3. (a) The magnetic field of a wire with radius $R$ is
\[ B_{\text{inside}} = \frac{\mu_0 I}{2\pi} \left( \frac{r}{R^2} \right), \quad B_{\text{outside}} = \frac{\mu_0 I}{2\pi r}. \]
We see that the magnetic field inside increases as $r$ increases and is maximal at $r = R$. The magnetic field outside decreases as $r$ increases and is maximal at $r = R$. Thus the magnetic field is greatest at $r = R$.

(b) The maximum magnetic field is
\[ B_{\text{max}} = \frac{\mu_0 I}{2\pi R} \text{ at } r = R. \]

(c) The minimum magnetic field is
\[ B_{\text{min}} = 0 \text{ at } r = 0 \text{ and } r = \infty. \]

(d) \[ \frac{\mu_0 I}{2\pi R} \]

12. The sheet may be thought of as an infinite number of parallel wires. The figure shows a view looking directly at the current. If we consider a point above the sheet, the wire directly underneath produces a magnetic field parallel to the sheet. By considering a pair of wires symmetrically placed about the first one, we see that the net field will be parallel to the sheet. Below the sheet, the field will be in the opposite direction.

We apply Ampere's law to the rectangular path shown in the diagram. For the sides perpendicular to the sheet, $\vec{B}$ is perpendicular to $d\vec{s}$. For the sides parallel to the sheet, $\vec{B}$ is parallel to $d\vec{s}$ and constant in magnitude, because the upper and lower paths are equidistant from the sheet. We have
\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}; \]
\[ \int B \cdot ds + \int_{\text{lengths}} B \cdot ds = 0 \]
\[ \int_{\text{lengths}} B ds = B2L = \mu_0 hL. \]
This gives
\[ B = \frac{\mu_0 I}{2} \text{ parallel to sheet and perpendicular to current (opposite directions on the two sides)}. \]
14. The magnetic field will be the sum of the two fields from the sheets. Because the magnitudes are the same, we have
\[B = \mu_0 j \parallel \text{the sheets in the region between the sheets;}
\[B = 0 \text{ outside the sheets.}
\]
If we reverse \(h_2\) to make the currents parallel, the direction of \(B_2\) will reverse, and we will have
\[B = 0 \text{ in the region between the sheets.}
\[B = \mu_0 j \parallel \text{the sheets outside the sheets (opposite directions on the two sides).}
\]

17. From the cylindrical symmetry, we know that the magnetic field will be tangent to a circle centered on the axis of the cylinders and will depend only on the distance from the axis. We use Ampere’s law for a circular path of radius \(r\) midway between the inner and outer surfaces:
\[B \cdot d\vec{s} = \mu_0 I_{\text{enclosed}} ;
\[B 2\pi r = \mu_0 I, \text{ which gives}
\[B = \frac{\mu_0 I}{2\pi r}
\]
\[(4\pi \times 10^{-7} \text{T} \cdot \text{m/A})(10 \text{ A})/2\pi(0.3 \times 10^{-2} \text{ m}) = 6.7 \times 10^{-4} \text{T circular.}
\]

21. At a distance \(x\) from the wire, the magnetic field is directed into the paper with magnitude
\[B = \frac{\mu_0 I}{2\pi x}.
\]
Because the field is not constant over the square, we find the magnetic flux by integration. We choose a differential element parallel to the wire at position \(x\) with area \(a\) \(dx\):
\[\Phi_B = \int \int \vec{B} \cdot d\vec{A} = \int \int B \, dA = \int_{a}^{a+d} \frac{\mu_0 I}{2\pi x} \, dx = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{a + d}{d}\right).\]
39. The magnitude of the field from each segment of the wire will be one-half the field from an infinite wire:

\[ B = \frac{\mu_0 I}{4\pi H} \]

The directions are shown in the figure, so we have

\[ B = \sqrt{2}\left(\frac{\mu_0 I}{4\pi H}\right) \]

in the xy-plane 45° from the –x-axis and the –y-axis.

53. Because the point \( P \) is along the line of the two straight segments of the wire, there is no magnetic field from these segments. The magnetic field at the point \( P \) is the field of the semicircle:

\[ B = B_{\text{semicircle}} = \frac{1}{2} \left( \frac{\mu_0 I}{2R} \right) \]

\[ = \frac{1}{2} \left( 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}(8 \text{ A})/2(1.2 \times 10^{-2} \text{ m}) \right) \]

\[ = 2.1 \times 10^{-4} \text{T} \text{ into the page} \]

65. The current configuration is equivalent to a pair of infinitely long, parallel wires plus a circular current loop. The magnetic field at point \( P \) due to each infinitely long wire is

\[ B_1 = \frac{\mu_0 I}{[2\pi(D/2)]}, \text{ pointing into the paper;} \]

while that due to the circular current loop is

\[ B_2 = \frac{\mu_0 I}{[2(D/2)]}, \text{ also pointing into the paper.} \]

The total magnetic field at point \( P \) is then

\[ B = 2B_1 + B_2 \]

\[ = \frac{2\mu_0 I}{[2\pi(D/2)]} + \frac{\mu_0 I}{[2(D/2)]} \]

\[ = \left( \frac{\mu_0 I}{D} \right)(2/\pi + 1), \text{ pointing into the paper.} \]

