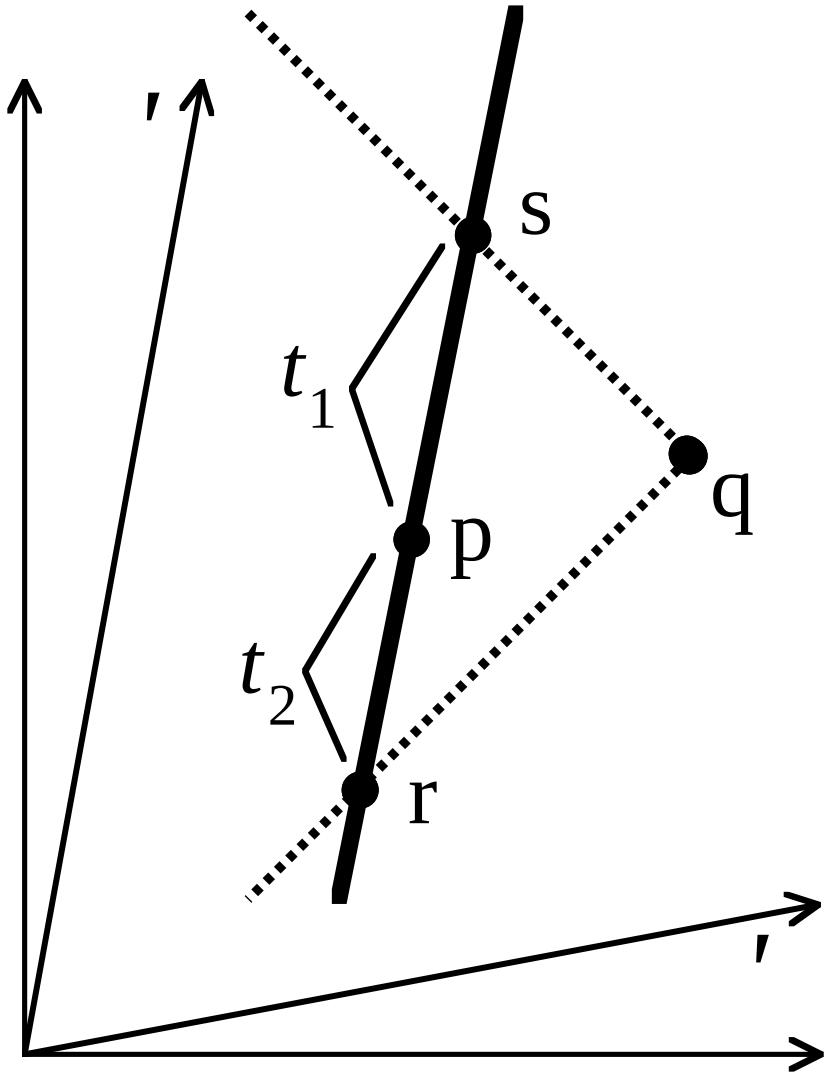
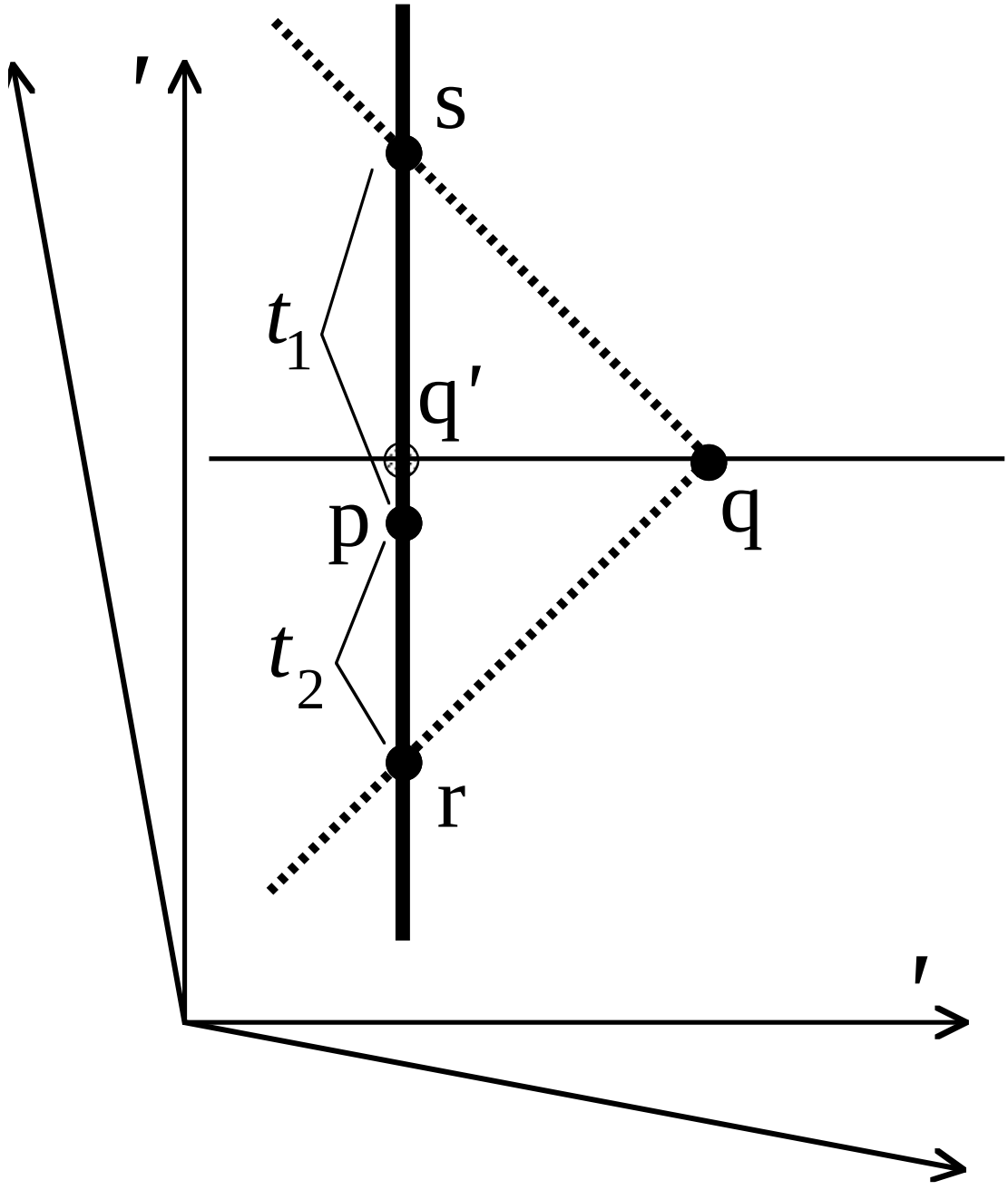
Building SR from Scratch: The Interval



