP imply

$$\sum_{i=1}^{I} \delta_i \left[ \phi'_i(q^*) - c'(q^*) \right] = 0.$$ 

Recalling $\phi'_i > 0$ and $c' > 0$, this implies that whenever $I > 1$ and $q^* > 0$ we have

$$\sum_{i=1}^{I} \phi'_i(q^*) > c'(q^*)$$

Solutions:

1. optimal direct provision:-- govt chooses to produce $q^0$.
2. subsidize private provision.

E.g. Suppose $\phi_i(q) = \ln q$, and $c(q) = q^2/2$.

Efficient level given by solution to:

$$\max_q I \ln q - q^2/2 \Rightarrow \frac{I}{q^0} - q^0 = 0 \Rightarrow q^0 = I^{1/2}$$

Market solution: $(x^*, p, q^*)$, where

1. Preference maximization: $x_i^* = x^*/I$ is solution to

$$\max_x \ln \left( x + \frac{[I - 1]}{I} x^* \right) - p^*x \quad \Rightarrow \quad \frac{1}{Ix} - p^* = 0$$

$$\Rightarrow \quad x^* = \frac{1}{p^*}$$

2. Profit maximization: $q^*$ solution to

$$\max_q p^*q - q^2/2 \Rightarrow p^* - q^* = 0 \Rightarrow q^* = p^*$$

3. Market-clearing: $x^* = q^* \Rightarrow p^* = 1 = x^* = q^*$
**Lindahl Equilibrium** – firm charges each consumer $p_i^{**}$

$$\max_{x_i \geq 0} \phi_i(x_i) - p_i^{**}x_i$$

FONC $x_i$ : $\phi_i'(x_i^{**}) \leq p_i^{**}$ with equality if $x_i^{**} > 0$.

Firm solves

$$\max_{q \geq 0} \left( \sum_{i=1}^{I} p_i^{**}q \right) - c(q)$$

FONC $q^{**}$ : $\sum_{i=1}^{I} p_i^{**} \leq c'(q^{**})$ with equality if $q^{**} > 0$

“Mkt-clearing” $x_i^{**} = q^{**}$ for all $i$.

$$\sum_{i=1}^{I} \phi_i'(q^{**}) \leq c'(q^{**}) \text{ with equality if } q^{**} > 0$$

i.e. $q^{**} = q^0$.

For example above with $\phi_i(q) = \ln q$, and $c(q) = q^2/2$, in Lindahl Equilibrium $p_i^{**} = I^{-1/2}$.

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**Multilateral Externalities**

*Depletable Externalities* – [experience of externality by one agent reduces the amount that will be felt by other agents]

$J$ firms generating externality ($\pi_j(.)$) and $I$ consumers ($\phi_i(.)$)

Assume $\pi_j' > 0$, $\pi_j'' < 0$, $\phi_i'(.) < 0$ & $\phi_i''(.) < 0$

Firms: $h_j : \pi_j'(h_j^*) \leq 0$ with equality if $h_j^* > 0$.

Pareto optimal allocation: $\left(\tilde{h}_1^0, \ldots, \tilde{h}_I^0, h_1^0, \ldots, h_J^0\right)$ that solves

$$\max_{(\tilde{h}_1, \ldots, \tilde{h}_I; h_1, \ldots, h_J) \geq 0} \sum_{i=1}^{I} \phi_i(\tilde{h}_i) + \sum_{j=1}^{J} \pi_j(h_j) \text{ s.t. } \sum_{i=1}^{I} \tilde{h}_i = \sum_{j=1}^{J} h_j$$
Letting $\mu$ be multiplier on this constraint, FONC

$$\tilde{h}_i : \phi_i' (\tilde{h}_i^0) \leq \mu \text{ with equality if } \tilde{h}_i^0 > 0, \ i = 1, \ldots, I$$

$$h_j : \mu \leq -\pi_j' (h_j^0) \text{ with equality if } h_j^0 > 0, \ j = 1, \ldots, J.$$

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Non-depletable Externalities: $\tilde{h}_i = \sum_{j=1}^{J} h_{j}$, for all $i = 1, \ldots, I$

$$\max_{(h_1, \ldots, h_J) \geq 0} \sum_{i=1}^{I} \phi_i \left( \sum_{j=1}^{J} h_{j} \right) + \sum_{j=1}^{J} \pi_j (h_j)$$

FONC $h_j : \sum_{i=1}^{I} \phi_i' \left( \sum_{j=1}^{J} h_{j} \right) \leq -\pi_j' (h_j^0) \text{ with equality if } h_j^0 > 0$

- mkt-based solution would require “personalized” prices as in Lindahl equil.
- But given sufficient info. (i.e. work out optimal agg. level of externality) Govt can achieve optimality using quota.

Suppose $h^0 = \sum_{j=1}^{J} h_j^0$ permits issued, firm $j$ receives $\tilde{h}_j$ & $\sum_{j=1}^{J} \tilde{h}_j = h^0$. 
Each firm’s demand for permits, $h_j$, solves

$$\max_{h_j \geq 0} \pi_j(h_j) + p_h^*(\bar{h}_j - h_j)$$

FONC $h_j: \pi'_j(h_j) \leq p_h^*$ with equality if $h_j > 0$.

Mkt-clearing: $\sum_{j=1}^J (\bar{h}_j - h_j) = 0$ & equil. price $p_h^* = -\sum_{i=1}^I \phi'_i(h^0)$

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Private Information. Suppose $\phi(h, \eta)$ where $\eta \in \mathbb{R}$, is consumer’s type, and $\pi(h, \theta)$ where $\theta \in \mathbb{R}$, is firm’s type.

Decentralized Bargaining – suppose only 2 possible levels of externality 0 or 1.

Consumer makes ‘take-it-or-leave-it’ offer. Set,

$$b(\theta) : = \pi(1, \theta) - \pi(0, \theta) > 0 \text{ measure of firm’s benefit}$$

$$c(\eta) : = \phi(0, \eta) - \phi(1, \eta) > 0 \text{ measure of consumer’s cost}$$

Denote by $G(b)$, CDF of $b$ (density $g(b)$) & $F(c)$, CDF of $c$ (density $f(c)$),

$$G(0) = F(0) = 0 \text{ and } G(\bar{b}) = F(\bar{c}) = 1.$$
Given consumer’s cost $c > 0$, she chooses value of $T$ to solve

$$\max_T [1 - G(T)] [T - c]$$

FONC \[1 - G(T)] - g(T)(T - c) = 0 \Rightarrow \frac{T - c}{T} = \frac{1 - G(T)}{g(T)T}

Solution has $T^*_c > c$.

No bargaining procedure can lead to efficient outcome for all values of $b$ and $c$ in this setting.

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Groves-Clark Mechanism

Revelation Mechanism

Firm announces $\hat{b}$ & receives $T_F$ from government, and consumer announces $\hat{c}$ and receives $T_C$ from government.

Where govt implements rule: allow pollution iff $\hat{b} > \hat{c}$ & transfers given by

$$T_F = \begin{cases} -\hat{c} & \text{if } \hat{b} > \hat{c} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad T_c = \begin{cases} \hat{b} & \text{if } \hat{b} > \hat{c} \\ 0 & \text{otherwise} \end{cases}$$

Weakly dominant for firm to announce $\hat{b} = b$ and for consumer to announce $\hat{c} = c$.

Optimal amount of pollution but government runs deficit whenever $\hat{b} > \hat{c}$.