From the Aristotelian to Galilean Space-Time

Recap:

Last time we have completed our study of the Aristotelian space-time framework. This framework endows space-time with a rather rich structure that allows one to characterize all events in the world in terms of their spatial and temporal locations. This enables one to provide a precise geometrical characterization of all physical processes in terms of the corresponding collections of events in space-time. Thus we represent particles (including "particles" of light) by their world-lines, their collisions by intersecting world-lines, ropes and sticks by world-surfaces, etc. Once we have drawn these configurations, we know everything there is to know about life histories of these objects.

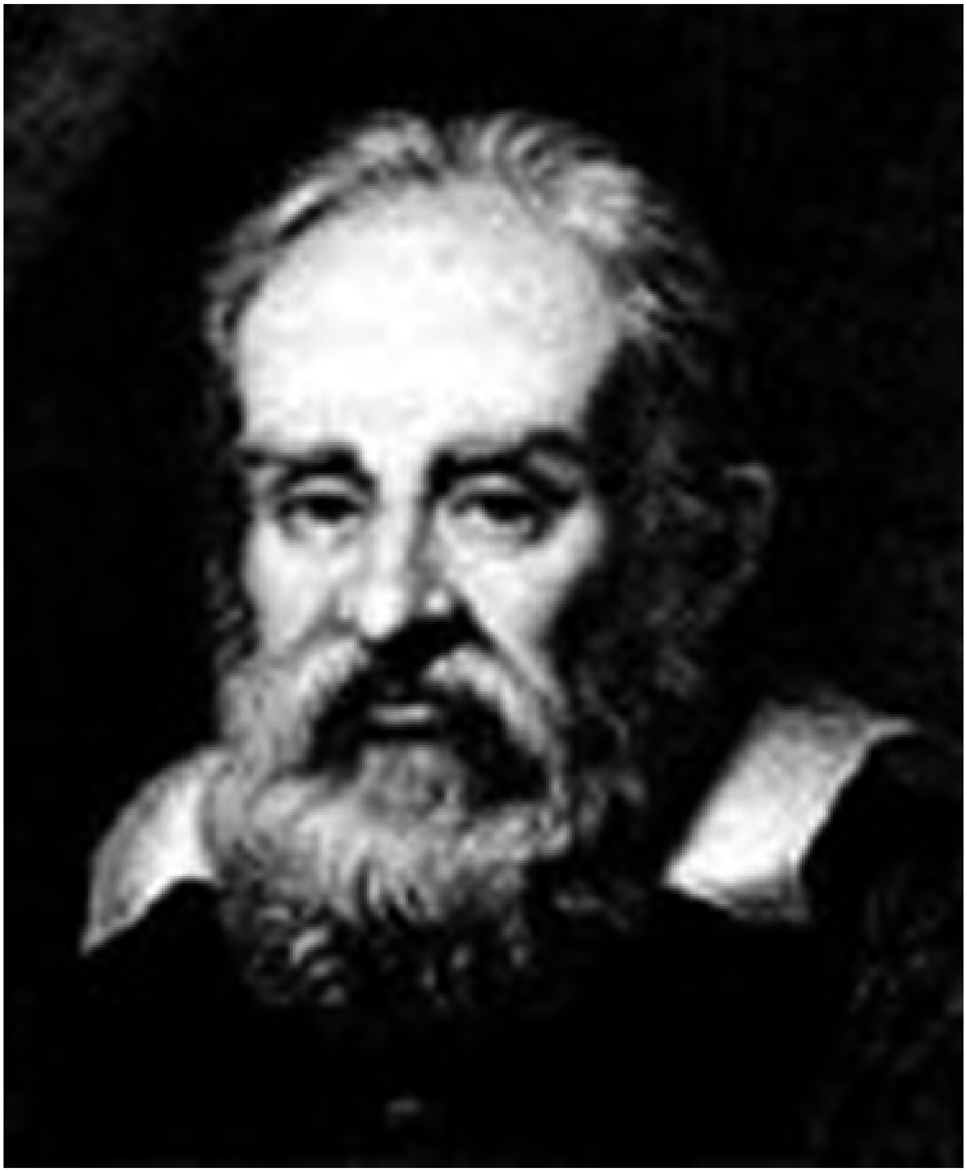
Now the facts just noted: that the life histories of particles are represented by lines, and their collisions by intersecting lines, etc.—these facts have nothing Aristotelian about them. They have a universal significance, because they express very general ideas about geometrical constructions in a space-time framework as such, not any particular view of its structure.

