Renewable Technology Adoption and the Macroeconomy*

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Abstract

We develop a dynamic general equilibrium integrated assessment model to study the role of technological progress on the optimal transition to a renewable energy fueled world economy. Renewable firms can invest in improving their productivity. This improvement is subject to spillovers and involves an opportunity cost resulting of scrapping existing capital in order to adopt a new vintage. In the presence of environmental externalities, the optimal allocation can be implemented through a Pigouvian tax on emissions in conjunction with a policy that promotes new technology adoption. We use world-economy data to calibrate our model and characterize the optimal energy transition. We find that when it comes to long-term growth and welfare, energy considerations are of first-order importance. Incentivizing penetration by renewables in the absence of a carbon tax can provide only small benefits, and may even be detrimental in the short run.

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1 Introduction

Global economic growth relies on vast amounts of energy. Since the industrial revolution, this need has been met largely by the use of fossil fuel. In recent decades, other energy sources, such as solar and wind, have been increasing their share in the energy supply of many nations. The transition toward renewable energy is expected to accelerate, as concerns about climate change become more prevalent, and as technological progress reduces the cost of green technologies relative to that of fossil fuel.

A common belief holds that markets currently under-invest in renewable energy. The reasons for under-investment can range from the externalities resulting from greenhouse gases (GHG), to spillovers associated with innovation. While some advocate abandoning fossil fuel and switching to renewable sources as soon as possible, the best policy from an optimal growth perspective needs to carefully evaluate the trade-offs involved. The shale revolution has created an abundance of oil and natural gas that has vastly increased the available resources of hydrocarbons, and coal resources around the world remain vast. Of course, fossil fuels contribute to climate change through the negative externality from GHG emissions. Does optimal growth imply that fossil fuel reserves will continue to be used along a long transition, or are the relative costs, including those from environmental damages, such that we should abandon their use long before they are exhausted? How does the optimal adoption depend on technological progress and on the scrapping costs associated with installing new vintage capital in the green sector? What combinations of policies best accommodate technological and environmental externalities?

In this paper, we investigate the optimal transition from a primarily fossil-fueled, to a primarily renewable energy-fueled world economy. This transition depends on several factors. First, although fossil fuel sources are plentiful, they constitute an exhaustible resource. A second consideration involves environmental concerns. As fossil fuel generates externalities through increasing the stock of GHG emissions, the need for a clean substitute becomes increasingly apparent. In addition, spillovers imply that investment in renewable technology creates a positive externality. One contribution of our paper lies in exploring the quantitative significance of an additional, not well-studied factor in the process of new renewable technology adoption. In the presence of fast technological progress and large capital costs, new technology adoption involves scrapping existing equipment. More precisely, when productivity is embedded in the capital stock, replacing existing capital in order to install more productive, newer vintage equipment implies certain additional costs.[1] In principle, there is an option value to delaying the adoption of a new vintage when rapid technological

[1] These include costs associated with decommissioning, scrapping, or recycling old equipment, adjustment costs resulting from the switch, legal and transaction costs associated with financing, selling, purchasing, and installing new equipment, etc. Mauritzen (2012) discusses scrapping patterns for less productive wind turbines in Denmark. To our knowledge, our paper is the first to study the implications of these costs for optimal renewable energy investment in the context of a dynamic general equilibrium integrated assessment model.

progress will necessitate the need for a yet newer vintage before too long. The greater the speed of improvements, the more it makes sense to delay adoption. Our modeling of this effect applies more generally, but we believe it is particularly relevant for energy investments, as they tend to be capital intensive and as renewable energy technologies are new and subject to relatively rapid technological progress as compared to fossil fuel.[2] We explore the significance of this argument in the context of a dynamic general equilibrium integrated assessment model (IAM).

As an example of the kind of effect we have in mind, consider the market for electric vehicles (EV). Batteries are a significant fraction of the cost (often close to 30-40%) of purchasing an EV. By any measure (including increased energy density, reduced cost, etc.) batteries have been steadily improving in recent years. Yet, EV purchases have remained relatively flat over the same time horizon. It is important to note that for virtually all EV, the battery technology at the time of purchase is "embedded" in the vehicle and cannot be upgraded, unless the vehicle is scraped and replaced with a new vintage. It is reasonable to expect that, concerned about characteristics such as overall range, consumers might prefer to wait until sufficient progress justifies an EV purchase. This would be in line with our reasoning: there are costs associated with early adoption.[3]

While Pigouvian taxes on carbon emissions have several theoretical advantages and are generally favored by economists, they have proved difficult to implement in practice. As a partial substitute, many advocate policies that promote penetration of renewables. A second focus of our paper concerns the degree of substitutability between carbon taxes and policies that internalize spillovers, thus, promoting renewable energy.

In our model, energy together with capital and labor, is an input in the production of final consumption goods. Energy can be produced from either fossil or renewable sources. Both require capital, which is also used in the production of the final good. At each point in time, productivity in the renewable energy production can increase as a result of replacing capital with a new vintage. The actual improvement is subject to a spillover, as it depends on the aggregate investment in the renewable sector. This spillover effect alone would lead to an under-investment in renewable energy. In addition, investment creates certain costs associated with scrapping and replacing older equipment. We model these costs as a temporary adverse productivity shock to the production function of renewable energy firms. Importantly, these costs are assumed to be proportional to the size of the capital replacement.

We use an IAM to characterize the optimal transition to a renewable energy-

[2]We do not model technological progress in fossil fuel explicitly.

[3]We use the EV market as an illustration of the effect we have in mind. As is common with IAM, our approach is aggregate and does not consider specific markets. For more on EV see, for example, https://www.eia.gov/todayinenergy/detail.php?id=36312#, https://www.researchgate.net/figure/Evolution-of-battery-energy-density-and-cost-Global-EV-outlook-2016_fig3_309313559, and https://www.statista.com/statistics/797638/battery-share-of-large-electric-vehicle-cost/. EV purchases are often subsidized, but we ignored this effect for this discussion.

fueled economy. The model is based on Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2014), who in turn build on Nordhaus's pioneering work in climate economics. We find that the efficient energy mix involves a gradual decline in the use of fossil fuel and demonstrate how to decentralize the optimum by a policy that accommodates both externalities. This involves a Pigouvian tax on GHG emissions together with a revenue neutral tax/subsidy on renewable technology adoption. We then calibrate the model using world economy data.

The extent to which the world economy will use its available fossil fuel reserves depends on the assumption about the total endowment of accessible hydrocarbons. For realistic parametrizations, the transition is made well before exhaustion of these resources. We characterize the optimal policy combination and find that, when it comes to long-term growth and welfare, energy considerations are of first-order importance. We then examine each of the two policies in isolation. We find that carbon taxes and policies that promote renewable energy by eliminating spillover externalities are best thought of as complements rather than substitutes. In the absence of a Pigouvian tax on carbon emissions, policies that incentivize penetration by renewables can provide relatively small benefits, and can even be detrimental to economic growth in the short run. Thus, when it comes to social welfare, such policies cannot provide a suitable alternative to a carbon tax in our model. While theoretically desirable, Pigouvian taxes are generally considered difficult to implement in practice. For example, a short-lived government might choose a lower Pigouvian tax (perhaps zero), as it is effectively more impatient than the representative agent. In the Appendix, we briefly discuss how the theoretical model can be extended to account for such a possibility.

1.1 Quantitative Findings

Our calibration strategy involves several steps. We will focus on four parameters: (1) the level of the spillover externality in renewable energy production, (2) the current "Pigouvian tax rate" (the ratio of the actual carbon tax relative to its optimal value), (3) the initial resources of fossil fuel, and (4) the productivity in the renewable energy technology. To calibrate these parameters, we will use observations from the world economy, together with theoretical relationships derived in the context of our model. The change in the renewable energy share decreases in the level of the spillover externality. To get a handle on the current Pigouvian tax rate, we use the current level of fossil fuel consumption and the implication that an increase in the Pigouvian rate would result in a decrease in fossil fuel use. Last, to calibrate the productivity in the renewable energy production, we use the current share of renewables together with the fact that productivity in the renewable energy sector is increasing in their share.

Our calibrated model allows us to study several interesting connections between scrapping costs and the two externalities, Pigouvian taxation, growth, and welfare. We investigate the relative quantitative importance and potential complementarity between the two policy instruments, by studying their effects in isolation and in tandem. Our findings suggest that scrapping costs are of quantitative importance when it comes to new technology adoption along the energy transition. They also point to complementarities between carbon taxes and internalizing spillovers, with sizable welfare gains only when both policies are present.

1.2 Related Literature

Our paper contributes to the growing literature that uses IAM to study energy transitions, innovation, and growth. In the economic growth literature, Parente (1994) studied a model in which firms choose to adopt new technologies as they gain specific expertise through learning-by-doing. He identified conditions under which equilibria exhibit constant per capita output growth. As in most of the literature on innovation and growth, Parente abstracted from issues related to energy and the environment, which are the focus of our study. Atkeson and Burstein (2015) study the impact of policy-induced changes in innovative investment and the implications for medium-and long-run innovation and growth. They too abstract from energy and environmental considerations.

Nordhaus (1994) pioneered the study of climate factors in dynamic economic modeling. Traditionally, most economic analysis of energy and environmental issues focuses on computable general equilibrium models which often abstract from endogenous technological progress.[4] Acemoglu et al (2012) study a growth model that takes into consideration the environmental effect from operating "dirty" technologies. They consider policies that tax innovation and production in the dirty sectors. They find that subsidizing research in the "clean" sectors can speed up environmentally friendly innovation without the corresponding slowdown in economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production. We view our paper as complementary to theirs. We do not model directed technical change and we instead introduce the scrapping channel associated with new vintage adoption. While we think that their main recommendations are likely to remain valid in the presence of scrapping, our quantitative findings suggest that the optimal rates of new technology adoption might be affected if we take such costs into account.

GHKT (2014) build a tractable dynamic general equilibrium model that incorporates the use of energy and the resulting environmental consequences. They derive a formula and numerical values for the optimal tax on carbon emissions. They, however, abstract from the costs associated with endogenous technological progress and new vintage adoption. We will employ several elements from their work in what follows, including the tractable modeling of the environmental externality. Van der Ploeg and Rezai (2016) extended the model in GHKT in several ways. They allow for general fossil fuel extraction costs, a negative impact of climate change on growth, mean reversion in climate damages, labor-augmenting and green technology progress, and a direct effect of the

[4] See, for example, Nordhaus and Boyer (2000) and references therein.

emissions stock on welfare. They characterize the social optimum, as well as the optimal carbon tax and the renewable energy subsidies.[5]

Acemoglu, Akcigit, Hanley, and Kerr (2016) use the structure in GHKT to study questions related to the transition to clean technologies. They employ a "ladder" model to study technological progress in both the clean and the dirty sectors, and they estimate their model using R&D and patent data. They assume that increased representation of fossil fuel encourages further use, and that fossil fuel use stops after 200 years, regardless of whether fossil fuel is exhausted at that time. They conclude that both Pigouvian taxation and renewable energy subsidies are needed in order to make the (optimal) transition sooner rather than later. The reason is that subsidies encourage technological progress without overtaxing short-run future output.