$$t_2 = t\,(\mathrm{p}) - t\,(\mathrm{r})$$
$$t_1 = t\,(\mathrm{s}) - t\,(\mathrm{p})$$

- How to obtain Δx and Δt between p and q based on <u>just</u> t_1 and t_2?



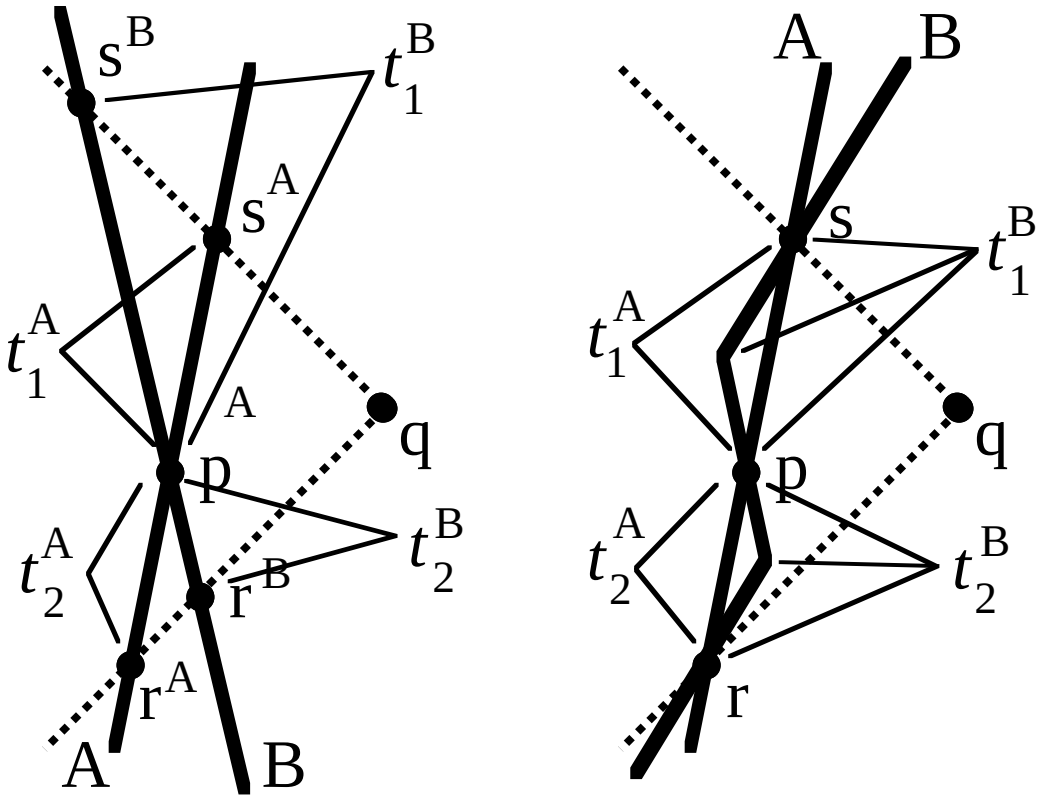
$$\Delta x = 1/2c\ (t_1 + t_2)$$

$$\Delta t = 1/2(t_1 + t_2) - t_2 = 1/2t_1 + 1/2t_2 - t_2$$
$$= 1/2t_1 - 1/2t_2 = 1/2(t_1 - t_2)$$

- "Spatial distance between p and q" (<u>according to **this** clock</u>) = $1/2c\ (t_1 + t_2)$
- "Elapsed time between p and q" (<u>according to **this** clock</u>) = $1/2(t_1 - t_2)$

Are t_1 and t_2 *intrinsic* to space-time? — NO!

(1) (2)



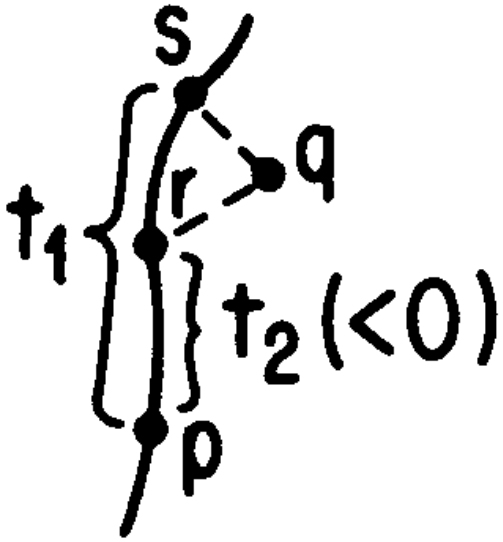
- t_1 and t_2 "encapsulate" features (of the relationship between p and q) intrinsic to ST *and* those associated with a particular choice of clocks. [Analogy with distance]

- We now need to “separate out the former” (i.e., features of the relationship between p and q intrinsic to ST) and “clearly post it within ST.”

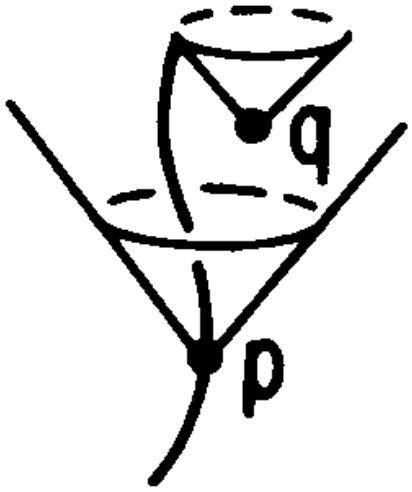
Recipe:

- Suppose you are given two arbitrary events, p and q (and nothing else!).
- Help yourself to a certain clock (i.e., a world-line with time function t (x) attached to it; ‘x’ just denotes a point on the clock line) and light pulses (i.e., dashed world-lines *always* inclined (we know it from experiment) at 45° in ST).
- Get $t_1 = t$ (s) – t (p) and $t_2 = t$ (p) – t (r).
- We know that:
  - given these arrangements, t_1 and t_2 are *uniquely* “generated” by p and q and, hence, *somehow* describe the relationship between these events in ST; but
  - this description is “contaminated” by “extrinsic” elements peculiar to a particular choice of the clock.
- Try to make a good guess as to how to *disentangle* this extrinsic admixture = try to guess at some *combination* of t_1 and t_2 that would ***not*** depend on the choice of a clock.