What IS Aristotelian about a particular set-up with which we've been working so far is the fact that it is a very personalized set-up, closely connected with the location of a particular single observer or, rather, a particular set of observers (remember all those badges and watches). Associated with this set is a particular coordinate system that allows them to assign spatial coordinates to every event and then use their watches to assign the times of their occurrences.

We have finished last time by noting two things: (1) that this Aristotelian set-up reflects perfectly well common-sense, kitchen-type intuitions about space and time but (2) that it is inadequate for the purposes of physics. The reason it is inadequate is, roughly, that it's very undemocratic: it represents things from a single point of view. It thereby gives undue importance to a particular set of Aristotelian observers: they tend to consider themselves as having a privileged status in the universe, because everything else is described in reference to them. But physically speaking, there is nothing special about them. Any other similar set of observers would do just as well. For example, observers moving with respect to the first set with a particular velocity have precisely the same right to consider themselves as having a privileged status. We noted that one way to proceed might be just to let them do so, that is, to incorporate into the physical picture of the world the whole multitude of various Aristotelian descriptions of the same situation or phenomenon, from all possible points of view. Every point of view would then carry its own space-time structure, featuring straight lines as markers of positions and horizontal 3-planes as markers of times. For example, to characterize the motion of a certain particle with a constant velocity, we would have to provide an infinite number of space-time diagrams, each representing the world-line of the particle in a particular Aristotelian setting. In one such diagram that line would be exactly vertical, thus representing the particle at rest. In others, it would be inclined at different angles and in various directions. It is important to understand that we would need ALL such diagrams to represent a single phenomenon or process, because none of those diagrams would be any "better" than any other and, hence, none could to be preferred to any other. Such a description would be infinitely cumbersome and incomprehensible. Even more importantly, it would be physically unreasonable. In physics, we try to come up with an economical description of nature. And we do so by extracting from the phenomena their most general, universal features and codifying them in the form of laws, such as the laws of motion. It would thus be most reasonable to choose a space-time structure that would be adequate for incorporating these universal features. But the structure of the Aristotelian space-time is simply too individualized to be able to do it. Instead of an economical universal description it gives us an infinite plurality of highly individualized descriptions.

It is a law of classical physics, for example, that a free particle (that is, a particle unaffected by any forces) must move along a straight line with constant velocity. It is NOT a law of physics that such a particle must move with any PARTICULAR constant velocity. But any individual Aristotelian framework is set up in such a way as to always attribute a particular velocity to a particle, by representing its world-line as inclined at a certain angle. In general, the Aristotelian space-time has too much intrinsic structure, a lot more than is needed to “bring out” the universal laws of nature. What we now want to do is to take some of this structure away, namely, a part of it that is associated with individual Aristotelian set-ups. We want to impose a democracy on these set-ups. We want to de-individualize it. The result, as you might guess, will be what Geroch calls the Galilean space-time, or framework.

Galileo Galilei (1564–1642)



Galileo’s Principle of the Relativity:

The laws of physics are the same in any inertial reference frame.

- Inertial reference frame = a reference frame at rest or in motion with constant velocity with respect to another inertial frame

☛ Nature does not “discriminate” among inertial reference frames

Fragment from *Dialogues Concerning the Two Chief World Systems,* by Galileo Galilei (tranls. By Stillman Drake):

Salviati:

Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals.

Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it.

With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin.

The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need to throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction.

When you have observed all of these things carefully (though there is no doubt that when the ship is standing still everything must happen this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that.

You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.

In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than towards the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump.

In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite.

The droplets will fall as before into the vessel beneath without dropping towards the stern, although while the drops are in the air the ship runs many spans.

The fish in the water will swim towards the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl.

Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. . . .

Sagredo:

Although it did not occur to me to put these observations to the test when I was voyaging, I am sure that they would take place in the way you describe.

In confirmation of this I remember having often found myself in my cabin wondering whether the ship was moving or standing still; and sometimes at a whim which I have supposed it going one way when its motion was the opposite.