Gowrisankaran and Rysman (2012) explore a similar argument to ours in a different context. They study optimal decisions by consumers choosing the timing of purchase among an expanding set of available camcorders. As prices, quality, and variety improve over time, waiting is valuable in their model. In addition, while consumers usually hold only one camcorder at a time, they may substitute a new camcorder by scrapping an old one. They find that an important component for why the initial market share for digital camcorders was not higher was because forward-looking consumers were rationally expecting that cheaper and better players would appear in the future. A related effect is explored in Manuelli and Seshadri (2014). They study technology diffusion of tractors in American agriculture during the first part of the 20th century. They argue that part of the reason for the slow rate of adoption was that tractor quality kept improving over that period. As a result, farmers chose to postpone their purchase, rather than investing in a tractor that would soon become obsolete. The main contribution of our paper is to explore this channel in the context of renewable energy technology adoption.

The paper proceeds as follows. Section 2 introduces the model. Sections 3 and 4 discuss efficiency, equilibrium, and optimal policy. Section 5 presents our calibration and quantitative findings. A brief conclusion follows. Technical material is dedicated to the Appendix.

2 The Model

We build an IAM that incorporates a version of the neoclassical growth model, together with energy, technology, and environmental factors. Time is discrete and the horizon is infinite, $t = 0, 1, \ldots$. There is a single consumption good per period, and all markets are competitive. The economy is populated by a

[5] Other related papers include Stokey (1998), who considers growth under environmental constraints, Goulder and Schneider (1999), who study endogenous innovations in abatement technologies, Van der Zwaan et al. (2002), who study the impact of environmental policies in a model with learning-by-doing, and Popp (2004), who studies innovation in the energy sector and the costs of environmental regulation. See also Hartley et al. (2016), who study technological progress and the optimal energy transition, and Van der Ploeg and Withagen (2011), who study the possibility of a green paradox in this context.

representative infinite-lived household. The household discounts the future at rate $\beta \in (0,1)$ and values period-t consumption, c_t, through a utility function $u(c_t)$. We assume that u is smooth, strictly increasing, strictly concave, and that the usual Inada conditions hold. The labor endowment is normalized to 1, and labor is supplied inelastically. There are three different types of firms, all owned by the household: the final-good firm, and two types of intermediate-good firms that produce energy from fossil fuel and from renewable sources, respectively. In each period, the household chooses how much capital, k_t, to rent at rate r_t and receives profits resulting from the firms' activities. All capital depreciates at rate $\delta \in (0,1)$.

The representative final-good-producing firm produces output, y_t, using capital, k_t^g, labor, l_t, and energy, e_t. In addition, production can be affected by environmental quality, Γ_t. This, in turn, depends on the total stock of greenhouse gas (GHG) in the atmosphere. Ignoring environmental damages, the final-good production function is given by

$$y_t \leq F\left(k_t^g, l_t, e_t, \Gamma_t\right). \tag{1}$$

We assume that environmental quality, Γ_t, affects output through a damage function $D_t(\Gamma_t)$, and that damages are multiplicative. Thus, the final good production function becomes

$$F\left(k_t^g, l_t, e_t, \Gamma_t\right) = (1 - D_t(\Gamma_t))\widetilde{F}\left(k_t^g, l_t, e_t\right), \tag{2}$$

where $1 - D_t(\Gamma_t) = \exp\left[-\pi_t\left(\Gamma_t - \overline{\Gamma}\right)\right]$. Here, $\overline{\Gamma}$ represents the pre-industrial GHG concentration in the atmosphere, and π_t is a variable that parametrizes the effect of higher GHG concentrations on damages. The function D captures the mapping from the stock of GHG, Γ_t, to economic damages measured as a percentage of output. We assume that $\widetilde{F}\left(k_t^g, l_t, e_t\right)$ has a Cobb-Douglas form:

$$\widetilde{F}\left(k_t^g, l_t, e_t\right) = \widetilde{A}_t(k_t^g)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta}, \tag{3}$$

where $\widetilde{A}$ is a productivity parameter, while θ, θ_k, $\theta_l \in (0,1)$, and $\theta_k + \theta_l = \theta$. Thus, the final good production function can be rewritten as

$$y_t \leq A_t(k_t^g)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta}, \tag{4}$$

where $A_t \equiv (1 - D_t(\Gamma))\widetilde{A}_t$. The level of GHG evolves according to

$$\Gamma_t - \overline{\Gamma} = \sum_{n=0}^{t-T}(1 - d_n)f_{t-n}, \; t \geq T, \tag{5}$$

where $d_n \in [0,1]$, and f_{t-n} indicates the anthropogenic GHG emissions in period $t - n$. The variable $1 - d_n$ represents the amount of carbon that remains in the atmosphere n periods into the future. The value of the preindustrial stock of GHG in the atmosphere is denoted by $\overline{\Gamma}$, and T defines the start of industrialization.

The depreciation structure in (5) is characterized by three-parameters. It is assumed that a fraction φ_L of emitted carbon stays in the atmosphere forever, while a fraction $(1-\varphi_0)$ of the remaining emissions exit into the biosphere. The remaining part decays at geometric rate φ. Thus,

$$1 - d_n = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^n. \tag{6}$$

The level of the GHG concentration can be then decomposed to a permanent part, Γ_t^p, and a decaying part, Γ_t^d:

$$\Gamma_t = \Gamma_t^p + \Gamma_t^d, \tag{7}$$

where

$$\Gamma_t^p = \Gamma_{t-1}^p + \varphi_L f_t, \tag{8}$$

$$\Gamma_t^d = (1-\varphi)\Gamma_{t-1}^d + (1-\varphi_L)\varphi_0 f_t. \tag{9}$$

Energy can be produced by using fossil or renewable sources. We assume that the two types of energy are perfect substitutes in the production of the final good.[6] As we measure fossil fuel use in units of carbon content, the flow of anthropogenic GHG emissions equals f_t, the fossil fuel used in energy production in period t. Let ϖ_t denote the available stock of fossil fuel in period t. Given ϖ_0, the law of motion for ϖ_t is

$$\varpi_{t+1} \leq \varpi_t - f_t. \tag{10}$$

The fossil-fuel-derived energy production function uses fossil fuel and capital as inputs

$$e_t^f \leq A_f (f_t)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f}, \tag{11}$$

where $A_f > 0$ and $\alpha_f \in (0,1)$. This specification captures that, by using additional capital, the representative firm can extract more energy from the remaining fossil fuel reserves.

There is a competitive sector of renewable energy-producing firms. As these are heterogenous, we will need to keep track of the identity of each individual firm. The renewable energy production for firm j is given by

$$e_{j,t}^r \leq \Psi(i_{j,t}) \left(\mathcal{E}_{j,t}\right)^{1-\alpha_r} \left(k_{j,t}^r\right)^{\alpha_r}, \tag{12}$$

where $\mathcal{E}_{j,t}$ is firm j's productivity parameter, $\mathcal{E}_{j,0}$ is given, for all j, and $\alpha_r \in (0,1)$. We interpret $i_{j,t}$ as the *new technology adoption rate* by firm j in period t. We wish to capture that the process of adopting a new "vintage" is costly

[6] This assumption can be relaxed at the cost of computational complexity. A high substitutability across energy sources seems a reasonable benchmark when considering long-run effects. For example, substitutability is justified in the presence of energy storage. While not widely available currently, there are strong indications that such storage technologies will enter commercialization in the next decade or so. Our analysis also abstracts from short-run fluctuations in the supply and demand for energy and from the corresponding short-run volatility in energy prices.

and can result in temporary disruptions. We will model these costs as directly reducing output. In other words, while it boosts future productivity, new vintage adoption in our model has a cost in terms of a contemporaneous output loss. More precisely, we assume that investment in new vintage, i, reduces firm j's current output by a factor $\Psi(i_{j,t}) \geq 0$, where $\Psi(\cdot)$ is such that $\Psi(0) = 1$, $\Psi'(\cdot) < 0$, $\Psi''(\cdot) < 0$, and $\Psi(\bar{i}) = 0$, for some $\bar{i}$.[7]

We will consider the possibility of a spillover effect, where aggregate technology adoption also affects the productivity of each individual firm. Put differently, as more firms adopt new technologies, the benefits affect the entire renewable energy sector. This creates an externality, leading to a discrepancy between equilibrium and desirable levels of new vintage adoption. We consider this effect to be especially relevant, as investments in the energy sector tend to be capital intensive. Thus, if innovators do not expect to capture the resulting returns, under-adoption of new technologies relative to the optimum is likely to occur.[8] More precisely, the productivity of firm j evolves according to

$$\ln \mathcal{E}_{j,t+1} \leq \xi i_{j,t} + (1-\xi)\left(\int_0^1 i_{j,t} k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj\right) + \ln \mathcal{E}_{j,t}, \tag{13}$$

where $0 \leq \xi \leq 1$ parametrizes the strength of the spillover effect. The case where $\xi = 1$ corresponds to no spillovers, while $\xi = 0$ implies that productivity is entirely determined by spillovers. In the above expression, we normalize each firm's technology adoption by its capital stock in order to abstract from any size-dependent advantage to firms.

Each period, the production factors are allocated freely across sectors. Total capital used in the economy cannot exceed the total supply; i.e., for all t,

$$k_t^g + k_t^f + \int_0^1 k_{j,t}^r dj \leq k_t, \tag{14}$$

and the energy used in the production of the final good cannot exceed the total supply of energy:

$$e_t \leq e_t^f + \int_0^1 e_{j,t}^r dj. \tag{15}$$

The next section discusses desirable allocations for our model economy.

3 Efficiency

We begin by characterizing allocation efficiency in terms of some key relationships. We will later compare efficient outcomes to market allocations. The social

[7] Admittedly, innovation also takes place in the fossil fuel sector. Mainly for simplicity, in this paper we will concentrate on technological progress in the renewable sector.

[8] These effects appear to be particularly relevant for the energy sector. Bosettia et al. (2008) argue that international knowledge spillovers tend to increase the incentive to free-ride, thus decreasing investments in energy R&D. Braun et al. (2009) perform an empirical study of spillovers in renewable energy. They document significant domestic and international knowledge spillovers in solar technology innovation, as well as significant international spillovers in wind.

planner chooses a sequence $\{c_t, k_t^g, k_t^f, f_t, e_t^f, \Gamma_t^p, \Gamma_t^d, \{i_{j,t+1}, k_{j,t}^r,\ \mathcal{E}_{j,t+1}, e_{j,t}^r\}_{j\in[0,1]}\}_{t=0}^{\infty}$ to solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to (8), (9), (10), (11), (12), (13), (14), (15) and

$$c_t + k_{t+1} \leq (1 - D_t(\Gamma_t^p + \Gamma_t^d)) \left[\widetilde{A}_t (k_t^g)^{\theta_k} (l_t)^{\theta_l} (e_t)^{1-\theta}\right] + (1-\delta) k_t, \tag{16}$$

as well as nonnegativity constraints and given the initial values for the stock variables.

We let μ_F denote the Lagrange multiplier on the production constraint in the fossil fuel sector (equation (11)), and μ_r^j be the multiplier on the production constraint for firm j in the renewable sector (equation (12)). Similarly, let $\mu_{\mathcal{E}}^j$ be the multiplier for the evolution of firm j's productivity in the renewable sector (equation (13)). Finally, μ_K and μ_E are the multipliers associated with the distribution of the capital stock across sectors, and with the supply of energy (equations (14) and (15)), respectively.

