COMP 210, FALL 2000
Lecture 3: Conditional Logic

Things to do
- Explain homework hand-in procedure…
- Reading sections 1-5

Review
Last class, we:
1. Discussed the *rewriting* approach to evaluating Scheme programs.
2. Saw that complex programs can be built on the shoulders of smaller programs. This led to the 1\textsuperscript{st} principle of COMP 210: *Always reuse an existing function if one exists.*
3. Introduced a design methodology for programs
   a) Write a contract, purpose, and header
   b) Develop several examples, chosen to elucidate the program’s behavior
   c) Write the program’s body
   d) Test the program to ensure that it behaves as specified

This class, we’ll apply the methodology and extend it with more detail.

In-class Exercise (3—5 minutes)
Recall our function Area

\begin{verbatim}
(define (Area Radius)
    (* pi (* Radius Radius)))
\end{verbatim}

In your New Media Design Class, you are looking at some of the tradeoffs that arise in designing a new physical format for reusable optical disks. A critical parameter is the useful area of the optical disk. We can model its area as a disk with a hole in the middle.

Design a program, named Area-of-disk-with-a-hole, that calculates the area of an optical disk. It should take two ARGUMENTs, the radius of the disk and the radius of the hole in its center. Recall the Area program that we wrote earlier.

Work with the person seated next to you. One of you will get to write your solution on the board.
Another Example

Let’s go back to paying for your pizza. Of course, even work-study jobs require you to pay taxes. Since the actual tax system is too complex for a second program, let's assume that Steve Forbes wins the Iowa caucuses tonight, then goes on to triumph in the November elections, and becomes President. Congress adopts his flat tax, at a 17% rate.

Can we write a program that calculates our tax burden? We already wrote one that calculates wages

; Wage: num -> num
; Purpose: calculate gross wage from # hours worked
;         assume pay rate is $6 per hour
(define (Wage Hours)
  (* 6 Hours))

The tax on wages of W is just (* W (/ 17 100)), or

; Taxes: num -> num
; Purpose: compute taxes on a gross wage
;         assume a 17% flat tax a la Steve Forbes
(define (Taxes W)
  (* W (/ 17 100)))

Finally, we might want to know our take home pay, so that we can figure out how much pizza to order.

; Take-home-pay: num -> num
; Purpose: compute take home pay, under Forbes’ flat tax, from hours worked
(define (Take-home-pay Hours-worked)
  (- (Wage Hours-worked) (Taxes (Wage Hours-worked)) ))

Together, these functions compute gross wages, net taxes, and net income. If we need to change the minimum wage, or the tax rate, each occurs in exactly one place.

Writing good programs requires good organization. Re-use existing programs, or break a program’s task into smaller ones.

The Real World

One of the hardest aspects of working for pizza is computing your net pay, as opposed to your gross pay. Recall that your gross pay could be computed as
; Wage: num -> num
; Purpose: compute gross pay from hours worked
; assume a $6 per hour pay rate
(define (Wage H)
  (* 6 H))

Assuming that your pay rate is $6 per hour, and that H is the number of
hours worked during a week.

Under the current tax code, wages up to $500 per week are taxed at 15%,
above $500 to $1,500 per week are taxed at 28%, and wages above $1,500
per week are taxed at 33%. (To keep this example simple, we assume that
the 36 and 39% brackets do not exist.)

How do we write our tax program to handle this more complex tax scenario?
In other words, if I give you the amount of weekly earnings, how do we
figure out the taxes? This is the program “Taxes” in the previous example.

If WW stands for the weekly wage, then we first determine which tax bracket
includes WW, then we multiply it by the proper tax rate.

In math, we know that we can test equality and various inequalities.

\[
\begin{align*}
3 < 10 & \Rightarrow \text{true} \\
10 \leq 3 & \Rightarrow \text{false} \\
3 = 4 & \Rightarrow \text{false}
\end{align*}
\]

In Scheme, we can write expressions that test equalities and inequalities and
return either true or false

\[
\begin{align*}
(< \ 3 \ 10) \\
(\leq \ 10 \ 3) \\
(= \ 3 \ 4)
\end{align*}
\]

We can combine trues and falses using and, or, and not. We write true
and false, in Scheme as #t and #f.

To use this insight in a program, we need to lay out the intervals on the
number line, and then construct an appropriate set of tests.

\[
\begin{array}{ccc}
0 & \rightarrow & 500 \rightarrow & \rightarrow & 1500 \rightarrow & \rightarrow & 1500
\end{array}
\]

\[
\begin{array}{ccc}
15\% & \rightarrow & 28\% & \rightarrow & 33\%
\end{array}
\]

If WW stands for our weekly wage, then the appropriate conditions are

\[
\begin{align*}
(\leq \ WW \ 500) \\
(\text{and} \ (< \ 500 \ WW) \ (\leq \ WW \ 1500))
\end{align*}
\]
To put this together in Scheme, we use a construct called the **cond** construct (for *conditional*).

```scheme
(define (Real-Tax WW)
  (cond
    ((<= WW 500) (* (/ 15 100) WW))
    ((and (< 500 WW) (<= WW 1500)) (* (/ 28 100) WW))
    ((< 1500 WW) (* (/ 33 100) WW))
  ))
```

Work an example, like `(Real-Tax 300)`.

```scheme
(Real-Tax 300)
= (cond
    ((<= 300 500) (* (/ 15 100) 300))
    ((and (< 500 300) (< 300 1500)) (* (/ 28 100) 300))
    ((< 1500 300) (* (/ 33 100) 300))
  )
= (cond
    (#t (* (/ 15 100) 300))
    (#f (* (/ 28 100) 300))
    (#f (* (/ 33 100) 300))
  )
= (* (/ 15 100) 300)
= 45
```

When a problem specifies different behaviors for different sets of numeric data, draw an interval picture, translate the interval into conditions, and use a **cond** expression to formulate the program.