Isaac Newton (1642–1727)

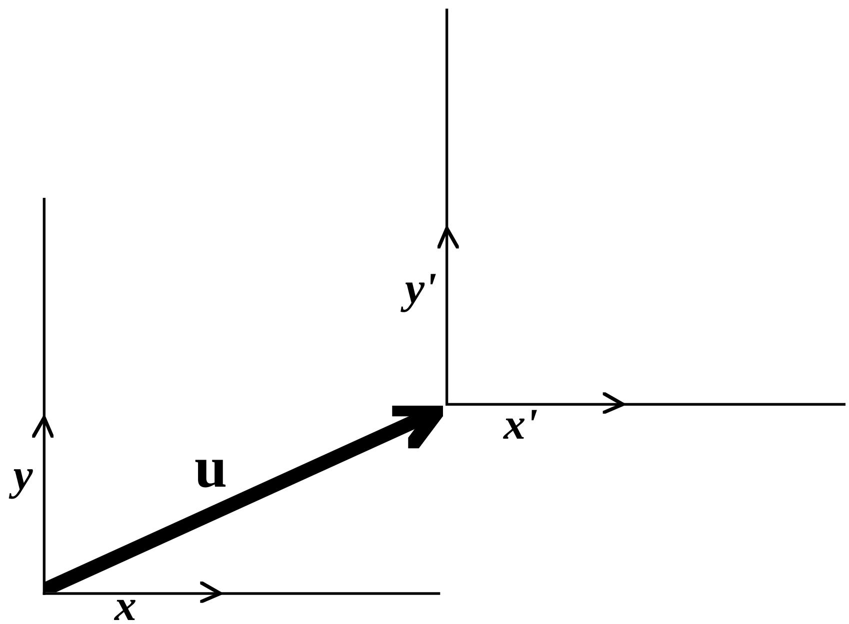
Newton's First Law of Motion:

A body unaffected by force retains its state of rest or motion with constant velocity.

Newton's Second Law of Motion:

The acceleration of a body is proportional to the net force exerted on it: F = ma

Inertial (Galilean) Coordinate Transformation



$$x' = x - u_x t \qquad v_x' = v_x - u_x$$
$$y' = y - u_y t \qquad v_y' = v_y - u_y$$
$$z' = z \qquad v_z' = v_z$$
$$t' = t$$

$$a_x' = \frac{dv_x'}{dt'} = \frac{d(v_x - u_x)}{dt} = \frac{dv_x}{dt} - \frac{du_x}{dt} = a_x - 0 = a_x$$
$$a_y' = a_y$$
$$a_z' = a_z$$

- ☛ Newton's laws of motion don't make reference to any individual Aristotelian set-ups.
- ☛ Nature cares about the general laws of motion, not about individual Aristotelian perspectives.

We need to find a space-time framework where we could "accommodate" these laws as part of the general structure of space-time.

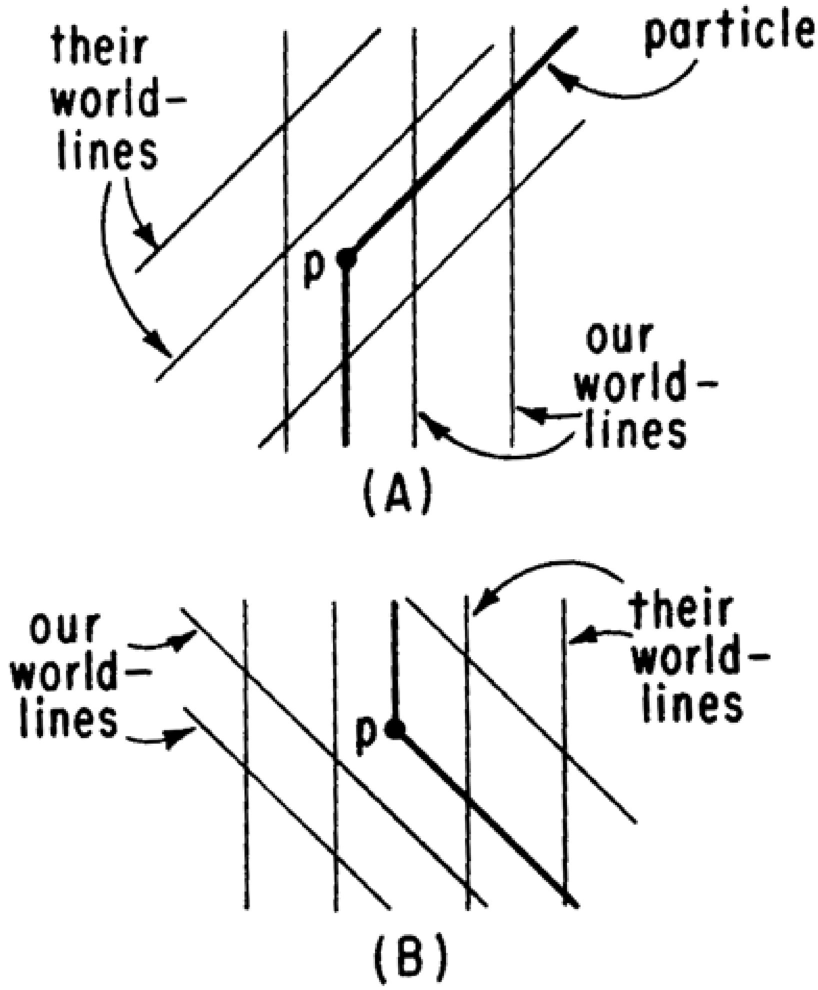
This, of course, is the Galilean space-time.