The first-order condition with respect to e_{jt}^r gives

$$\int_0^1 \mu_{r,t}^j dj = \mu_{E,t}. \tag{17}$$

Moreover, the marginal utility from producing an extra infinitesimal amount of renewable energy should be equal across firms; i.e.,

$$\mu_{r,t}^j = \mu_{r,t}^h, \text{ for any two firms } j \text{ and } h. \tag{18}$$

The first-order condition with respect to $k_{j,t}^r$ gives

$$(1-\xi)\left(\frac{i_{j,t} - \overline{i_t}}{\overline{k_t^r}}\right) \int_0^1 \mu_{\mathcal{E},t}^j dj + \Psi\left(i_{j,t}\right) \alpha_r \mu_{r,t}^j \left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r} = \mu_{K,t}, \tag{19}$$

where $\overline{k_t^r} = \int_0^1 k_{j,t}^r dj$, and $\overline{i_t} = \int_0^1 i_{j,t} k_{j,t}^r dj / \int_0^1 k_{j,t}^r dj$. Since (17) and (18) give that $\mu_{r,t}^j = \mu_{E,t}$, (19) implies that the only non-aggregate variable that influences $i_{j,t}$ is $\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}$.

The first order condition with respect to $i_{j,t}$ gives

$$-\mu_{r,t}^j \Psi'(i_{j,t}) \left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r} = \xi \frac{\mu_{\mathcal{E},t}^j}{k_{j,t}^r} + (1-\xi) \frac{\int_0^1 \mu_{\mathcal{E},t}^j dj}{\overline{k_t^r}}. \tag{20}$$

Finally, the first-order condition with respect to e_t^f gives

$$\mu_{E,t} = \mu_{F,t}. \tag{21}$$

Since $\mu_{r,t}^{j} = \mu_{E,t} = \mu_{F,t}$, and $i_{j,t}$ is a function of $\frac{\mathcal{E}_{j,t}}{k_{j,t}^{r}}$, we have that $\frac{\mu_{\mathcal{E},t}^{j}}{k_{j,t}^{r}}$ is also a function of $\frac{\mathcal{E}_{j,t}}{k_{j,t}^{r}}$.

Keeping track of the distribution of vintages across heterogeneous firms tends to be complex.[9] The following result greatly simplifies our analysis. It asserts that if $\mathcal{E}_{j,t}$ and $k_{j,t}^{r}$ are proportional to the initial values of $\mathcal{E}_{j,0}$, then $i_{j,t} = i_t$, for all j and t. In other words, although renewable energy-producing firms are heterogeneous, efficiency implies that they choose identical levels of i_t.

Proposition 1 *In an efficient allocation,* $\frac{k_{j,t}^{r}}{\mathcal{E}_{j,t}} = \frac{k_{t}^{r}}{\mathcal{E}_{t}}$ *and* $i_{j,t} = i_t$*, for all* j*.*

Proof. For any initial values of $\mathcal{E}_{j,0}$, there is a solution such that $\mathcal{E}_{j,t}$, $k_{j,t}^{r}$, $\mu_{\mathcal{E},t}^{j}$, and $\mu_{r,t}^{j}$ are proportional to the initial values of $\mathcal{E}_{j,0}$. Then (20) implies that $i_{j,t} = i_t$, for all $j \in [0,1]$. From (19), $\frac{\mathcal{E}_{j,t}}{k_{j,t}^{r}}$ is a function of $i_{j,t}$ only. As $i_{j,t} = i_t$, we have $\frac{\mathcal{E}_{j,t}}{k_{j,t}^{r}} = \frac{\mathcal{E}_{t}}{k_{t}^{r}}$. ■

4 Equilibrium and Optimal Policy

Turning first to competitive equilibrium, we derive the FOC for consumers and firms in the Appendix. Using these, we first characterize the equilibrium choice of investment in the renewable technology. In what follows, we let $\Phi_t(i_{j,t})$ stand for the government policy conditional on a renewable firm's investment. We establish that, provided that $\xi < 1$, and there is no subsidy, this investment will be lower than optimal. Of course, the magnitude of the distortion depends on the level of the externality, ξ.

Proposition 2 *In a competitive equilibrium with* $\Phi_t(i_{j,t}) = 0$*,* $i_{j,t}$ *is lower than optimal when* $\xi < 1$*.*

The proof is given in the Appendix. Next, we discuss optimal policy. This needs to take into account two distortions. First, there is under-investment in i_t, due to the spillover effects. The second distortion is due to the social costs associated with the environmental externality from GHG emissions. The next Proposition demonstrates that both distortions can be fully accommodated through the use of two instruments. First, a policy that taxes firms in proportion to their under-investment in i_t restores optimal investment by making firms indifferent between paying the tax or pursuing the optimal level of investment. Second, a Pigouvian tax internalizes the externality from carbon emissions. As in GHKT (2014), under the special assumptions of log utility and 100% depreciation of capital, the Pigouvian tax imposed on the fossil fuel firms does not depend on the growth rate of the economy.

[9] There is extensive literature on dynamic vintage-capital-related models. See, for example, Benhabib and Rustichini (1991), Chari and Hopenhayn (1991), Greenwood, Hercowitz, and Krusell (1997), and Jovanovic (2012). Boucekkine, De La Croix, and Licandro (2017) provide a recent review.

Proposition 3 *(1) The optimal allocation can be supported by a combination of a revenue-neutral policy,* $\Phi_t^j(i_{j,t}) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$*, imposed on renewable firms, together with a Pigouvian tax on fossil fuel use,* $\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j)$*, where* $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the planner's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$*. (2) If* $u(c) = \log(c)$*,* $\alpha_r = \alpha_f = \alpha$*,* $\pi_t = \pi$*, all* t*, and* $\delta = 1$*,* $\tau_t^f = y_t\pi\left[\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)\beta}\right]$ *does not depend on the growth rate of the economy.*

The proof is given in the Appendix. The optimal policy in our model has several interesting implications. First, the policy on renewable energy firms generates no revenue, but it reduces the household's profits from the renewable sector, as a result of inducing additional innovation compared to the competitive equilibrium. Second, the Pigouvian tax reduces the household's profits from the fossil fuel sector. However, the household receives a lump-sum transfer of equal magnitude; thus, its budget constraint remains unchanged. Finally, there is a separation between the two schemes, as the total effect on the household's budget is the same as the resource cost of innovation in the planner's problem.

For the remainder of the paper we will assume that $u(c) = \log(c)$ and $\delta = 1$. Moreover, we will assume that the stock of fossil fuel is large enough so that consumption of fossil fuel is not constrained. In the Appendix we solve the planner's problem backward, from a final state, where only renewable energy is used. We show that the total consumption of fossil fuel is endogenously bounded. In other words, the transition to renewable energy takes place prior to the exhaustion of fossil fuel resources. This is because the growing productivity in the renewable sector eventually surpasses a threshold that makes using fossil fuel a less efficient source of energy. While allocating additional capital to the fossil fuel sector increases the production of energy per unit of fossil fuel, the present value of the marginal environmental damages limits the overall benefit from fossil fuel use. In the next section we calibrate our model in order to study the optimal timing of the transition to a renewable energy regime, as well as the effects of the GHG accumulation prior to this transition. This will also allow us to explore the quantitative significance of the scrapping effect on optimal policy and welfare.

5 Calibration

In this section we calibrate our model in order to study the transition from the current, predominantly fossil fuel economy, to an economy that fully relies on renewable energy. We use the calibrated model to evaluate the interaction between the two policy instruments: (i) Pigouvian taxation of carbon emissions and (ii) technology adoption targeting for renewable energy firms. In particular we evaluate how the two policies would affect the share of renewable energy, the accumulation of GHG, global temperatures, economic growth, and welfare, first in isolation, and then in tandem. This will allow us to quantify the significance of the scrapping effect and to explore the potential substitutability between the two policy tools.

The model's parameters can be divided into four categories related to preferences, technology, environmental damages, and the current (status quo) policies in place. We will assume a log utility function and a benchmark annual discount rate of 4%, which gives $\beta = 0.96^{10}$, as a period is calibrated to 10 years.[10] Given the length of the period, there is some justification in considering the benchmark case of full depreciation of capital, $\delta = 1$. Turning to the aggregate Cobb-Douglas production function, we set the share of capital and labor, respectively, to $\theta_k = (1/3) \times 0.95$ and $\theta_l = (2/3) \times 0.95$, which implies an energy share of $1 - \theta = 1 - (\theta_k + \theta_l) = 0.05$. We set the productivity growth rate in the final good sector so that the balanced growth rate is 2%, while the long-run population growth rate is zero; i.e., $g^l = \exp(0)$.

We assume the following form for the renewable technology adoption cost function, Ψ:

$$\Psi(i) = \left(1 - \left(\frac{i}{\bar{\bar{i}}}\right)^{\psi}\right)^{1/\psi}.$$

This functional form satisfies the earlier assumptions that $\Psi\left(0\right) = 1$, $\Psi'\left(\cdot\right) < 0$, $\Psi''\left(\cdot\right) < 0$, and $\Psi\left(i\right) = 0$, for $i = \bar{i}$. Moreover, the elasticity of the technology adoption cost with respect to the adoption rate is given by

$$-\frac{\Psi'(i)}{\Psi(i)} = \frac{1}{i} \times \frac{(i/\bar{\imath})^{\psi}}{1 - (i/\bar{\imath})^{\psi}}. \tag{22}$$

As shown in the Appendix, this elasticity plays an important role in determining both the long-run and the transitional technological adoption rate in the renewable sector. The parameter ψ provides us with a degree of freedom to match a long-run adoption rate that is consistent with the long-run growth rate of the economy. To calibrate Ψ, we need to assign values to two parameters: $\bar{i}$ and ψ.

We use the fact that if $\beta e^{\bar{i}} > 1$ then growth would be unbounded, to set $\bar{i} = -\log(\beta) = 0.4082$. To calibrate ψ, we use its effect on the optimal asymptotic long-run growth rate of i^l, the productivity in the renewable sector. As we show in the Appendix, if the spillover of the renewable technological adoption is fully endogenized, in the long-run i^l is given by

$$-\frac{\Psi'(i^l)}{\Psi(i^l)} \quad = \quad \frac{\beta}{1-\beta}\left(1-\alpha\right).$$

Combining the above equation with (22) gives:

$$\psi = \log\left(\frac{\beta/(1-\beta)(1-\alpha)i^l}{(1+\beta/(1-\beta)(1-\alpha)i^l)}\right)/\log(i^l/\bar{\imath}). \tag{23}$$

[10]Most macroeconomic studies use yearly discount rates between 2% and 5%.

To determine i^l, note that an asymptotically balanced growth path requires equal asymptotic growth rates between the renewable energy sector (which is the only source of energy in the long-run) and the final good sector. This equality implies $i^l = \log\left(g^l \times (g^g)^{\frac{1}{\theta_l}}\right) = 0.198$. Using (23), we set $\psi = 2.498$.

We follow GHKT (2014) in our calibration of the environmental damage parameters and the computation of the Pigouvian carbon tax. In particular, we set $\pi = 2.379 \times 10^{-5} \times 10$, $\varphi = 0.0228$, $\varphi_L = 0.2$, and $\varphi_0 = 0.393$. The optimal carbon tax follows from the last part of (83) in the Appendix and is given by

$$\mathcal{T}_P/y = \pi\left[\frac{\beta\varphi_L}{(1-\beta)} + \frac{(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))}\right]. \tag{24}$$

Given our calibration, this equation implies that $\mathcal{T}_P/y = 3.55 \times 10^{-4}$, which is equivalent to a tax of \$24.9 per ton, which is broadly consistent with the climate economics literature given the assumed level of discounting.

