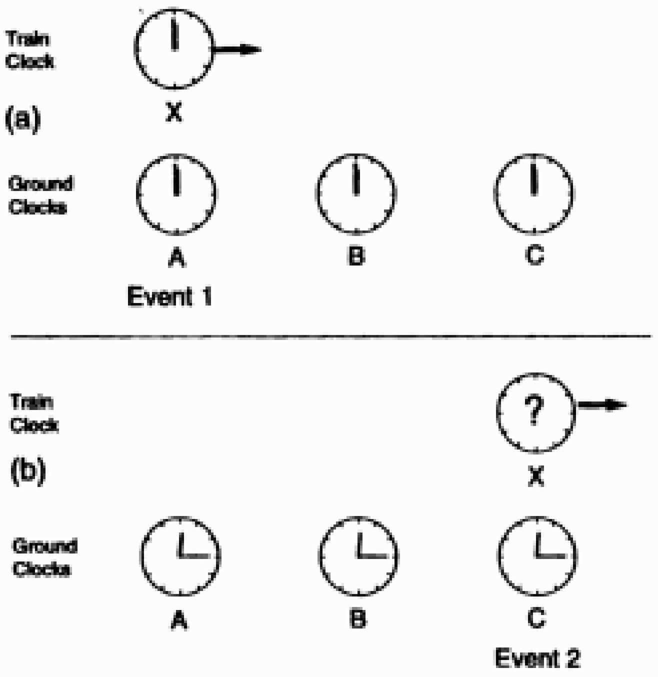
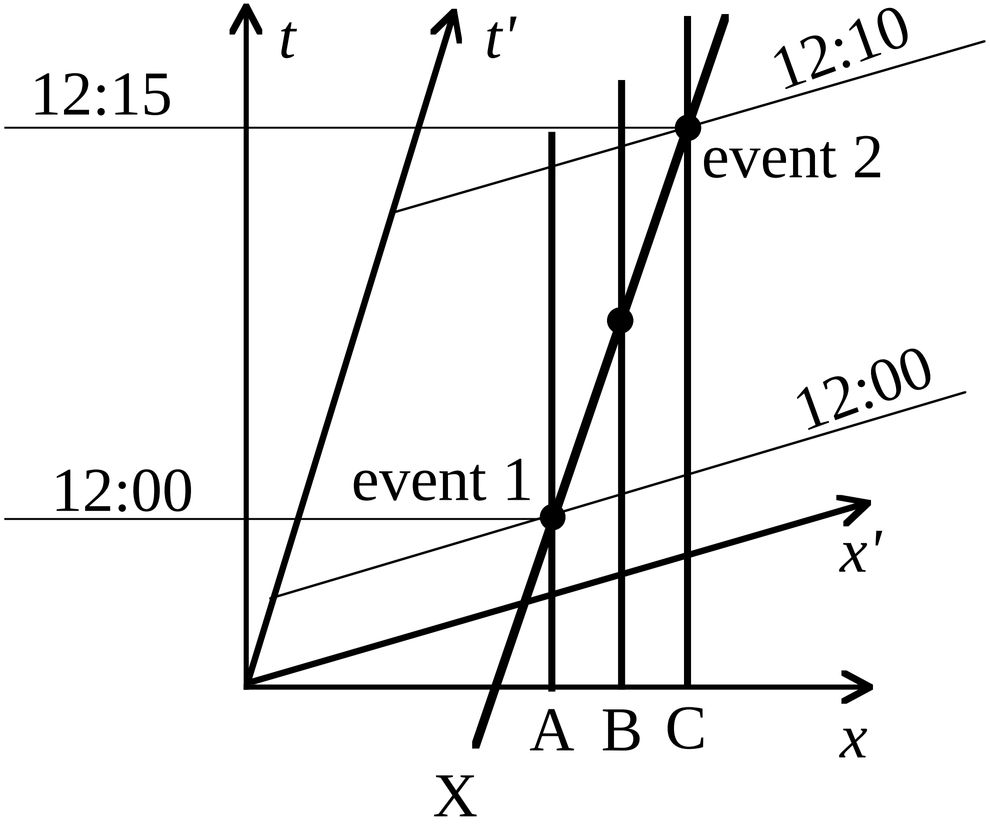


Fig. 3.10. Relativistic description of a moving clock viewed by a sequence of stationary observers. (a) Train clock *X* is adjacent to ground clock *A* (event 1); both clocks read 12:00. (b) Some time later, *X* is adjacent to ground clock *C* (event 2). The difference between the reading of *C* in sketch (b) and that of *A* in sketch (a) represents the interval between events 1 and 2 in the ground frame. With the help of fig. 3.11, one finds that *X* has lost time between the two events: it is running slow.



