Experiment 3  Lenses and Images

Who shall teach thee, unless it be thine own eyes?

Euripides (480?-406? BC)

OBJECTIVES

To examine the nature and location of images formed by lenses.

THEORY

Lenses are frequently found in both nature and technology, wherever it is useful to form an image. They are usually made from transparent material with an index of refraction that is different from the surrounding material and have more or less spherical surfaces. Here we will consider only thin lenses, for which the distance light travels within the lens is small compared to other distances in the problem. We will also limit ourselves to the paraxial approximation. This means that all the light rays propagate within a small angle of the optical axis, the line extending perpendicularly through the center of the lens. Not all optical systems can be described in detail within these limits, but they do simplify the problem for an initial study.

A. Basic image formation

Figure 3-1 defines the basic nomenclature used for lenses. The lens itself can have a positive focal length, in which case it brings parallel light rays together at a common point, or a negative focal length, which causes parallel light rays to appear to diverge from a common point. Either kind of lens can form real or virtual images from real or virtual objects.

A real object, shown in Fig. 3-1, emits or reflects light rays that travel toward the lens, defining the propagation direction for the lens or group of lenses. Rays from a virtual object also travel in the propagation direction, but away from the lens. Such an object would be located to the right of the lens in Fig. 3-1. Obviously it cannot be a material object, but it could be an image formed by a previous lens.

A real image, also shown in Fig. 3-1, occurs where rays from a particular point on the object converge again to a point. A screen at this location will show an illuminated image of the object. A virtual image is created when rays from a point on the object continue to diverge after passing

\[ h_o \]

\[ h_i \]

\[ s_o \]

\[ s_i \]

\[ \text{propagation direction} \]

\[ \text{optical axis} \]

Fig. 3-1 Object and image positions for a simple converging lens.
through the lens. Extrapolating the rays back to a common point locates the virtual image. Since the rays diverge, a screen will show only a blur, regardless of where it is placed. Observation of a virtual image requires a camera, eye or similar instrument that can analyze diverging rays and infer the source location.

The actual location of the image is related to the focal length and location of the object by

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \tag{3-1}$$

where $s_i$ is the distance of the image from the lens, $s_o$ the distance of the object and $f$ the focal length. The magnification, $M$, of the image is defined to be the ratio of image height to object height. By tracing a ray through the center of the lens, it is easy to show that the magnification is given by

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o} \tag{3-2}$$

which can also be expressed in terms of $f$ and $s_o$ using Eq. 3-1

$$M = \frac{1}{1-\left(s_o/f\right)} \tag{3-3}$$

We claim that these equations correctly predict image position and magnification within the thin lens, paraxial approximation, provided we use the following conventions for the signs of the various quantities.

- $f > 0$ for a converging lens, convex surfaces (thicker in the middle)
- $f < 0$ for a diverging lens, concave surfaces (thicker at the edges)
- distances $s_i$, $s_o$ for real images (objects)
- distances $s_i$, $s_o$ for virtual images (objects)
- $M < 0$ indicates an inverted image, relative to the object
- $M > 0$ indicates a non-inverted image

The results of Eqs. 3-1 and 3-3 are summarized in Fig. 3-2 for both converging and diverging lenses. The solid lines in the image position plot show, for example, that a converging lens produces a real image of a real object if the object is farther than $f$ from the lens. Depending on the object distance, the image will be found somewhere between the focal point and infinity. An object between the focal point and the lens produces a virtual image located between the lens and infinity.
Similar considerations apply to the diverging lens. The dashed line to the right of the y-axis shows that a diverging lens always produces a virtual image of a real object, and that the virtual image is always located between the lens and the focal point. The dashed line to the left of the y-axis asserts that a virtual object placed between the lens and the focal point would lead to a real image between the lens and infinity. We will see later that it is possible to use a converging lens to produce a virtual object and create that real image.

EXPERIMENTAL PROCEDURE

The experimental work consists of forming images with converging and diverging lenses, and determining the location and magnification of the images for comparison with Eqs. 3–1 and 3–3. By using a second lens, it is possible to extend the measurements into the virtual-object region.

A. Converging lens

Arrange the components on the optical bench as shown in Fig. 3-3, using the lens marked +15 cm. To maintain the paraxial approximation, adjust the lens holder so that the lens is at the same height as the center of the light-box target and the optical axis is parallel to the long axis of the

Fig. 3-3 Optical bench arrangement for finding the real image of a converging lens. Start with the arrow and cross-line target on the small light box.
optical bench. If you now position the lens so that the object distance is greater than the focal length, about 15 cm, you should be able to focus an image of the light box onto the screen.