We chose 2015 as our base year. Four parameters related to the energy sector remain to be calibrated: the current stock of fossil fuel, W_0, the current productivity of the renewable sector, $\mathcal{E}_0$, the current Pigouvian tax level, τ^f, and the spillover from the renewable technology adoption, ξ. For our baseline calibration we set $W_0 = 666GtC$.[11]

We set $\mathcal{E}_0$, τ^f, and ξ to match three data moments: (i) the current share of renewable energy in total energy production, s_0, (ii) the current consumption of fossil fuel, f_0, and (iii) the change in the share of renewable energy in the last period (10 years), $s_0 - s_{-1}$. The current productivity of the renewable sector affects the renewable share in total energy production. In turn, the spillovers from the renewable technology adoption affect the change in the productivity of the renewable sector, and thus the change in the share of renewable energy. In addition, the Pigouvian tax affects the use of fossil fuel and, consequently, the share of fossil fuel and renewable sources in total energy production. In what follows, we denote by τ^f the value of the Pigouvian tax as a percentage of its optimal level, τ^*. Setting $\mathcal{E}_0 = 13.25$, $\tau^f = 0.62 \cdot \tau^*$, and $\xi = 0.46 \cdot \xi^*$, our model matches $f_0 = 100\ GtC$,[12] $s_0 = 10.2\%$, and $s_0 - s_{-1} = 2.3\%$.[13]

Calibration of the policy parameters (the tax rate, τ^f, and the technology adoption, ξ) allows us to evaluate the effect of the carbon tax and the renewable adoption policy in isolation, as well as in tandem. We simulate our model considering different scenarios for the two policy parameters. Figure 1 below shows the paths for the share of renewable energy (top), accumulated fossil fuel consumption (middle), and global temperatures (bottom), in each respective policy

[11]See Section 4.3 in Li, Narajabad, and Temzelides (2016). As estimates vary widely, comparative statics with respect to this value can be used as a partial substitute for explicitly introducing technological progress in the fossil fuel sector.

[12]See EPA: https://www.epa.gov/ghgemissions/global-greenhouse-gas-emissions-dataTrends.

[13]This includes all modern plus traditional renewables (including biomass). We calculated an initial 10-year growth rate of 4.7% for renewables, with a corresponding rate of 2% for the entire energy sector. We excluded nuclear energy from this calculation. See https://www.ren21.net/reports/global-status-report/.

scenario. The dotted, dashed, dot-dashed, and solid lines indicate the status quo benchmark (business as usual), optimal technology adoption, optimal Pigouvian tax, and combined optimal policies, respectively. Clearly, the outcomes under either an optimal Pigouvian tax policy alone or the optimal technology adoption policy alone are different from the outcome when both policies are present. This thought-experiment helps us understand how the two policies interact in the presence of the scrapping channel. We comment on each panel individually.

The first panel gives the share of renewables in energy production as a function of time under the different policy scenarios. Setting technology adoption to its optimal level in the absence of the optimal Pigouvian tax *reduces* the share of renewables in the short run relative to the status quo. At the same time, the energy production becomes fully renewable somewhat earlier than in the benchmark case. In contrast, setting the Pigouvian tax to its optimal level in the absence of a policy inducing optimal technology adoption increases the share of renewable energy immediately. In the full optimum, setting both policies to their combined optimal levels does not change the share of renewable energy immediately. However, the transition to a fully renewable global economy takes place by 2080, the earliest date among the four scenarios.

The second panel describes the evolution of cumulative fossil fuel consumption in the same four scenarios. Interestingly, absent a tax on GHG emissions, the cumulative fossil fuel consumption is initially somewhat more intense if the technology externality is internalized than in the benchmark status quo case. This is because the faster growth in renewable energy productivity allows the economy to rely fully on renewable energy earlier. Similarly, at the full optimum where, in addition to the Pigouvian tax, the technology adoption is also set to its optimal level, the economy reaches the fully renewable energy state sooner and more fossil fuel is left unused. Consistent with the "green paradox," this also implies a heavier use of fossil fuel initially than in the case where the Pigouvian tax is in place but the renewable policy is absent.

The third panel shows the path for global temperatures under our four policy scenarios. In order to map carbon concentrations into global temperatures, T, we use the following expression from GHKT (2014):

$$T(S_t) = 3 \ln \left(\frac{S_t}{\overline{S}} \right) / \ln(2),$$

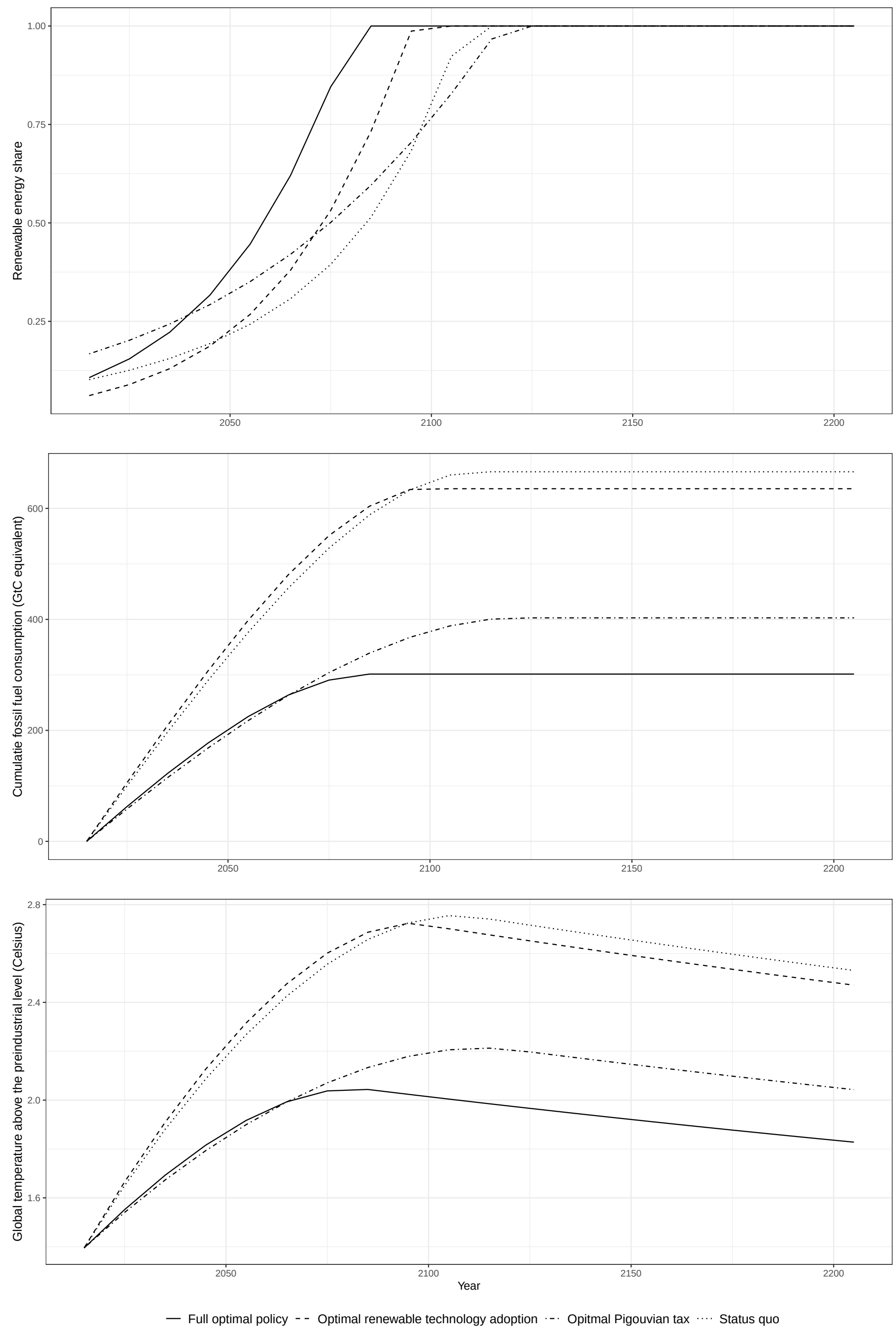
where $\overline{S}$ is the pre-industrial level of atmospheric carbon concentration. Consistent with the fossil fuel use in the top panel, the global temperature increases under both the business-as-usual and the optimal technology adoption scenario, reaching short of 2.8 degrees Celsius above the pre-industrial level. The temperature under the technology policy alone (in the absence of a Pigouvian tax) later falls slightly faster than in the benchmark case. Under the optimal Pigouvian tax and in the fully optimal case, global temperatures peak at around 2.2 and 2.0 degrees Celsius above the pre-industrial level, respectively, and then decline over time.

The three panels in our Figure 2 describe the over-time contribution of certain key variables to related growth rates under our four policy scenarios. The top panel shows the period-by-period difference in related damages caused by GHG emissions. Of note, the implementation of the technology policy in the absence of a carbon tax leads to a higher damages to growth than the status quo. This is due to the green paradox reasoning we discussed before, as the fully renewable state is reached earlier than in the status quo. After the full transition takes place, the contribution to growth is positive (if small) in all four cases, due to the gradual decline in the stock of emissions. The second panel plots the contribution of the energy sector to economic growth. The status quo scenario results in a sizable negative contribution to growth. This is partly due to damages and partly due to scarcity and an increasing shadow price of fuel. As the resource constraint on fossil fuel is far from binding under the fully optimal policy scenario, equation (68) in the Appendix implies that the net contribution of energy to growth is positive, and increasing during the energy transition.

