3. Solution.

The equation of state function of van der Waals gas is

\[ P = \frac{RT}{V-b} - \frac{a}{V^2} \]

Multiplying through by \( V^2 (V-b) \) produces a cubic equation in \( V \):

\[ PV^3 - pV^2 b = RTV^2 - a(V-b) \]

So we should expect curves that have a maximum and minima, as cubic equations often do. However, the previous statement for some \( T \) (i.e., the critical temperature), the max. and min. coalesce. So we expect an isotherm with an inflection point.

Above these isotherms (\( T > T_c \)) the curve should be monotonic in \( V \). Also notice that at \( V = b \), there is a divergence in the equation of state, so we expect the slope of the \( p-V \) curve will be very steep at small \( V \).

0) At the critical point.

The \( p-V \) curve has an inflexion point where \( \left( \frac{\partial^2 P}{\partial V^2} \right)_T = 0 \) and \( \left( \frac{\partial^3 P}{\partial V^3} \right)_T = 0 \)

From \( P = \frac{RT}{V-b} - \frac{a}{V^2} \) van der Waals

\( \left( \frac{\partial^2 P}{\partial V^2} \right)_T = -\frac{RT}{(V-b)^3} + \frac{2a}{V^3} = 0 \Rightarrow RTcV_c^3 = 2a(V_c-b)^3 \)

\( \left( \frac{\partial^3 P}{\partial V^3} \right)_T = \frac{2RT}{(V-b)^4} - \frac{6a}{V^4} = 0 \Rightarrow 2RTcV_c^4 = 6a(V_c-b)^3 \)

\( \frac{0}{\Theta} \Rightarrow 2V_c = 3(V_c-b) \Rightarrow \boxed{\frac{V_c}{b} = 3} \)