Building SR from Scratch: The Interval

- Build SR "from scratch"? Why?

☛ Lessons from the study of the Aristotelian and Galilean space-times:

- ST can be endowed with intrinsic structure.
- The question of how much intrinsic structure ST has is intimately related to the question of what statements about physical objects and relationships between events make and which don't make sense: the more structure ST has, the more such statements make sense.
- The structure must "reflect" the laws of nature.

Example: Aristotelian Galilean

- Works well in mechanics: Netwonian mechanics is Galilean-invariant.
- But some electromagnetic phenomena are *not*: apparently, they allow one to distinguish the state of rest from the state of motion (by associating the state of rest with the reference frame of the ether).
- But the Galilean space-time does *not* have enough structure to "support" that distinction.

Options:

- Select electromagnetic phenomena for special treatment, leave them non-invariant and hence "non-accommodatable" within space-time;
- Try to make both mechanics and electromagnetism invariant by reconsidering, once again, the intrinsic structure of ST.

- Adopting (2) leads to counter-intuitive ideas about space and time and to questioning implicit assumptions about physical measurements of relationships between events (e.g. simultaneity).
- All such assumptions, from now on, have to be grounded in *precisely specified operational procedures*. Can't rely anymore on rods, clocks (assumed to tick uniformly and be always in sync).
- On the other hand, the propagation of light displays a remarkable constancy and uniformity in its behavior in all inertial reference frames.
- We want now to have it do the job of demarcating the structure of space-time anew for us, without making any problematic or questionable assumptions about its physical behavior.

Building “from scratch”—a Cartesian project?

- Previous steps: began with a very rigid structure of the Aristotelian space-time; then moved to a more flexible Galilean structure by allowing arbitrary “beveling-of-the-deck” transformations.
- Now need to really go to the extreme in this direction of depriving space-time of its intrinsic “rigidity.”
- “Rubber sheet” geometry of space-time: can be “stretched, pulled, and bent” in all directions.

Retain, at this point, only the broad qualitative features of the 4D space-time manifold:

- The notion of an event and the idea of assembling all events into a 4D manifold;
- Particles are represented by lines in ST, ropes by world-surfaces, etc., collisions of particles are represented by intersections of world-lines, etc.

- Our manifold is still 4D (i.e. we need four numbers to characterize an event), but we don’t know yet how to get these numbers = Events are “there” to be labeled with coordinates, but we don’t know (yet) how to label them.
- Need to use non-trivial procedures to “rigidify” our “rubber sheet” (i.e. to somehow label events with coordinates), procedures that will rely on the propagation of light and on measuring local times.
- Need to be guided by empirical considerations: Whatever structure we may eventually build must be consistent with observations and experiment.

"We must somehow work into this broad framework some hard, numerical, geometrical information about space-time. We need something to replace the old spatial distances, elapsed times, and so forth. Here is where we wish to be ultracautious. The problem (it is now known) is that we were entirely too cavalier in describing, for example, the original Aristotelian setup. We allowed ourselves "watches" without further discussion, and we implicitly assumed a number of properties of these "watches" (some of which, as we shall see shortly, are just not true in our world). We allowed ourselves "meter sticks" … with implicitly assumed properties. We in effect allowed ourselves to "just know" about space-time around us without a careful physical prescription for what is to be done" (Geroch, p 69).

From now on:

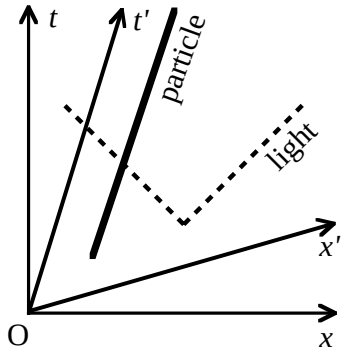
- Will be very selective in our choice of measuring instruments.
- Will require that the instruments *themselves* be represented within our space-time.

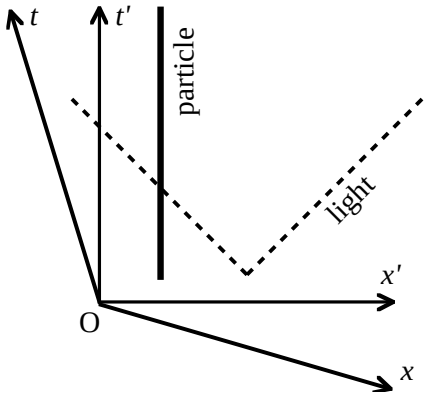
Need two types of instruments:

- One, to reach out into ST, away from our world-line, to "collect information" about ST, and to carry it back to us;
- Another, to "record" the information brought back.

- Meter sticks? (No good. Why?)
- Particles? (No good. Why?)

- Light rays! (Yes! Why?)