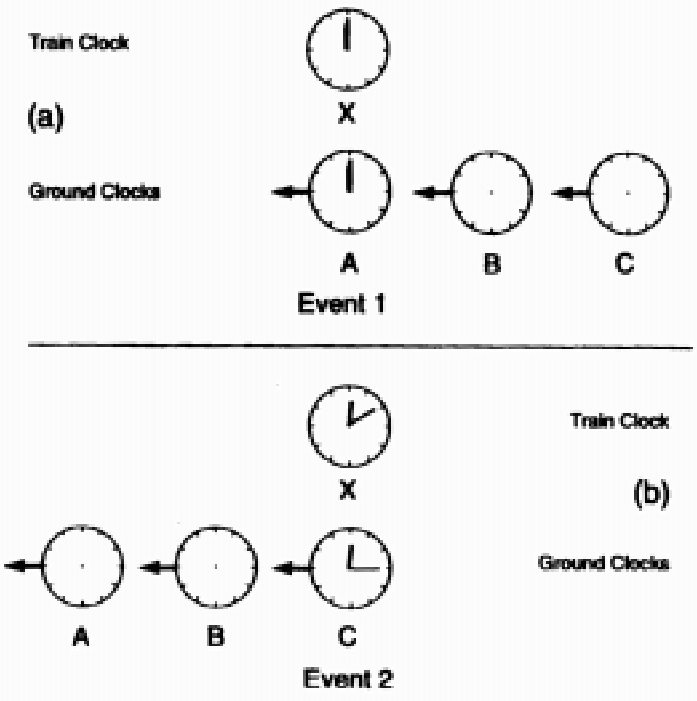
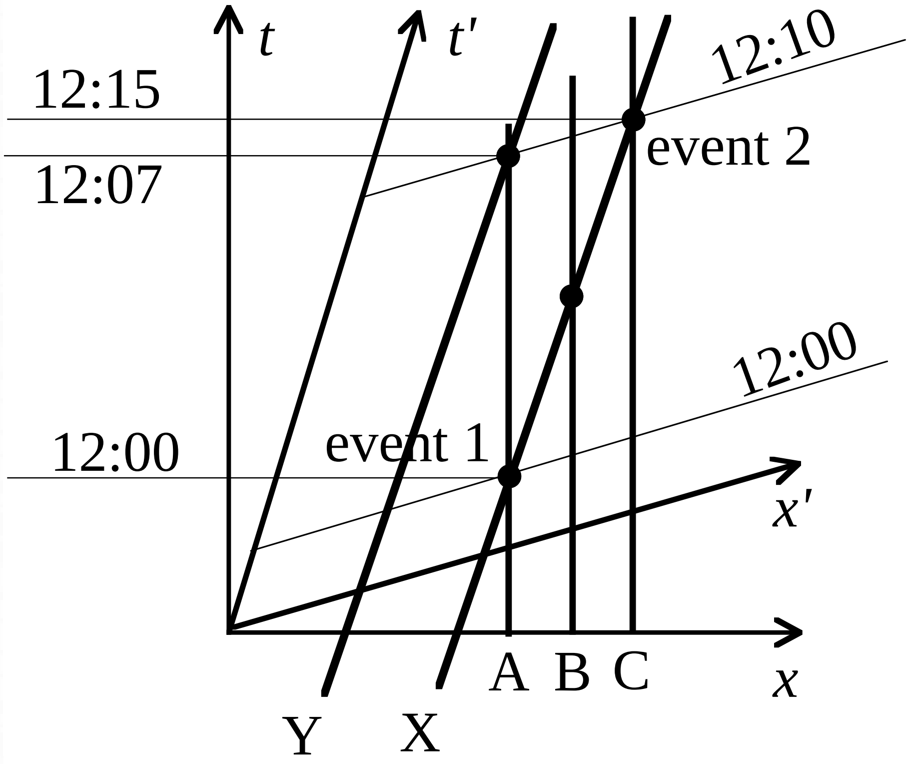
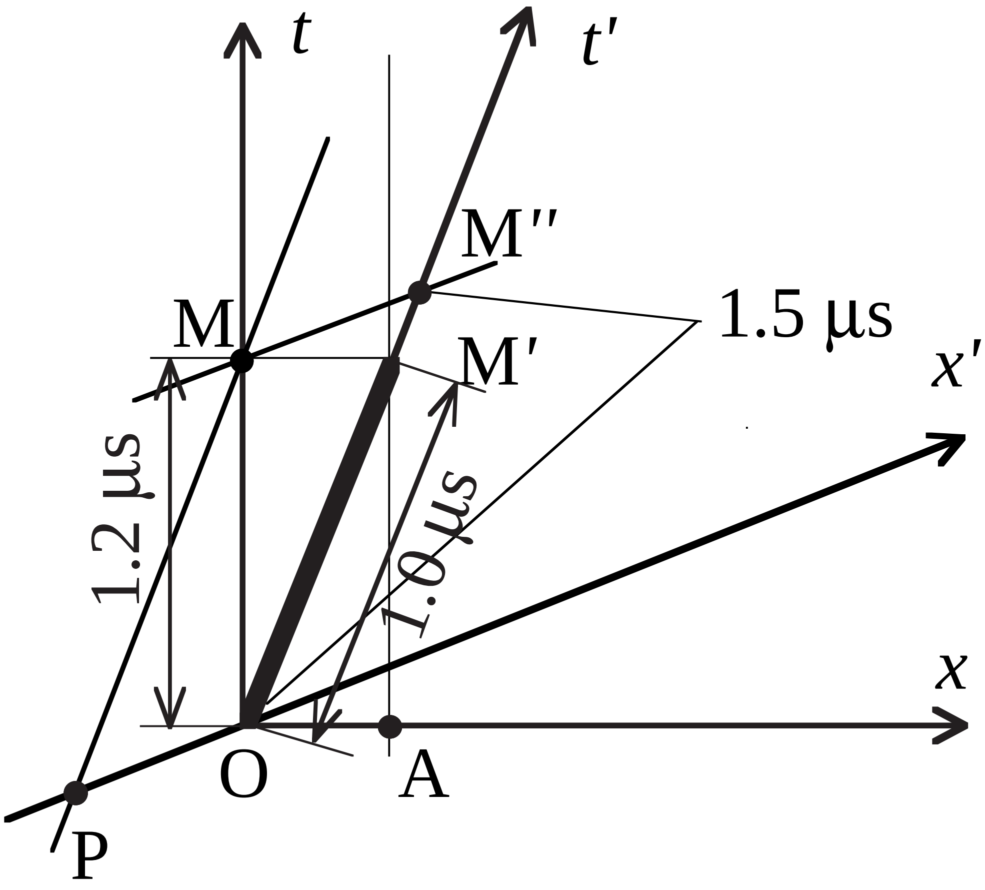