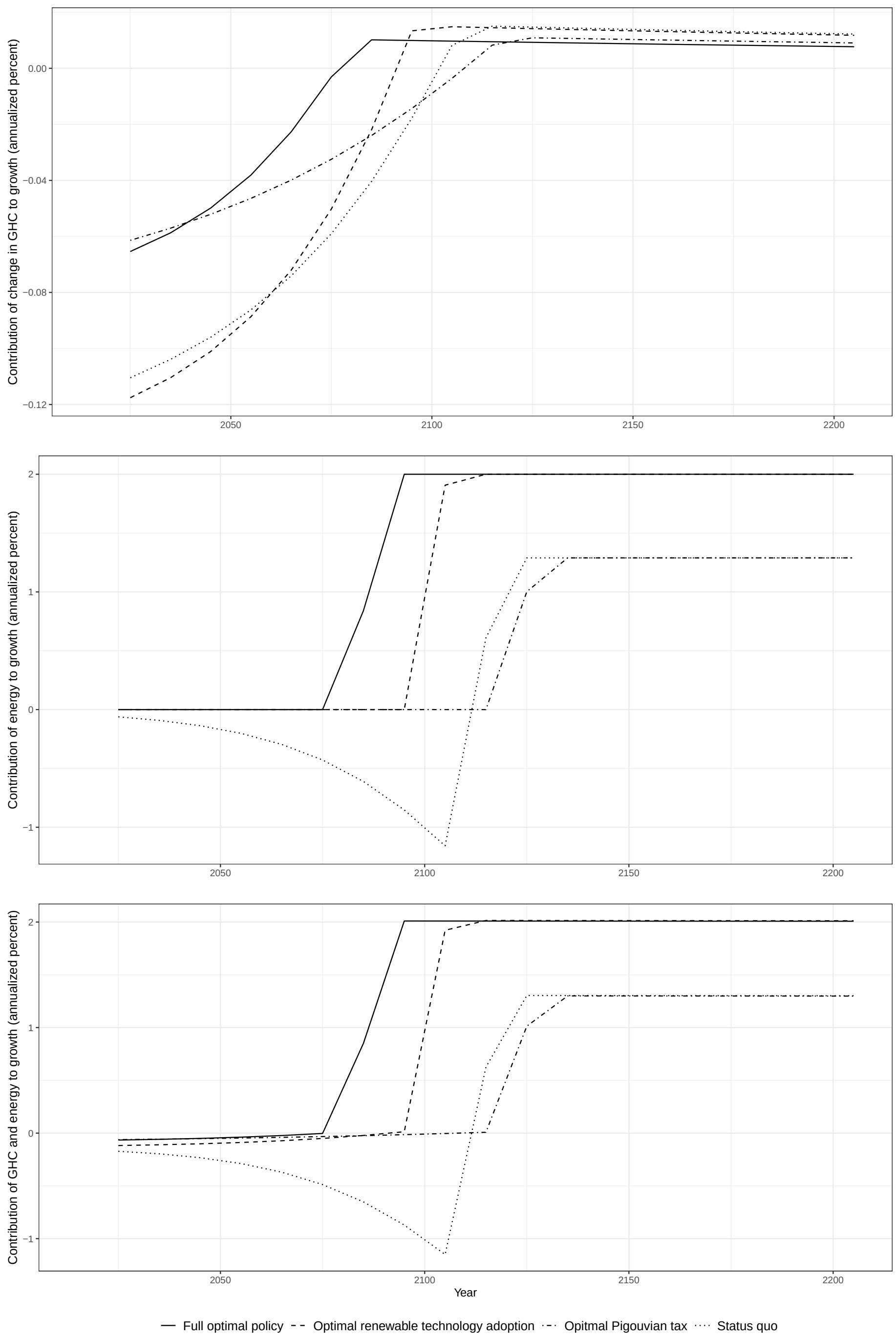
Next, we turn our attention to welfare across these scenarios. Following Lucas (1987), we report the consumption-equivalent percentage welfare gain from these policies, over the business-as-usual benchmark. Moving from the status quo to optimal technology adoption alone (but no Pigouvian tax) would imply a 0.0136% consumption-equivalent gain, while the optimal Pigouvian tax alone would result in a gain of 0.9698%, confirming the relative importance of the carbon tax. Comparing the status quo to the situation where both policies are applied results in a consumption-equivalent welfare gain of 1.257%. The difference between the welfare gain from applying either policy in isolation versus implementing both amounts to about 0.27% increase in consumption, suggesting a sizeable complementary between the two policies.

In summary, these findings assert that, when it comes to long-term growth and welfare, energy supply considerations are of first order importance. In addition, while they are often viewed as a suitable, easier to implement alternative to carbon taxes, our model suggests that, in the presence of scrapping costs, policies that incentivize penetration by renewables in the absence of a Pigouvian tax on carbon emissions can provide relatively small benefits. Indeed, our model points to strong complementarities between the two policies. Sizable welfare gains can be explored only when the policies are adopted in tandem.

To further identify the role of the scrapping channel in these findings, we run the model under the same parametrization as before, but with the scrapping costs "shut down;" i.e., $\Psi = 1$. In the absence of scrapping, we set the growth rate in the renewable technology equal to its long-run value, as implied by the balanced growth path. We then target $f_0 = 100\ GtC$, after which the model generates $s_0 - s_{-1} = 2.1\%$. The Pigouvian tax rate is close to the one under scrapping: $\tau^f = 0.62 \cdot \tau^*$. The implied dynamics for the share of renewables, fossil fuel consumption, and global temperature, as well as the corresponding effects on growth, are reported in Figures 3 and 4. By comparing to the case with scrapping, we notice a number of differences. First, when it comes to the renewable energy share (top panel of Figure 3), under no scrapping, the optimal renewable energy growth rate is always equal to its long-run level. Optimal penetration by renewables starts lower in the case with scrapping, but it soon overtakes, and the transition to the fully renewable state occurs earlier in



1.00
0.75
0.50
0.25
Renewable energy share
2050
2100
2150
2200
600
400
200
0
Cumulatie fossil fuel consumption (GtC equivalent)
2050
2100
2150
2200
2.8
2.4
2.0
1.6
Global temperature above the preindustrial level (Celsius)
2050
2100
2150
2200
Year
Full optimal policy
Optimal renewable technology adoption
Opitmal Pigouvian tax
Status quo

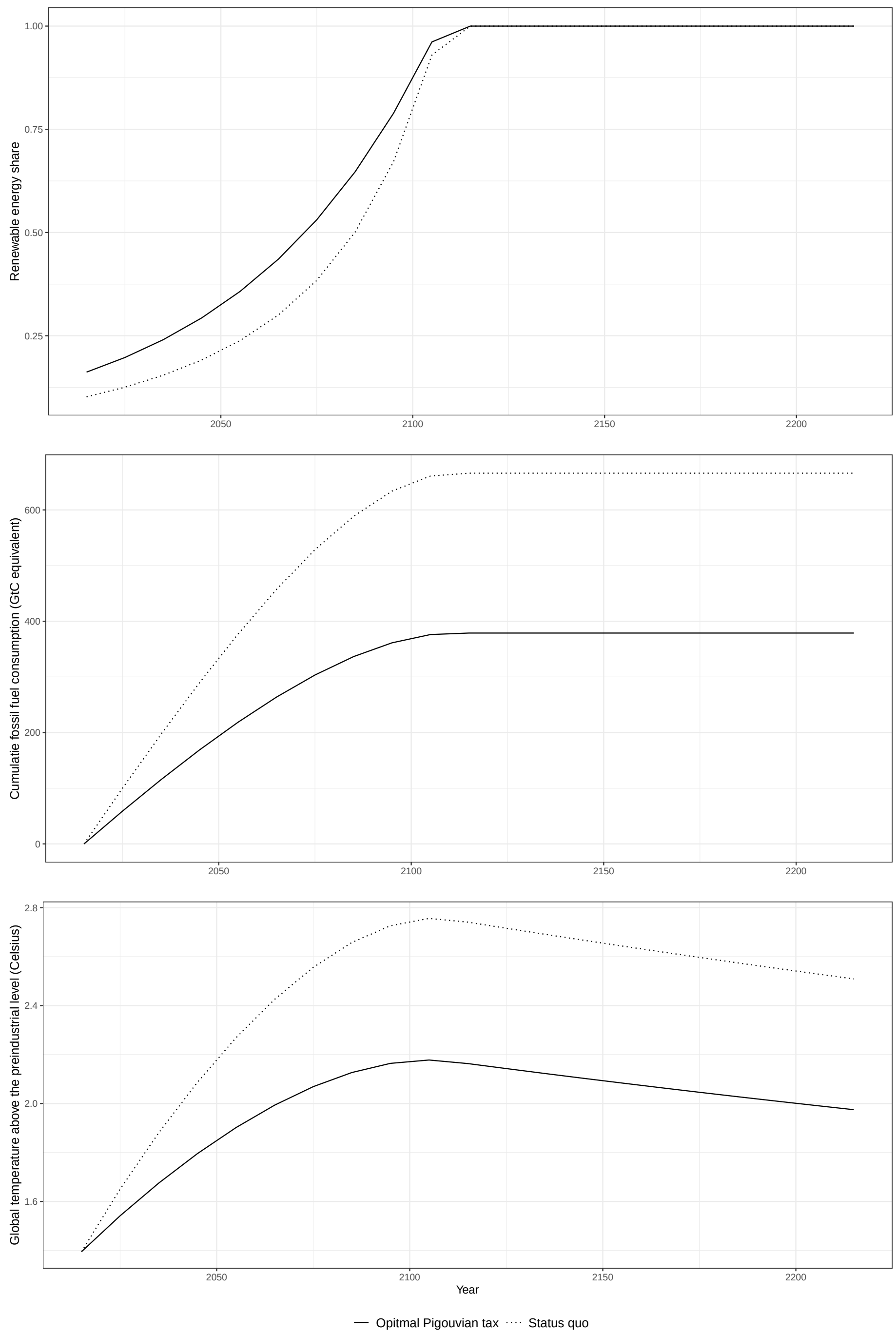

Contribution of change in GHC to growth (annualized percent)
0.00
−0.04
−0.08
−0.12
2050
2100
2150
2200
Contribution of energy to growth (annualized percent)
2
1
0
−1
2050
2100
2150
2200
Contribution of GHC and energy to growth (annualized percent)
2
1
0
−1
2050
2100
2150
2200
Year
Full optimal policy
Optimal renewable technology adoption
Opitmal Pigouvian tax
Status quo

this case. Fossil fuel consumption and global temperatures demonstrate corresponding differences. In the case without scrapping, the consumption-equivalent welfare gain from setting the Pigouvian tax to its optimal level corresponds to an 1.05% increase in consumption. We conclude that the scrapping channel plays a significant role when we quantify the optimal energy transition.