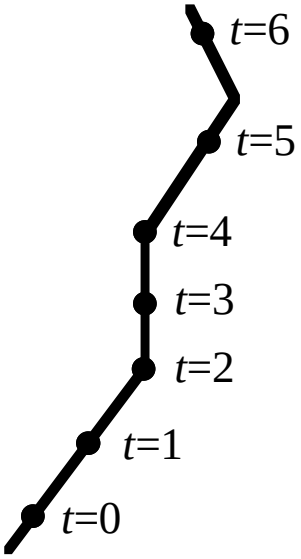


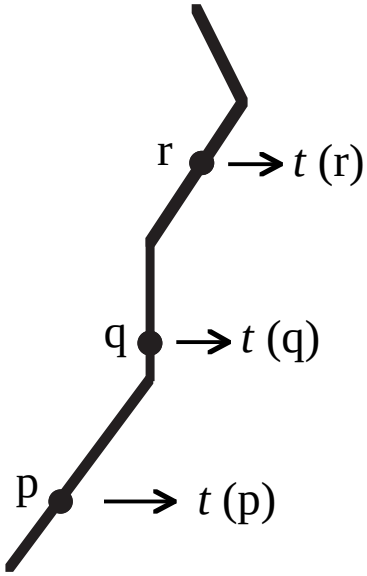
- Local clocks (very small—point-like and perfectly similar to one another)

- A light ray (pulse) is represented by a (dashed) world-line.
- A (local) clock is represented by a world-line endowed with a function that assigns a number (i.e., a clock reading: the time, according to this clock, at event in its immediate vicinity) to each point of the clock's world-line.
- Clocks only assign times to events on their world-lines.
- All world-lines of particles acquire time-functions.

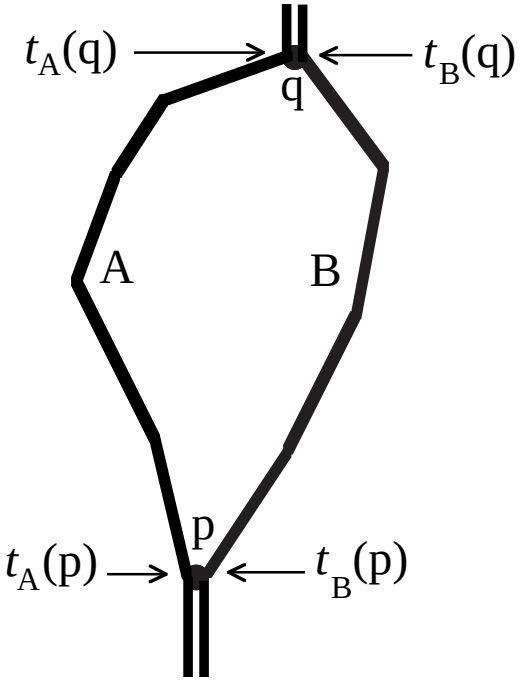
[Straight vs. curved lines]

Clocks = world-lines with time functions attached to them





Time elapsed between p and q = t (q) – t (p)



Time elapsed between p and q, *as measured by clock A*:

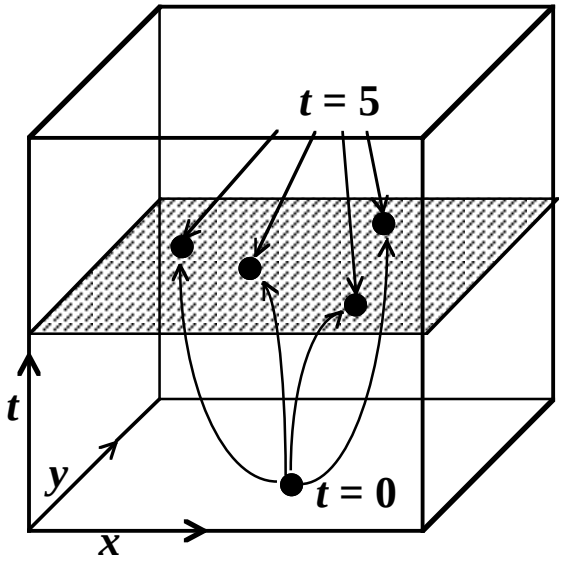
$$t_A = t_A\,(q) - t_A\,(p)$$

Time elapsed between p and q, *as measured by clock B*:

$$t_B = t_B\,(q) - t_B\,(p)$$

$$\boxed{t_A = t_B?}$$

☛ $t_A = t_B$ would lead to reinstating absolute time (i.e., the structure of the "decks") and, hence to absolute simultaneity of spatially separated events.

