### Spending and Income

- Suppose Cathy consumes two goods 1 and 2.
  - Quantity of good 1 consumed is $x_1$
  - Quantity of good 2 consumed is $x_2$
  - Let $p_1$ and $p_2$ denote the prices of good 1 and good 2, respectively
  - Let $m$ be Cathy’s (money) income

- The amount she spends is $p_1x_1 + p_2x_2$
- Ignoring the possibility of borrowing, she cannot spend more than her income
  - That is: $p_1x_1 + p_2x_2 \leq m$

### The Budget Line

- The line $p_1x_1 + p_2x_2 = m$ is often referred to as the budget line.
  - It shows the maximum possible amounts that can be spent on the two goods.

- The Feasible Set is the set of all affordable consumption bundles
  - That is all bundles $(x_1, x_2)$ such that $p_1x_1 + p_2x_2 \leq m$
  - And $x_1 \geq 0$ and $x_2 \geq 0$.

### Budget Constraint: Intercepts

**Budget constraint is**

\[ p_1x_1 + p_2x_2 = m \]

**Intercepts are** $m/p_1$ and $m/p_2$
Budget Constraint for Two Goods

\[ p_1 x_1 + p_2 x_2 = m \]

Budget Set, Constraint for Two Goods

Collection of all affordable bundles.

Budget Constraint for Three Goods

\[ p_1 x_1 + p_2 x_2 + p_3 x_3 = m \]
Slope of the Budget Constraint

Since \( p_1x_1 + p_2x_2 = m \)
Then \( x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1 \)
Or \( \frac{dx_2}{dx_1} = -\frac{p_1}{p_2} \)

- We can interpret this as the opportunity cost of a good
- If I want more of good \( x_2 \), I must give up \( \left( \frac{p_1}{p_2} \right) \) units of good \( x_1 \) to get it.

Changes in Budgets

- What happens if:
  - Income increases
  - Income decreases
  - Price of good 1 increases
  - Price of good 2 decreases
  - All prices and income increase by 10%

Composite Commodities

- We can only conveniently analyze two goods in a budget set diagram.
- In practice people consume a wide variety of goods.
- Often we are interested in describing how some change in price or income affects the amount of one good that can be purchased e.g. loaves of bread.
- To consider this case, it is often convenient to treat all goods other than the good that is of interest as a single composite commodity whose quantity is measured in dollars.

Composite Commodities (cont)

- Let \( x_1 \) represent loaves of bread
- Let \( x_2 \) represent dollars spent on everything else
- As before, we have \( p_1x_1 + p_2x_2 = m \)
- Dividing through by \( p_2 \) we get:
  - \( \left( \frac{p_1}{p_2} \right)x_1 + x_2 = \left( \frac{m}{p_2} \right) \)
- One conclusion of this is that we can define prices based on any numeraire we want so long as we are consistent about it
- Also note that measuring the amount of other goods in dollars is valid only if the relative prices of these other goods is not changing
Non-Linear Budget Constraints

- Not all budget constraints are linear
- People may be prohibited from buying all of a good they can afford
- Prices may (and often do) vary depending on quantity purchased
- Example: the Food Stamp Program

The Food Stamp Program

- Popular income support program
- Coupons given to poor (used to be sold)
- Can be legally exchanged only for food
- Popular with some donors
- Popular with agricultural interests

The Food Stamp Program

- What is effect on budget constraint?
  - Suppose $m = $100
  - Price of food: $p_F = $1
  - Price of “all other goods”: $p_G = $1
  - Other goods is "numeraire" good
  - Budget constraint is $F + G = 100$
  - Key factor: Income available for “other goods” does not change with receipt of food stamps
  - Suppose receive food stamps for $40 worth of food
The Food Stamp Program

F + G = 100: before stamps.

Budget set after 40 food stamps issued.

Welfare up since budget set is enlarged

The Food Stamp Program

- If food stamp program is generous, families may be at “kink” of budget set
- What if food stamps can be traded on a black market for $0.50 each?

The Food Stamp Program

Black market trading expands budget set further

Budget constraint with black market trading

Quantity-Based Prices

- Price may be a function of quantity
  - Quantity discounts for large buyers
  - Penalties for buying “too much”
- Budget constraints “kinked” where $p$ changes
- Suppose quantity discount:
  - $p_2$ constant at $1$ and $m = 100$
  - $p_1 = $2 for $0 \leq x_1 \leq 20; \ -p_1 / p_2 = -2$
  - $p_1 = $1 for $x_1 > 20; \ -p_1 / p_2 = -1$
  - What does the budget set look like?
Budget Constraints w/ Quantity Discount

\[ m = 100 \]

20 units @ \( p = 2 \)
60 units @ \( p = 1 \)

Slope = -2

Slope = -1

Quantity Restrictions

• Suppose as follows:
  - \( p_1 = 2 \)
  - \( p_2 = 1 \)
  - \( m = 100 \)
  - But not allowed to buy more than $25 of good 1

• What does the budget set look like?

More General Choice Sets

• Other constraints on choices
  - Time constraints (labor choice)
  - Other resource constraints

• “Available” bundle must meet all relevant constraints simultaneously

• Budget set is intersection of each set formed by each separate constraint

Example

• Instead of budgeting Money, let’s budget time
  - Erika is choosing how much time to work per week
  - If she works, she earns a wage of $25 / hour
  - In addition, she has $250 in non-wage income

• What would this budget set look like?