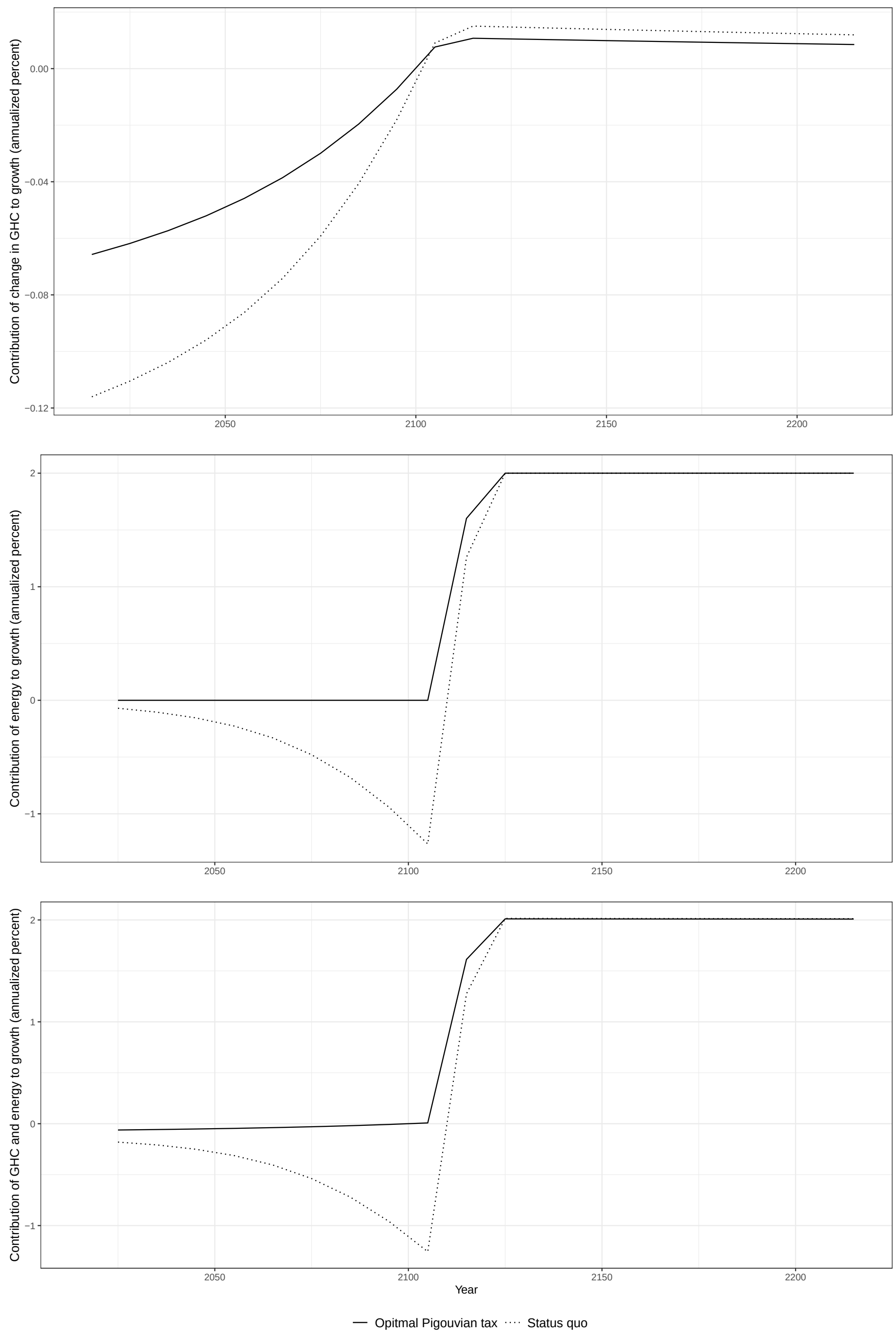
6 Conclusion

We studied the interplay between optimal growth, carbon taxes, and policies related to the adoption of new renewable technologies in a IAM framework where energy is an input in the production of final goods and where fossil fuel use creates damages resulting from GHG emissions. We considered a new channel by incorporating the scrapping costs associated with productivity advancements and new vintage capital in the renewable energy sector. As renewable technologies are relatively new, such advancements can be frequent. We modeled the corresponding scrapping costs as temporary adverse TFP shocks in the renewable energy sector's production function and investigated the quantitative significance of these shocks for the optimal energy transition. Importantly, we also took into consideration that productivity improvements are subject to spillovers.

Efficiency in our model requires a policy that promotes the adaptation of new technologies by subsidizing investment in renewables, as well as a Pigouvian tax on the environmental externality. Energy supply considerations were found to be of first-order importance for growth and welfare. Policies that promote penetration by renewables, in our case by internalizing spillover effects, are often viewed as a suitable, easier to implement alternative to carbon taxes. Our model suggests that the two policies are better thought of as being complements rather than substitutes. In the absence of a carbon tax, promoting maximum penetration by renewables can provide only relatively small benefits, and can even be detrimental to economic growth in the short run.



Renewable energy share
1.00
0.75
0.50
0.25
2050
2100
2150
2200
Cumulatie fossil fuel consumption (GtC equivalent)
600
400
200
0
2050
2100
2150
2200
Global temperature above the preindustrial level (Celsius)
2.8
2.4
2.0
1.6
2050
2100
2150
2200
Year
Opitmal Pigouvian tax
Status quo



Contribution of change in GHC to growth (annualized percent)
0.00
−0.04
−0.08
−0.12
2050
2100
2150
2200
Contribution of energy to growth (annualized percent)
2
1
0
−1
2050
2100
2150
2200
Contribution of GHC and energy to growth (annualized percent)
2
1
0
−1
2050
2100
2150
2200
Year
Opitmal Pigouvian tax
Status quo

7 Appendix

7.1 Household's and Firms' Optimization

The representative household owns the firms, as well as the stock of capital and the stock of fossil fuel. It rents capital to firms and sells fossil fuel to the non-renewable sector. The representative household's problem is given by

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t.}$$
$$\sum_{t=0}^{\infty} p_t \left[c_t + k_{t+1} - (1-\delta)k_t\right] \leq$$
$$\sum_{t=0}^{\infty} p_t \left[r_t k_t + w_t l_t + p_t^f f_t + \pi_t^g + \pi_t^f + \int_0^1 \pi_{j,t}^r dj + z_t\right],$$
$$\varpi_{t+1} \leq \varpi_t - f_t, \tag{25}$$

where p_t is the Arrow-Debreu price of the period t final good, δ is the depreciation rate of capital, r_t is the rental price of capital, w_t is the wage rate, p_t^f is the price of fossil fuel, π_t^g, π_t^f, and $\int_0^1 \pi_{j,t}^r dj$ stand for the profits of firms in the various sectors of the economy, and z_t are lump-sum transfers from the government. The government runs a balanced budget. It collects taxes from the fossil fuel energy sector and rebates them lump-sum to households.

The first order conditions, which are also sufficient for a maximum, imply:

$$1 - \delta + r_{t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)}, \tag{26}$$

and

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{p_{t+1}}{p_t}. \tag{27}$$

Equation (26) says that the rental price of capital plus the non-depreciated part of capital must equal the marginal rate of substitution between consumption in two consecutive periods. Equation (27) says that the marginal rate of substitution between consumption in period t and consumption in period $t+1$ must equal the relative price of the respective consumption goods.

The final good-producing firms rent capital, hire labor and buy energy in competitive markets, at prices w_t, r_t, and p_t^e, respectively. The representative firm in the final good sector solves:

$$\max \left[A_t \cdot (k_t^g)^{\theta_k} (l_t)^{\theta_l} (e_t)^{1-\theta} - r_t k_t^g - w_t l_t - p_t^e e_t\right].$$

The first-order conditions imply that the marginal input productivities equal their respective prices:

$$\theta_k A_t (k_t^g)^{\theta_k - 1} (l_t)^{\theta_l} (e_t)^{1-\theta} = r_t, \tag{28}$$

$$\theta_l A_t \left(k_t^g\right)^{\theta_k} \left(l_t\right)^{\theta_l - 1} \left(e_t\right)^{1-\theta} = w_t, \tag{29}$$

and

$$(1-\theta) A_t \frac{\left(k_t^g\right)^{\theta_k} \left(l_t\right)^{\theta_l}}{e_t^{\theta}} = p_t^e. \tag{30}$$

Firms in the fossil fuel sector rent capital and buy fossil fuel. Additionally, they pay a per unit tax on the energy sold, τ_t. The representative firm in this sector solves:

$$\max \left[\left(p_t^e - \tau_t\right) A_f \left(f_t\right)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f} - r_t k_t^f - p_t^f f_t \right].$$

The first-order conditions imply that the value of the marginal input productivities equal their respective prices:

$$\left(p_t^e - \tau_t\right) \alpha_f A_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} = r_t, \tag{31}$$

and

$$\left(p_t^e - \tau_t\right) \left(1-\alpha_f\right) A_f \left(\frac{k_t^f}{f_t}\right)^{\alpha_f} = p_t^f. \tag{32}$$

The renewable energy firms' production function is given by (12). It depends on the firm's productivity, the firm's technology adoption rate, and the capital used. The firms in this sector rent capital and receive a subsidy, $\Phi\left(i_{j,t}\right)$, which is a function of the firm's technology adoption rate, $i_{j,t}$. We allow $\Phi\left(i_{j,t}\right)$ to be negative and assume it is differentiable. In each period t, the renewable firm j maximizes future discounted profits subject to (13):

$$\begin{aligned}
&\max \sum_{\tau=0}^{\infty} \beta^{t+\tau} u'(c_{t+\tau}) \left[p_{t+\tau}^e \Psi(i_{j,t+\tau}) \left(\mathcal{E}_{j,t+\tau}\right)^{1-\alpha_r} \left(k_{j,t+\tau}^r\right)^{\alpha_r} - r_{t+\tau} k_{j,t+\tau}^r + \Phi\left(i_{j,t+\tau}\right) \right] \\
&\qquad \text{s.t. } \ln \mathcal{E}_{t+1}^j \leq \ln \mathcal{E}_t^j + \xi i_{j,t} + (1-\xi) \left(\int_0^1 i_{j,t} k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj \right) \\
&\qquad\qquad i_{j,t} \geq 0, \text{ and } \mathcal{E}_0 \text{ given.}
\end{aligned} \tag{33}$$

Let $\lambda_{\mathcal{E},t}^j$ be the Lagrangian multiplier associated with equation (13). The first-order conditions of this problem are

$$p_t^e \alpha_r \Psi(i_{j,t}) \left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r} = r_t, \tag{34}$$

$$-\beta^t u'(c_t) \left[p_t^e \Psi'(i_{j,t}) \left(\mathcal{E}_{j,t}\right)^{1-\alpha_r} \left(k_{j,t}^r\right)^{\alpha_r} + \Phi'\left(i_{j,t}\right) \right] = \xi \lambda_{\mathcal{E},t}^j, \tag{35}$$

and

$$\lambda_{\mathcal{E},t+1}^j + \beta^{t+1} u'(c_{t+1}) p_{t+1}^e (1-\alpha_r) e_{j,t+1}^r = \lambda_{\mathcal{E},t}^j. \tag{36}$$

Equation (34) says that the value of the marginal productivity of capital should be equal to its rental price. Equation (35) says that the cost of increasing the adoption rate, which is the loss in production plus the marginal subsidy, should equal the benefit from increasing the adoption rate, which comes from the value of having a higher level of productivity next period. Equation (36) says that the value this period from relaxing constraint (13) should be equal to the value from relaxing that constraint next period plus the benefit of higher productivity next period.

7.2 Proof of Proposition 2

Proposition 2 in the text states:

Proposition 2: *In a competitive equilibrium with $\Phi\left(i_{j,t}\right)=0$, $i_{j,t}$ is lower than optimal when $\xi<1$.*

Proof. From Proposition 1, the social planner chooses $i_{j,t}=i_t$ and $\frac{k^r_{j,t}}{\mathcal{E}_{j,t}}=\frac{k^r_t}{\mathcal{E}_t}$. This, together with the first order condition (20), implies that

$$-\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r}\mu^j_{r,t}=\xi\mu^j_{\mathcal{E},t}+(1-\xi)\frac{k^r_{j,t}}{\overline{k^r_t}}\int_0^1\mu^j_{\mathcal{E},t}dj. \tag{37}$$

The first order conditions of the social planer's problem also give

$$\beta^t u'(c_t)\left(1-\theta\right)A_t\frac{\left(k^g_t\right)^{\theta_k}\left(L_t\right)^{\theta_L}}{\left(e_t\right)^{\theta}}=\mu_{E,t}=\mu^j_{r,t}. \tag{38}$$

Equation (37) together with (38) and (30) implies

$$-\beta^t u'(c_t)p^e_t\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r}=\xi\mu^j_{\mathcal{E},t}+(1-\xi)\frac{k^r_{j,t}}{\overline{k^r_t}}\int_0^1\mu^j_{\mathcal{E},t}dj. \tag{39}$$

The first-order condition with respect to $\mathcal{E}_{j,t+1}$ is

$$\mu^j_{\mathcal{E},t+1}\frac{1}{\mathcal{E}^j_{t+1}}+\mu^j_{r,t+1}(1-\alpha_r)\Psi(i_{j,t+1})\left(\frac{k^r_{j,t+1}}{\mathcal{E}_{j,t+1}}\right)^{\alpha_r}=\mu^j_{\mathcal{E},t}\frac{1}{\mathcal{E}^j_{t+1}}, \tag{40}$$

which can be rewritten using condition (12) as

$$\mu^j_{\mathcal{E},t+1}+\beta^{t+1}u'(c_{t+1})p^e_{t+1}(1-\alpha_r)e^r_{j,t+1}=\mu^j_{\mathcal{E},t}.$$