Five Cases
(A)



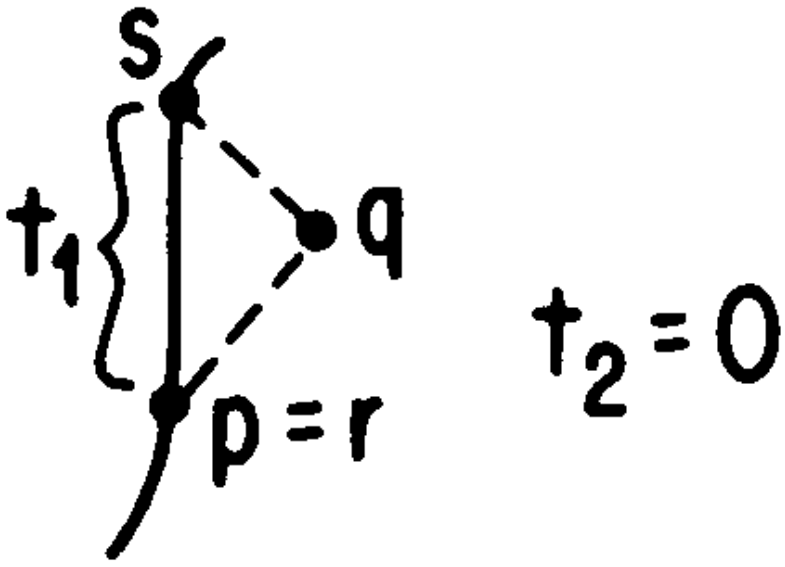
$t_1 = t\,(\mathrm{s}) - t\,(\mathrm{p}) > 0$
$t_2 = t\,(\mathrm{p}) - t\,(\mathrm{r}) < 0$



q is *inside* the light cone of p

Events p and q are *timelike* related (or *timelike* separated)

(B)



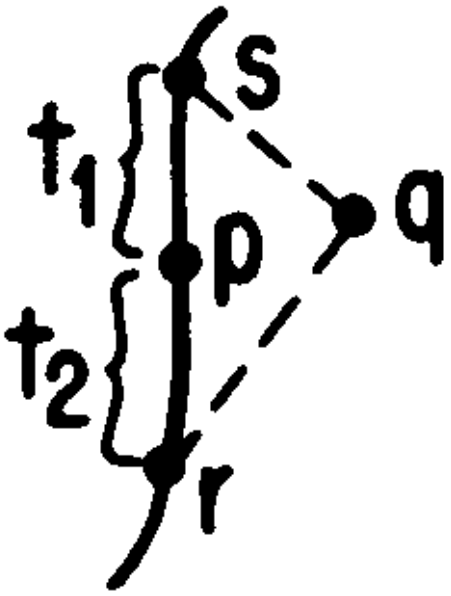
$t_1 = t\,(\mathrm{s}) - t\,(\mathrm{p}) > 0$
$t_2 = t\,(\mathrm{p}) - t\,(\mathrm{r}) = 0$

q is *on* the light cone of p

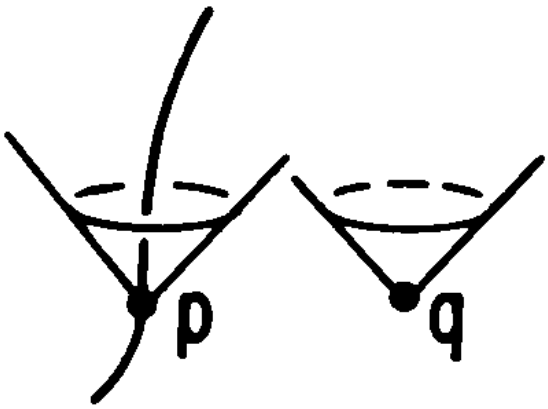
Light sent from p reaches q

Events p and q are *lightlike* related (or *lightlike* separated)

(C)