69. Because the magnetic field produced by the long wire depends only on the distance from the wire \( y \), \( B = \frac{\mu_0 I}{2\pi y} \), we find the force on the rectangular loop. The net force is

\[ \vec{F} = \vec{F}_a + \vec{F}_b + \vec{F}_c + \vec{F}_d. \]

For segments \( c \) and \( d \) of the loop, the symmetry of the field and the opposite directions of the current \( I_2 \) give

\[ \vec{F}_c + \vec{F}_d = 0. \]

Because the wires and the magnetic field are perpendicular, we have

\[ \vec{F} = \vec{F}_a + \vec{F}_b = I_2 L(\mu_0 I_1/2\pi a)(-\hat{j}) + I_2 L(\mu_0 I_1/2\pi b)(+\hat{j}) \]

\[ = (\mu_0/4\pi)(2I_1 L)(1/b - 1/a)\hat{j} \]

\[ = (10^{-7} \text{T} \cdot \text{m/A})(10 \text{ A})(5 \text{ A})(0.20 \text{ m})[1/(0.05 \text{ m}) - 1/(0.02 \text{ m})]\hat{j} \]

\[ = -6.0 \times 10^{-5} \text{ N} \hat{j} \text{ (attraction)}. \]
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73. (a) We choose $x = 0$ at the left coil. The magnetic fields on the axis from the coils are in the same direction, so we find the magnitude of the total field from

$$B(x) = \frac{\mu_0 I}{2R} \left\{ \frac{R^2}{(R^2 + x^2)^{3/2}} + \frac{R^2}{[R^2 + (R - x)]^{3/2}} \right\}$$

$$= \frac{\mu_0 I}{2R} \left\{ \frac{1}{1 + (x/R)^3} + \frac{1}{2 - 2(x/R) + (x/R)^3} \right\}.$$

At $x = 0$, we have $B(0) = (\mu_0 I/2R)[1 + (1/2^{3/2})] = 0.677 \mu_0 I/R$.

At $x = R/4$, we have $B(R/4) = (\mu_0 I/2R)((1/1 + (1/4)^{3/2}) + (1/2 - 2(1/4) + (1/4)^2)) = 0.713 \mu_0 I/R$.

At $x = R/2$, we have $B(R/2) = (\mu_0 I/2R)((1/1 + (1/2)^{3/2}) + (1/2 - 2(1/2) + (1/2)^2)) = 0.716 \mu_0 I/R$.

(b) When we differentiate the expression for $B$, we get

$$\frac{dB}{dx} = \frac{\mu_0 I}{2R} \left\{ \frac{(-3/2)2(1/R)(x/R)}{1 + (x/R)^3} + \frac{(-3/2)2(-1/R) + (1/R)(x/R)}{2 - 2(x/R) + (x/R)^3} \right\}$$

$$= \frac{3\mu_0 I}{2R^2} \left\{ \frac{(x/R)}{1 + (x/R)^3} + \frac{(x/R) - 1}{2 - 2(x/R) + (x/R)^3} \right\};$$

$$\frac{d^2 B}{dx^2} = \frac{3\mu_0 I}{2R^3} \left\{ \frac{1/R}{1 + (x/R)^3} + \frac{(-5/2)2(1/R)(x/R)^2}{[2 - 2(x/R) + (x/R)^3]^{3/2}} \right\}$$

$$+ \frac{1/R}{[2 - 2(x/R) + (x/R)^3]^{3/2}} + \frac{[x/(x/R) - 1][-5/2]2[(-1/R) + (1/R)(x/R)]}{[2 - 2(x/R) + (x/R)^3]^{7/2}} \right\}$$

$$= \frac{3\mu_0 I}{2R^3} \left\{ \frac{1}{1 + (x/R)^3} + \frac{5(x/R)^2}{[1 + (x/R)^3]^{7/2}} + \frac{1}{2 - 2(x/R) + (x/R)^3} - \frac{5[(x/R) - 1]^2}{[2 - 2(x/R) + (x/R)^3]^{7/2}} \right\}.$$
\[
\frac{dB}{dx} = -\frac{3\mu_0 I}{2R} \left\{ \frac{1/2}{1 + (1/2)^{3/2}} + \frac{(1/2) - 1}{2 - 2(1/2) + (1/2)^{3/2}} \right\} = -\frac{3\mu_0 I}{2R} \left\{ \frac{1/2}{(5/4)^{3/2}} + \frac{-(1/2)}{(5/4)^{3/2}} \right\} = 0;
\]

\[
\frac{d^2 B}{dx^2} = \frac{3\mu_0 I}{2R^3} \left\{ \frac{1}{1 + (1/2)^{3/2}} - \frac{5(1/2)^2}{[1 + (1/2)^{3/2}]^{3/2} + \frac{1}{2 - 2(1/2) + (1/2)^{3/2}} - \frac{5(1/2 - 1)^2}{[2 - 2(1/2) + (1/2)^3]^{3/2}} \right\} \]

\[
= \frac{3\mu_0 I}{2R^3} \left[ \frac{1}{(5/4)^{3/2}} - \frac{5/4}{(5/4)^{3/2}} + \frac{1}{(5/4)^{3/2}} - \frac{5/4}{(5/4)^{3/2}} \right] = 0.
\]