Solving for $\mu^j_{\mathcal{E},t}$, we obtain

$$\mu^j_{\mathcal{E},t}=\sum_{\tau=1}^{\infty}\beta^{t+\tau}u'(c_{t+\tau})p^e_{t+\tau}(1-\alpha_r)e^r_{j,t+\tau},\ \text{if}\ \lim_{\tau\to\infty}\mu^j_{\mathcal{E},\tau}=0. \tag{41}$$

Replacing (41) in (39) we obtain

$$-\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r} = \xi\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r + (1-\xi)\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)\frac{k_{j,t+\tau}^r}{\overline{k_{t+\tau}^r}}\int_0^1 e_{j,t+\tau}^r dj. \tag{42}$$

Solving equation (36) for $\lambda_{\mathcal{E},t}^j$, we obtain

$$\lambda_{\mathcal{E},t}^j = \sum_{\tau=1}^{\infty}\beta^{t+\tau}u'(c_{t+\tau})p_{t+\tau}^e(1-\alpha_r)e_{j,t+\tau}^r, \text{ if } \lim_{\tau\to\infty}\lambda_{\mathcal{E},\tau}^j = 0. \tag{43}$$

Finally, replacing (43) in equation (35) (with $\Phi(i_{j,t}) = 0$) gives

$$-\Psi'(i_{j,t})(\mathcal{E}_{j,t})\left(\frac{k_{j,t}^r}{\mathcal{E}_{j,t}}\right)^{\alpha_r} = \xi\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r. \tag{44}$$

It is straightforward to verify that the right hand side of equation (42) is larger than the right hand side of equation (44). Since $-\Psi'(i_{j,t})$ is increasing in $i_{j,t}$, everything else equal, the value of $i_{j,t}$ that satisfies (44) in the competitive equilibrium equation is lower than the i_t that satisfies (42) in the social planner's first order condition. ■

7.3 Proof of Proposition 3

Proposition 3 in the text states:

Proposition 3: (1) *The optimal allocation can be supported by a combination of a revenue-neutral policy,* $\Phi(i_{j,t}) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$, imposed on renewable firms, *together with a Pigouvian tax on fossil fuel use,*
$\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j)$,
where $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the planner's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$. *(2) If* $u(c) = \log(c)$, $\alpha_r = \alpha_f = \alpha$, $\pi_t = \pi$, all t, *and* $\delta = 1$, $\tau_t^f = y_t\pi\left[\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)\beta}\right]$ *does not depend on the growth rate of the economy.*

Proof. When $\Phi(i_{j,t}) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$ the firm j's first order condition (35) is

$$-\beta^t u'(c_t)\left[p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)\right] = \lambda_{\mathcal{E},t}^j, \tag{45}$$

Solving (36) for $\lambda^j_{\mathcal{E},t}$, we obtain

$$\lambda^j_{\mathcal{E},t} = \sum_{\tau=1}^{\infty} \beta^{t+\tau} u'(c_{t+\tau}) p^e_{t+\tau} (1-\alpha_r) e^r_{j,t+\tau}, \text{ if } \lim_{\tau\to\infty} \lambda^j_{\mathcal{E},\tau} = 0. \tag{46}$$

Combining this equation with (45) gives

$$-\Psi'(i^*_t)\left(\mathcal{E}_{j,t}\right)\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r} = \sum_{\tau=1}^{\infty} \beta^\tau \frac{u'(c_{t+\tau})p^e_{t+\tau}}{u'(c_t)p^e_t}(1-\alpha_r)e^r_{j,t+\tau}. \tag{47}$$

The social planner's problem gives rise to a similar condition, (42), which we repeat here:

$$\begin{aligned} -\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r} &= \xi \sum_{\tau=1}^{\infty} \beta^\tau \frac{u'(c_{t+\tau})p^e_{t+\tau}}{u'(c_t)p^e_t}(1-\alpha_r)e^r_{j,t+\tau} + (1-\xi) \\ & \sum_{\tau=1}^{\infty} \beta^\tau \frac{u'(c_{t+\tau})p^e_{t+\tau}}{u'(c_t)p^e_t}(1-\alpha_r)\frac{k^r_{j,t+\tau}}{\overline{k^r_{t+\tau}}}\int_0^1 e^r_{j,t+\tau}dj \end{aligned} \tag{48}$$

To show that these conditions are identical, thus implying the same i_t, it suffices to show that

$$\frac{k^r_{j,t}}{\overline{k^r_t}}\int_0^1 e^r_{j,t}dj = e^r_{j,t}. \tag{49}$$

This follows from

$$\begin{aligned} \frac{k^r_{j,t}}{\overline{k^r_t}}\int_0^1 e^r_{j,t}dj &= \frac{k^r_{j,t}}{\int_0^1 k^r_{j,t}dj}\int_0^1 \Psi(i_t)k^r_{j,t}\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r-1} dj \\ &= \frac{k^r_{j,t}}{\int_0^1 k^r_{j,t}dj}\Psi(i_t)\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r-1}\int_0^1 k^r_{j,t}dj \\ &= k^r_{j,t}\Psi(i_t)\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r-1} = e^r_{j,t}. \end{aligned} \tag{50}$$

Next, suppose that sellers of fossil fuel face a linear tax rate,

$$\tau^f_t = \sum_{j=0}^{\infty} \beta^j \frac{u'(c^*_{t+j})}{u'(c^*_t)} \pi_{t+j} y^*_{t+j}(1-d_j), \tag{51}$$

where $\{c^*_t, y^*_t\}^\infty_{t=0}$ solves the planner's problem, and $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$. Under this tax, the fossil-fuel-producers' optimal intertemporal substitution implies

$$u'(c_t)\left\{p^f_t - \tau^f_t\right\} = \beta u'(c_{t+1})\left\{p^f_{t+1} - \tau^f_{t+1}\right\}. \tag{52}$$

Using (32) for the price of fossil fuel and (51) for the tax, we obtain

$$\begin{aligned} & u'(c_t)\left\{MPF_t - \pi_t y^*_t\left(\varphi_L + (1-\varphi_L)\varphi_0\right)\right\} \\ & + \sum_{j=1}^{\infty} \beta^j u'(c^*_{t+j})\pi_{t+j}y^*_{t+j}((1-\varphi_L)\varphi_0(1-\varphi)^{j-1}\varphi) \\ = {} & \beta u'(c_{t+1})\left\{MPF_{t+1}\right\}, \end{aligned} \tag{53}$$

where MPF_t is the period-t marginal productivity of fossil fuel in units of the final good. Clearly, the claim follows if $\frac{y^*_{t+j}}{c^*_{t+1}} = \chi$, a constant. First, observe that $\frac{c_t}{y_t} = \chi \Leftrightarrow \frac{k^g_{t+1}}{y_t} = \theta^k \beta$. This equation follows from the FOCs of the social planner, which include

$$\frac{y_t}{c_t} = \frac{y_{t+1}}{c_{t+1}} \frac{\theta^k \beta y_t}{k^g_{t+1}}. \tag{54}$$

It remains to be shown that

$$\frac{k^f_{t+1}}{y_t} + \frac{k^r_{t+1}}{y_t} = 1 - \chi - \theta^k \beta \equiv \varrho, \tag{55}$$

where $k^r_t = \int k^r_{t,m} dm$. The social planner problem's FOCs with respect to $k^r_{j,t}$ implies

$$\alpha_r \Psi(i_t) \left(\frac{\mathcal{E}_{j,t}}{k^r_{j,t}} \right)^{1-\alpha_r} (1-\theta) \frac{y_t}{e_t} = \alpha_r \left(\frac{e^r_{j,t}}{k^r_{j,t}} \right) (1-\theta) \frac{y_t}{e_t} = \theta^k \frac{y_t}{k^g_t} \tag{56}$$

$$\alpha_r (1-\theta) \frac{e^r_{j,t}}{e_t} = \frac{1}{\beta} \frac{k^r_{j,t}}{y_{t-1}}. \tag{57}$$

The FOC with respect to k^f_t implies

$$\alpha_f (1-\theta) \frac{e^f_t}{e_t} = \frac{1}{\beta} \frac{k^f_t}{y_{t-1}}. \tag{58}$$

It is sufficient to show that

$$\beta (1-\theta) \left(\frac{\alpha_r e^r_{t+1} + \alpha_f e^f_{t+1}}{e_{t+1}} \right) = \varrho, \tag{59}$$

which is true if $\alpha_r = \alpha_f = \alpha$. ■

7.4 The Optimal Transition

Here we characterize the equilibrium allocation across the transition, and we derive some expressions that are used in our calibration. Let $V(k; A, L, \mathcal{E}, w; \Gamma^p, \Gamma^d)$ denote the value given k available units of capital and given that the aggregate productivity is A, the labor supply is l, the productivity in the renewable energy sector is $\mathcal{E}$, the stock of fossil fuel is w, and the stocks of permanent and depreciating emissions are Γ^p and Γ^d, respectively. We let g stand for the percentage productivity growth rate in the final good sector, while g^l is the population growth rate.

The optimal consumption and saving decision under log utility and full depreciation is given by $c = (1-\beta\Theta)y$, and $k' = \beta\Theta y$, where $\Theta = \theta_k + (1-\theta_k-\theta_\ell)\alpha$

is the marginal product of capital. The recursive formulation for $V(\cdot)$ is given by

$$\begin{aligned}
V\left(k; A, L, \mathcal{E}, w; \Gamma^p, \Gamma^d\right) \quad = \quad & \max_{i,f} \{\ln((1-\beta\Theta)y) \\
& +\beta V(\beta\Theta y; gA, g^l l, e^i\mathcal{E}, w-f; \Gamma^{p\prime}, \Gamma^{d\prime})\} \\
& \text{where} \\
& y = e^{-\pi(\Gamma^{p\prime}+\Gamma^{d\prime}-\bar{\Gamma})} A L^{\theta_\ell} \left(f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)^{(1-\alpha)(1-\theta_k-\theta_\ell)} k^{\Theta}, \\
& \Gamma^{p\prime} = \Gamma^p + \varphi_L f, \\
& \Gamma^{d\prime} = (1-\varphi)\Gamma^d + (1-\varphi_L)\varphi_0 f. \qquad (60)
\end{aligned}$$

Utilizing the envelope theorem, we have $V_k = \Theta\frac{1}{k} + \beta\Theta\frac{k'}{k}V'_{k'}$, which implies

$$kV_k = \Theta + \beta\Theta k' V_{k'}. \tag{61}$$

We guess that kV_k is a constant and we verify that

$$V_k = \frac{\Theta}{1-\beta\Theta}\frac{1}{k}. \tag{62}$$

Using the same method, we have that $V_A = \frac{1}{A} + \beta\left\{\frac{k'}{A}V'_{k'} + gV'_{A'}\right\}$, which, in turn, implies

$$AV_A = 1 + \beta\left\{\frac{\Theta}{1-\beta\Theta} + (gA)\,V'_{A'}\right\}. \tag{63}$$

Next, we guess that AV_A is a constant. As $A' = gA$, this equation allows us to verify that

$$V_A = \frac{1}{(1-\beta)(1-\beta\Theta)}\frac{1}{A}. \tag{64}$$

Similarly, we obtain

$$V_L = \frac{\theta_l}{(1-\beta)(1-\beta\Theta)}\frac{1}{L}, \tag{65}$$

$$V_{\Gamma^p} = \frac{1}{(1-\beta)(1-\beta\Theta)}(-\pi), \tag{66}$$

$$V_{\Gamma^d} = \frac{1-\varphi}{(1-\beta(1-\varphi))(1-\beta\Theta)}(-\pi). \tag{67}$$