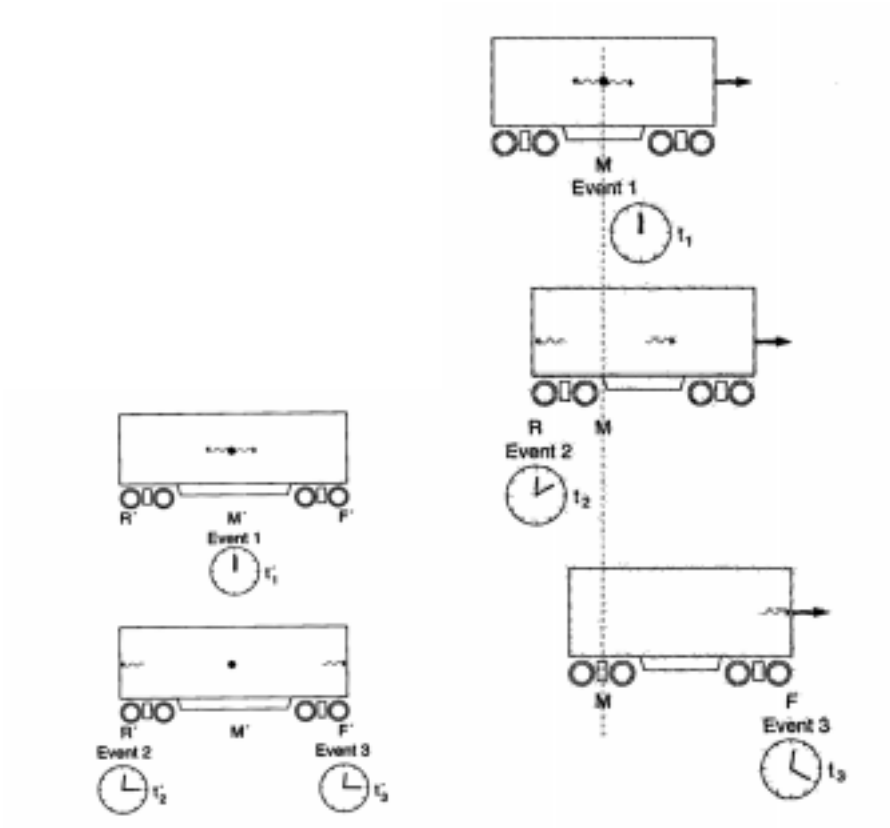


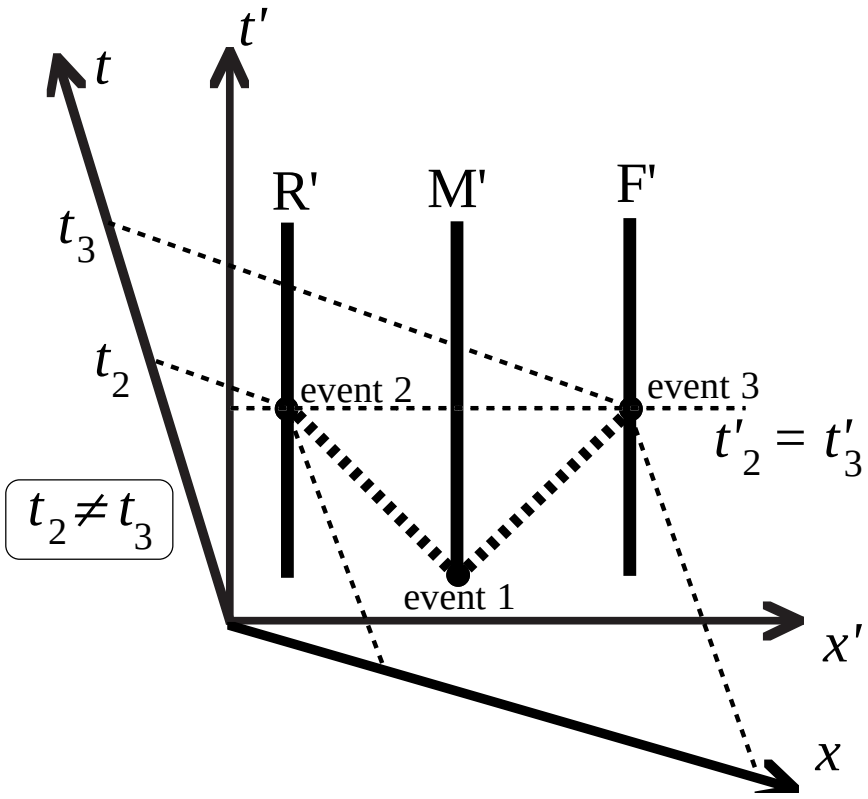
Moral:

- The motion of a clock *affects* its "ticking rate": Time depends on velocity. (Past history of a clock does *not* affect its present "ticking rate.")

Local or *proper* time for a particle = time measured by the clock moving with the particle.

"Given any world-line of a particle, one acquires an assignment of times to points of that world-line. (Physically, the assignment is obtained by carrying a clock alongside the particle, and using the readings of the clock to obtain the "times.") This assignment of "times" to the points of the world-line of any particle is unique as far as time differences are concerned" (Geroch, pp. 79–80).


M
Event 1
t_1
R
M
Event 2
t_2
M
F
Event 3
t_3
R'
M'
F'
Event 1
t'_1
R'
M'
F'
Event 2
t'_2
Event 3
t'_3


t'
t
R'
M'
F'
t_3
t_2
event 2
event 3
$t'_2 = t'_3$
$t_2 \neq t_3$
event 1
x'
x