(1) Free expansion - E is constant
(a) Express $dE$ as exact differential $E(T, V)$
\[
dE = \left(\frac{\partial E}{\partial T}\right)_V dT + \left(\frac{\partial E}{\partial V}\right)_T dV
\]
0 for constant energy process of free expansion
\[
\left(\frac{\partial E}{\partial V}\right)_T dV = -\left(\frac{\partial E}{\partial T}\right)_V dT
\]
=> \[
\left(\frac{\partial T}{\partial V}\right)_E = \frac{-\left(\frac{\partial E}{\partial V}\right)_T}{\left(\frac{\partial E}{\partial T}\right)_V}
\]

From HW #5, Problem #2:
\[
\left(\frac{\partial E}{\partial T}\right)_V = C_V
\]
\[
\left(\frac{\partial E}{\partial V}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_V - p
\]

(b) $dE = TdS - pdV$
0 for const. energy
\[
\left(\frac{\partial S}{\partial V}\right)_E = \frac{p}{T}
\]

(c) $dT = T_2 - T_1$ in free expansion from $V_1 \rightarrow V_2$
From (a),
\[
dT = \left(\frac{p - T \left(\frac{\partial p}{\partial T}\right)_V}{C_V}\right) dV
\]
\[
\Delta T = \frac{T_2 - T_1}{T_1} \int_{V_1}^{V_2} p - T \left(\frac{\partial p}{\partial T}\right)_V dV
\]