Examine the image carefully. Is it inverted or non-inverted with respect to the object? Is it reversed left to right? Since these terms are somewhat ambiguous, be careful to explain what you mean. Next, locate the metal plate with a hole in it at your lab station. Center the hole over the lens, and examine the image again. What changes? Now use the edge of the plate to cover about half of the lens. How does the image change? Is a whole lens needed to form an image? Does the size or shape of the lens aperture affect the size or shape of the image?

Now remove the screen and look back through the lens toward the light box. You will probably see an image that is more or less blurry, depending on how far you are from the lens and how strongly your eye can focus. Now move the light box toward the lens until the separation is less than the focal length, 15 cm. This should allow you to see a sharp virtual image within your range of focus. Is it inverted or non-inverted? Reversed? Again, be clear in your description. Is the change from a real to a virtual image consistent with Fig. 3-2, assuming the focal length is about 15 cm?

To quantify the relationships, we will measure $s_o$, $s_i$, $h_o$, and $h_i$ to compare with the predictions of Eqs. 3-1 and 3-3. Start by putting the screen back in place and locating the real image for as large a range of object positions as you can conveniently measure. The small light box is useful for short distances, but the large box makes a bigger image from longer distances. Be sure to record both the image distance and image height for each object distance.

Locating the virtual image is trickier. The classical method is to look through the lens with one eye, look around the lens at a movable target with the other eye, and position the target so that both images are in focus. Most people find this difficult, so we will instead use the arrangement of Fig. 3-4. A glass plate positioned near the lens and at 45º to the optical axis acts as a beam combiner. Some of the light coming from the screen passes through the glass plate, so it is visible to an observer positioned as shown. Some of the light coming through the lens is reflected from the glass, so the image is also visible to the same observer.

Looking through the plate as indicated you will see the lens image superimposed on the screen. Adjust the distance $b$ so that the grid pattern on the screen is in focus at the same time as the image seen through the lens. The exact position can be found by parallax: When the screen and image are at the same distance from the observer’s eye, small vertical or horizontal motions of the eye will not cause the images to shift relative to each other. Measure the distance from the glass to the screen for several object distances, and obtain the image distance as explained in Fig. 3-4. You can measure the image height by putting a ruler against the screen and reading the height directly.
You could compare your data with Eq. 3-1 and 3-3 by making a plot like Fig. 3-2, but it is easier to rewrite the equations in a linear form as

\[
\frac{1}{s_i} = \frac{1}{s_o} + \frac{1}{f} \quad \frac{1}{M} = -\frac{1}{f} + 1
\] (3-4)

The first equation says that if you plot $1/s_i$ vs $1/s_o$ you should get a straight line of slope $-1$ and intercept $1/f$. Similarly, a plot of $1/M$ vs $s_o$ should give a straight line with slope $-1/f$ and intercept $+1$. Are both plots straight lines with the expected slopes and intercepts and consistent values of $f$? Be careful with signs, or plotted points will go far astray.

**B. Diverging Lens**

Install the -20 cm diverging lens in place of the converging lens, adjust the height of the lens holder as needed, and look through the lens at the small light box. Do you see a virtual non-inverted image for all positions of the real object, as suggested by Fig. 3-2?

Measure the image distance for a very large and a very small object distance, using the virtual-image technique described above. Is the virtual image located within 20 cm of the lens in both cases, as claimed? Use Eq. 3-1 to estimate $f$ from one of your measurements. Do you get approximately -20 cm?

Fig. 3-2 claims that a diverging lens will form a real image of a virtual object that falls between the lens and the focal point. To test this, you can create a virtual object using the converging lens. Set up the arrangement of Fig. 3-5 and adjust the positions of the small light box.
and the converging lens to form a real image at a convenient location along the optical bench. Clamp down the components, and note the location of this real image, which becomes the virtual object for the diverging lens. Now put a holder with the diverging lens onto the optical bench, positioned between the converging lens and the virtual object location. You should be able to locate a real image by properly positioning the screen. Knowing the position of the virtual object, you can again measure $s_o$ and $s_i$ relative to the diverging lens, this time for a negative $s_o$. Are your measurements consistent with Eq. 3-1 and $f = -20$ cm? (This may not work accurately because it is difficult to determine $s_o$ with sufficient precision.)

**REPORT**

Your quantitative results will be summarized in the plots of image distance and magnification. Be sure to respond to all the qualitative questions in the text.