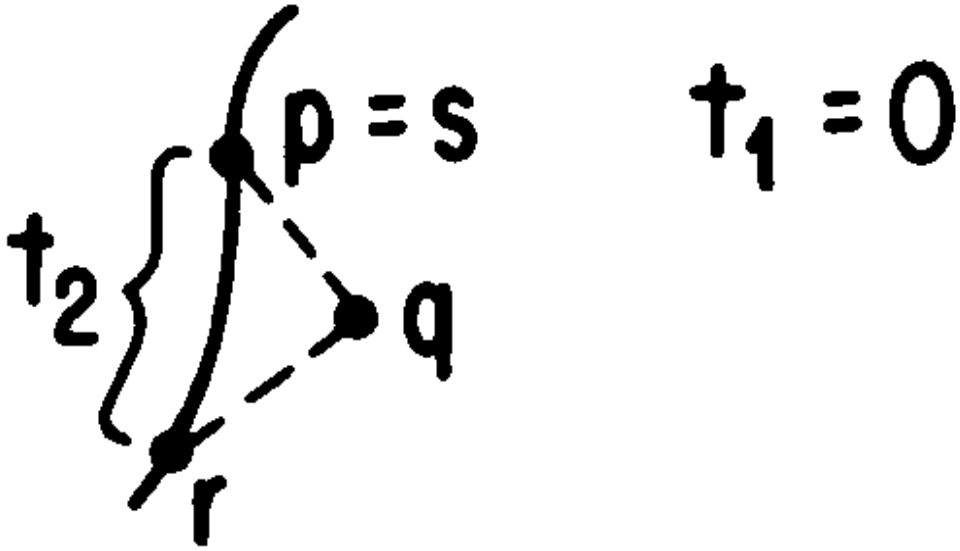
$t_1 = t\,(\mathrm{s}) - t\,(\mathrm{p}) > 0$
$t_2 = t\,(\mathrm{p}) - t\,(\mathrm{r}) > 0$



p and q are *outside* of each other's light cones

Events p and q are *spacelike* related (or *spacelike* separated)

(D)



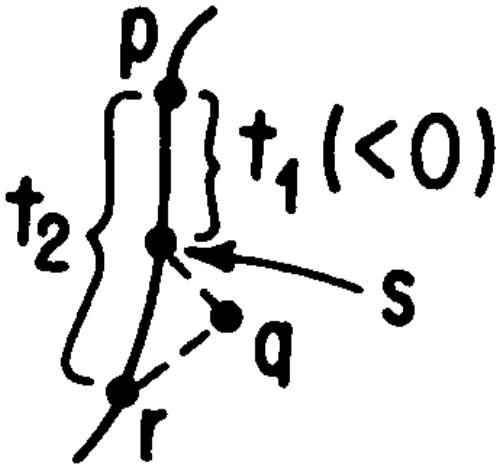
$t_1 = t\,(\mathrm{s}) - t\,(\mathrm{p}) = 0$
$t_2 = t\,(\mathrm{p}) - t\,(\mathrm{r}) > 0$

p is *on* the light cone of q

Light sent from q reaches p

Events p and q are *lightlike* related (or *lightlike* separated)

(E)



$t_1 = t\,(\mathrm{s}) - t\,(\mathrm{p}) < 0$
$t_2 = t\,(\mathrm{p}) - t\,(\mathrm{r}) > 0$

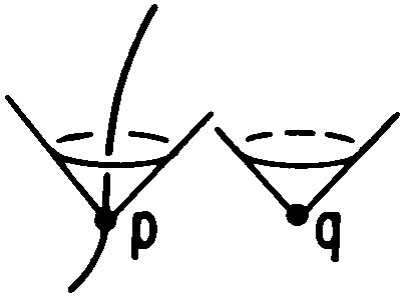
p is *inside* the light cone of q

Events p and q are *timelike* related (or *timelike* separated)

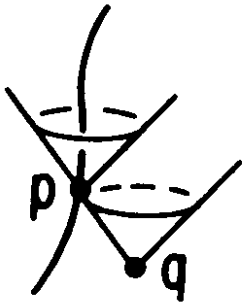
- $t_1 > 0$, $t_2 < 0$
- q is *inside* the light cone of p
- q and p *timelike* related

- $t_1 > 0$, $t_2 = 0$
- q is *on* the light cone of p
- q and p *lightlike* related

- $t_1 > 0$, $t_2 > 0$
- q and p are *outside* the light cones of each other
- q and p *spacelike* related



- $t_1 = 0$, $t_2 > 0$
- p is *on* the light cone of q
- q and p *lightlike* related



- $t_1 < 0$, $t_2 > 0$
- p is *inside* the light cone of q
- p and q *timelike* related

The Interval = $t_1 t_2$

p and q timelike related $\leftrightarrow$ $t_1 t_2 < 0$
p and q spacelike related $\leftrightarrow$ $t_1 t_2 > 0$
p and q lightlike related $\leftrightarrow$ $t_1 t_2 = 0$

- Whether p and q are time- or space- or lightlike related is *objective* and *does not depend* on the choice of a clock.
- Given p and q (and nothing else), there is a *fact of the matter* about whether they are time- or space- or lightlike related.
- The *sign* of the Interval (i.e., >0, <0, or =0) expresses this objective relationship between events p and q.

In fact, not only the sign, but also the *value* of the Interval is *invariant*, hence *objective*, hence *intrinsic* to ST.

[Geroch's handwaiving]