To make the transition from the Aristotelian to Galilean space-time, we need to learn, first, how to translate between different Aristotelian characterizations of the same physical phenomena.

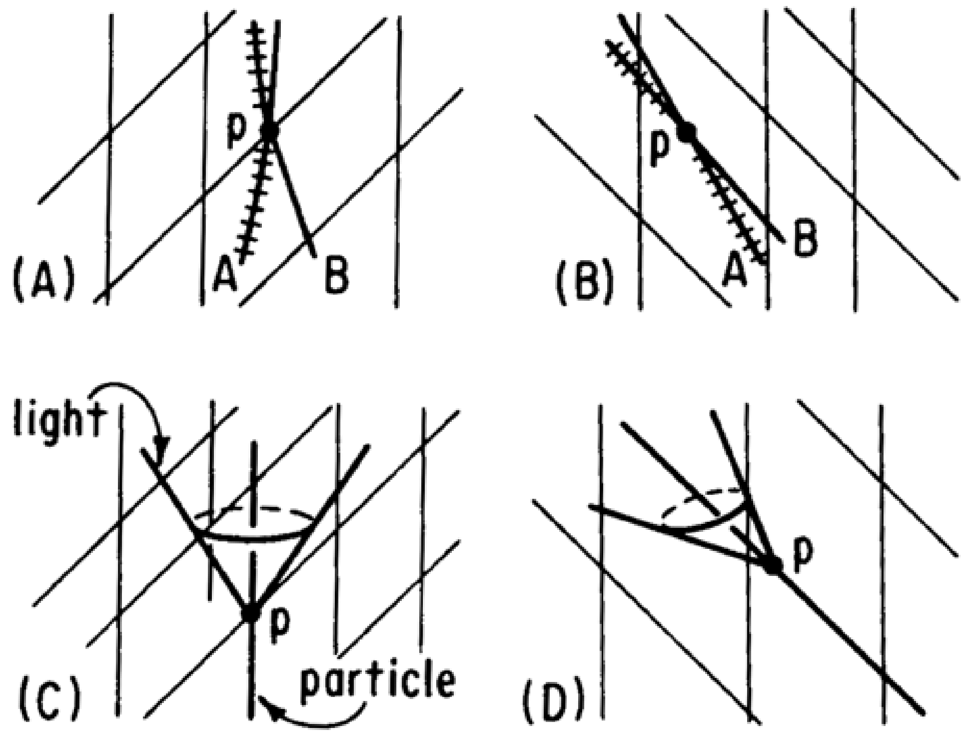
In order to be able to translate, we need some basis for comparison: We must agree on what we'll take to be common to all our observers. We will then be using such common notions to translate from one individual characterization to another.

Common notions:

- Events themselves *deprived* of their exact spatio-temporal determinations = events conceived as just points of the space-time manifold on which no particular coordinate "grid" is imposed
- Representation of particles by world-lines, ropes and sticks by world-surfaces, etc.
- Representation of certain complex processes by particular geometrical constructs in space-time: e.g., of a collision of two particles by the intersection of their world-lines



Geroch, 3-20, p. 44



Geroch, 3-21, p. 45