The last expression reflects the depreciation rate of the temporary part of the emissions stock. Finally, the marginal value of stock of fossil fuel is given by

$$V_w = \beta \cdot V'_{w-f}.$$

The optimal choice of f on the equilibrium path implies

$$\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}\left(1+\beta\cdot k'V'_{k'}\right) \leq V_w + \tau\left\{\begin{array}{l} \pi\cdot\left(\varphi_L + (1-\varphi_L)\,\varphi_0\right)\left(1+\beta\cdot k'V'_{k'}\right) \\ -\beta\left[\varphi_L V'_{\Gamma^{p\prime}} + (1-\varphi_L)\varphi_0 V'_{\Gamma^{d\prime}}\right] \end{array}\right\},$$

with equality when $f > 0$. The left hand side of the above inequality gives the marginal benefit from consumption and from future capital accumulation, respectively. The first term of the right hand side, $V_w = \beta V'_{w-f}$, is the price of fossil fuel. The second term is the tax on consumption of fossil fuel. Note that $\tau \in [0,1]$, therefore, this tax could take any value from zero to the total value of present and future damages resulting from emissions. We can rewrite the above inequality as follows:

$$f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E} \geq \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\} + (1-\beta\Theta)V_w}, \tag{68}$$

with equality for $f > 0$. For $f, f' > 0$, using $V_w = \beta V'_{w'}$ and (68) we obtain:

$$\left(f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)^{-1} \;=\; \beta\left(f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'\right)^{-1} + (1-\beta)\frac{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}{(1-\alpha)(1-\theta_k-\theta_\ell)}. \tag{69}$$

We use equation (69) to find the equilibrium path of fossil fuel consumption by solving this path backward. To do so, we also need to determine the equilibrium path of the renewable energy productivity.

The optimal choice for i, when the representative agent takes into account only ξ fraction of the benefit of higher i on future renewable productivity, implies

$$0 = (1-\alpha)(1-\theta_k-\theta_\ell)\frac{\frac{1}{1-\alpha}\Psi'(i)\Psi(i)^{\frac{1}{1-\alpha}-1}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}\left\{1 + \beta\frac{\Theta}{1-\beta\Theta}\right\} + \beta\cdot\xi\underbrace{e^i\mathcal{E}}_{\mathcal{E}'}V'_{\mathcal{E}'} \tag{70}$$

or

$$\frac{-\Psi'(i)}{\Psi(i)}\frac{(1-\theta_k-\theta_\ell)}{1-\beta\Theta}\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} = \beta\cdot\xi\mathcal{E}'V'_{\mathcal{E}'}. \tag{71}$$

Utilizing the envelope theorem, we have

$$\mathcal{E}V_{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{1-\beta\Theta}\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} + \beta\mathcal{E}'V'_{\mathcal{E}'}. \tag{72}$$

Combining the above equation with (71) , we obtain

$$\frac{-\Psi'(i)}{\Psi(i)} = \beta\cdot\frac{\frac{\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}{f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}}{\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}}\left(\xi(1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')}\right). \tag{73}$$

Equation (73) shows how the evolution of the elasticity of the technology adoption cost with respect to the adoption rate, $\frac{-\Psi'}{\Psi}$, in two consecutive periods depends on the corresponding ratio of the share of renewable energy, $\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}$.

To determine the path of i and f, we begin by determining $\widehat{i}$, the long-run i. On a long-run balanced growth path, we have $f = f' = 0$, and $i = i' = \widehat{i}$, where $\widehat{i}$ is determined by

$$\frac{-\Psi'(\widehat{i})}{\Psi(\widehat{i})} = \frac{\beta}{1-\beta} \cdot \xi(1-\alpha). \tag{74}$$

The minimum $\mathcal{E}$ for which f is zero follows from (68):

$$\Psi(\widehat{i})^{\frac{1}{1-\alpha}} \underline{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}. \tag{75}$$

Note that $\underline{\mathcal{E}} < \infty$ only if $\tau > 0$. The representative agent could exhaust the stock of fossil fuel before the productivity of the renewable energy reaches $\underline{\mathcal{E}}$, in which case the price of fossil fuel will be positive. But once $\mathcal{E} \geq \underline{\mathcal{E}}$, agents will not exhaust the stock of fossil fuel.

In the period right before agents stop using fossil fuel, that is $f > f' = 0$, following (73) and given that $i' = \widehat{i}$, we have:

$$\begin{aligned}
\frac{-\Psi'(i)}{\Psi(i)} &= \beta \cdot \frac{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} \left(\xi(1-\alpha) + \frac{-\Psi'(\widehat{i})}{\Psi(\widehat{i})}\right) \\
&= \beta \cdot \frac{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} \cdot \frac{\xi(1-\alpha)}{1-\beta}.
\end{aligned} \tag{76}$$

Substituting $f' = 0$ in (69) to solve for $f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}$, and noting that $\mathcal{E} = e^{-i}\mathcal{E}'$, the above equation gives:

$$\frac{-\Psi'(i)}{\Psi(i)} \cdot \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}} = \beta \cdot \frac{1}{\frac{\beta}{\Psi(\widehat{i})^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} \cdot \frac{\xi(1-\alpha)}{1-\beta}, \tag{77}$$

which uniquely determines i, given the next period's productivity, $\mathcal{E}'$. Given i and utilizing (69), we have:

$$\tilde{f} = \frac{1}{\frac{\beta}{\Psi(\widehat{i})^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} - \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}}. \tag{78}$$

Note that $\tilde{f}$ is the maximum level of fossil fuel consumption before stopping using fossil fuel. That is, if the stock of remaining fossil fuel was larger than $\tilde{f}$, then the agents would leave some of the fossil fuel for consumption in the next period.[14] Thus, it is possible that the stock of remaining fossil fuel is in fact

[14] Equation (69) holds when both f and f' are positive, but it also holds if $f = \tilde{f}$ and (68) holds with equality for $f' = 0$.

lower than $\tilde{f}$. In such cases, equation (69) does not hold, since $f' = 0$ and (68) is an inequality. Nevertheless, for any value of $f < \tilde{f}$ we can determine i simply by noting that $\mathcal{E} = e^{-i}\mathcal{E}'$ and using:

$$\frac{-\Psi'(i)}{\Psi(i)} \cdot \frac{\Psi(i)^{\frac{1}{1-\alpha}} e^{-i}\mathcal{E}'}{f + \Psi(i)^{\frac{1}{1-\alpha}} e^{-i}\mathcal{E}'} = \beta \cdot \frac{\xi(1-\alpha)}{1-\beta}. \tag{79}$$

When f, $f' > 0$, using (69) to solve for $f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}$ in (73) and noting that $\mathcal{E} = e^{-i}\mathcal{E}'$ we obtain:

$$\begin{aligned}\frac{-\Psi'(i)}{\Psi(i)}\Psi(i)^{\frac{1}{1-\alpha}}\underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}} \quad &= \quad \beta \cdot \frac{1}{\frac{\beta}{f+\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} \cdot \frac{\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}{f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'} \\ &\times \left(\xi(1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')}\right), \end{aligned} \tag{80}$$

which allows us to uniquely determine i for a given $\mathcal{E}'$, i', and f'. Then, using i and (69), we obtain the equilibrium path f:

$$f = \frac{1}{\frac{\beta}{f+\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} - \Psi(i)^{\frac{1}{1-\alpha}}\underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}}. \tag{81}$$

By using this backward calculation we can determine the entire equilibrium path for all possible initial stock of fossil fuel and renewable productivity levels.[15]

Finally, if $f = 0$, we have:

$$V_{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{(1-\beta)(1-\beta\Theta)}\frac{1}{\mathcal{E}}. \tag{82}$$

Hence,

$$\begin{aligned} V\left(k; A, L, \mathcal{E}; 0, \Gamma^p, \Gamma^d\right) \quad &= \quad C + \frac{\Theta}{1-\beta\Theta}\ln k \\ &+ \frac{1}{(1-\beta)(1-\beta\Theta)}\left\{\ln A + \theta_l \ln L + (1-\alpha)(1-\theta_k-\theta_\ell)\ln\mathcal{E}\right\} \\ &- \frac{\pi}{(1-\beta)(1-\beta\Theta)}\Gamma^p - \frac{\pi(1-\varphi)}{(1-(1-\varphi)\beta)(1-\beta\Theta)}\Gamma^d, \end{aligned} \tag{83}$$

where C is a constant. Note that log utility, full depreciation, and the structure of the damage function imply that the above expression is linear in Γ^p and Γ^d.

[15] We can show that going backward, i converges to $\hat{i}^f$ determined by

$$\frac{-\Psi'(\hat{i}^f)}{\Psi(\hat{i}^f)} = \frac{\beta e^{\hat{i}^f}}{1-\beta e^{\hat{i}^f}}(1-\alpha),$$

where $\hat{i}^f > \hat{i}$.

7.5 Lack of Commitment

In order to explore the implications of commitment, here we briefly consider an extension of the model in the main text. Suppose that the government experiences "electoral death" with probability ω in each period. More precisely, for any period t, with probability $\omega \in (0,1]$, the government learns at the end of t, that it will not be around at the beginning of period $t+2$. The implied discounting sequences for the government, β^G, and for the representative agent, β^A, respectively, are given by:

$$\begin{aligned} \beta^G &= \{1, \beta, \beta^2(1-\omega), \beta^3(1-\omega)^2, ...\} \\ \beta^A &= \{1, \beta, \beta^2, \beta^3, ...\} \end{aligned} \tag{84}$$

We assume lack of commitment.[16] Thus, at the beginning of each period, the taxes and subsidies are set for the current period. We examine the case where $\xi = 1$, so there are no technology spillovers and the only externality is the one associated with GHG emissions. In each period, the government chooses the Pigouvian tax rate on fossil fuel consumption, τ_t^f. Under no commitment, this tax needs to be set in a time-consistent fashion. The optimal allocation can again be supported by a Pigouvian tax on emissions. This tax is lower than the case studied in the earlier model.

The government's objective is a modified version of the planner's objective in the previous section and it is given by:

$$u(c_0) + \sum_{t=1}^{\infty} (1-\omega)^{t-1} \beta^t u(c_t) \tag{85}$$

All feasibility constraints remain the same. As a result, the optimal allocation is characterized by similar FOCs as in the previous section, with the only modification being in the discount sequence. We can demonstrate the following.

Proposition 4 *(1) The government's optimal allocation can be supported by a Pigouvian tax given by* $\tau_t^f = \pi_t y_t^*(1-d_0) + \sum_{j=1}^{\infty}(1-\omega)^{j-1}\beta^j \frac{u'(c_{t+j}^*)}{u'(c_t^*)} \pi_{t+j} y_{t+j}^*(1-d_j)$*, where* $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the government's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$*. (2) If* $u(c) = \log(c)$, $\alpha_r = \alpha_f = \alpha$, $\pi_t = \pi$*, all* t*, and* $\delta = 1$*, then* $\tau_t^f = y_t \pi \left[\varphi_L + (1-\varphi_L)\varphi_0 + \frac{\beta \varphi_L}{1-(1-\omega)\beta} + \frac{\beta(1-\varphi)(1-\varphi_L)\varphi_0}{1-(1-\omega)(1-\varphi)\beta}\right]$ *does not depend on the growth rate of the economy. (3) The tax is is strictly lower than the optimal Pigouvian tax of the previous section for all* t*.*

[16] See Harstad (2019) for a discussion of time inconsistency in a related model.

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