Fig. 3.11. The events of fig. 3.10 are shown as they appear in the train frame. The two events occur at the same place; hence the time interval between them is proper and is shorter than the interval between the same events measured in the ground frame. The proper time interval is read directly off a single clock, *X*.





OM′ — the world-line of the muon: is born at O, decays at M′.

In its rest frame (x',t'), the muon lives 1.0 μs, i.e. the time elapsed between events O and M′ is 1.0 μs when measured in (x',t') frame (say, by a clock moving with the muon).

We say that the muon time is *dilated* when measured in the Earth frame. How do we measure this effect? We need to compare the readings of the muon clock with the readings of our clocks. Since the muon is moving (this is the whole point, right?), we need *two* synchronized Earth clocks situated in such a way as to register the birth and the decay of the muon. These two events (i.e. O and M′) must occur in the Earth's clocks' "immediate vicinity." So we need to run two clocks that are at rest in the Earth frame through events O and M′: clocks OM and AM′. Since they are in sync in the Earth frame, we can compare their readings at points O and M′ and take the difference. This difference is equal to OM and is about 1.2 μs (let's say). (Thus OM is actually longer than OM′, even though it appears shorter in our Euclidean diagram). Based on our observations, we conclude that *time goes slower in the muon frame*, because it's moving with respect to us and we know that motion involves time dilation.

But the muon observer could object to this imperialist way of thinking by noting that he has equal rights to say that HE is at rest and OUR frame is moving (in the opposite direction). Hence, our time must be dilated. So WHOSE time is REALLY dilated?

The answer is that the question is ill-posed. The muon's time is dilated in OUR frame, and our time is dilated in the muon's frame. In order to measure how our time behaves in the muon frame, the muon observer must compare the time readings of two clocks stationary in ITS frame (and hence properly synchronizable) at events at which they meet the world-line of one of our clocks. Let our clock be that represented by OM and the events in question be O and M. The muon observer would have to run two *different* synchronized clocks that are at rest in his frame through O and M—say, clocks OM″ and PM, compare their readings at O and M, and then take the difference of those readings. This difference is represented by OM″ and is 1.44 μs. Thus our time is indeed dilated in the muon frame.