The Logic of Relativity

Postulate 1: The Principle of Relativity:

The laws of nature are the same in all inertial frames and there is no way of distinguishing absolute uniform motion.

Postulate 2: Constancy of the Velocity of Light:

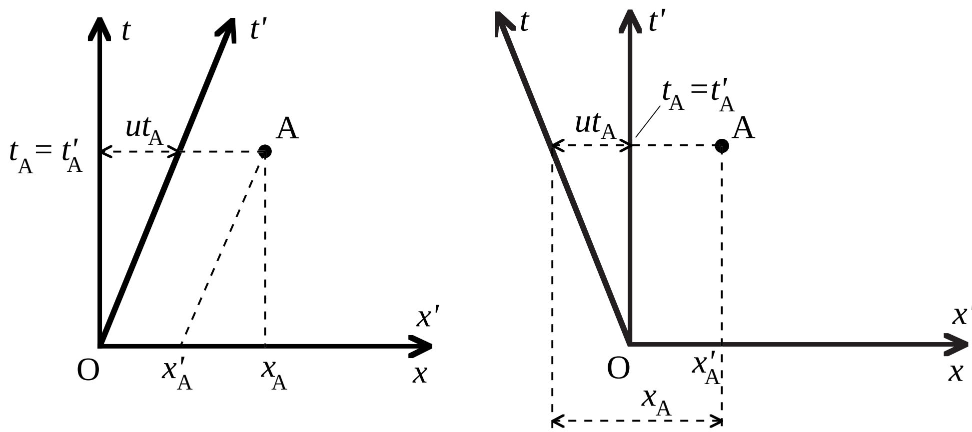
The speed of light *in vacuo* is an absolute constant of nature and is independent of the motion of the source or receiver of light.

☞ If "the laws of nature" include both mechanics and electrodynamics (propagation of light), Postulates 1 and 2 are apparently inconsistent:

- The speed of light must be *different* in different inertial frames (according to Galilean transformations), but it is *constant* (according to Postulate 2).
- Maxwell's equations are *not* covariant under Galilean transformations.

An easy way to see it: the magnetic component of the electromagnetic force acting on a charge depends on the *velocity* of the charge, and velocity is *different* in different frames.

Implicit assumption: Positions, velocities, and times are transformed from one inertial system to another according to the classical Galilean transformation rules:



From (x,t) to (x',t'): $x'_A = x_A - ut_A$ $v' = v - u$

$t'_A = t_A$ $a' = a$

Options:

1) The relativity principle (Postulate 1) does not apply to electromagnetism; Maxwell's equations are valid only in one special frame of reference (ether).
2) The relativity principle applies universally, the speed of light *is* a universal constant, and Maxwell's equations *are* covariant, but the Galilean transformation is *wrong*.

Nineteenth-century physicists preferred option (1); the ether frame was postulated to be the special frame in which Maxwell's equations hold.

Einstein rejected that view and boldly asserted that option (2), which appears the least plausible, is in fact correct. This assumption led him to special relativity.

Why Postulates 1 and 2 *force* us to accept the relativity of simultaneity

Both common sense and pre-relativistic physics took simultaneity of spatially separated events to be unproblematic. In fact, all temporal determinations of events were considered unproblematic: time was supposed to be absolute, i.e. independent of a reference frame. This belief found its expression in the Galilean "transformation" of time: $t'_A = t_A$.

We know that the Galilean transformation is wrong (from Postulates 1 and 2). Consequently, we cannot rely anymore on the temporal determinations of events used in classical mechanics (and in everyday life). This is especially true wrt spatially separated events: we simply have no simple way of ascribing temporal relations (such as "before than," "after than," and "simultaneous with") to them. In particular, we cannot speak of simultaneity as if it were unproblematic.

Einstein: "What, exactly, do we *mean* by simultaneity in speaking of events separated in space?"

- Two events are simultaneous just in case they have the same time coordinate? But *in what frame*? We can no longer speak as if it didn't matter.

☛ To ascribe *meaning* to simultaneity, one has to *specify conditions* under which it obtains, i.e. to introduce an (operational) method that, when applied in a particular case, would enable one to determine whether any two events are indeed simultaneous or not. In the absence of such a method, the notion of simultaneity is *meaningless*.

Proposal (naïve):

- Provide all "observers" constituting (i.e., densely populating) a certain reference frame with clocks.
- Ask the two observers that are in the "immediate vicinity" of the two events to check if their clocks have the same readings when the events in question occur.

Problem:

- Their clocks must be *synchronized*. How can we make sure that two spatially separated clocks are in fact synchronized?

Proposal (naive):

- Two spatially separated clocks are synchronized just in case they show the same reading (say, 3:00:00 pm) at the same time.

Problem:

- This brings us back to the original problem of simultaneity between two spatially separated events (the momentary readings of two clocks).

Moral:

> Simultaneity cannot be ascribed to spatially separated events without a prior synchronization of spatially separated clocks. Spatially separated clocks cannot be synchronized without first ascribing simultaneity to spatially separated events.

Proposal (informed):

- Let's *define* simultaneity by using light rays. This will take care of *both* simultaneity and the synchronization of spatially separated clocks.

Rationale:

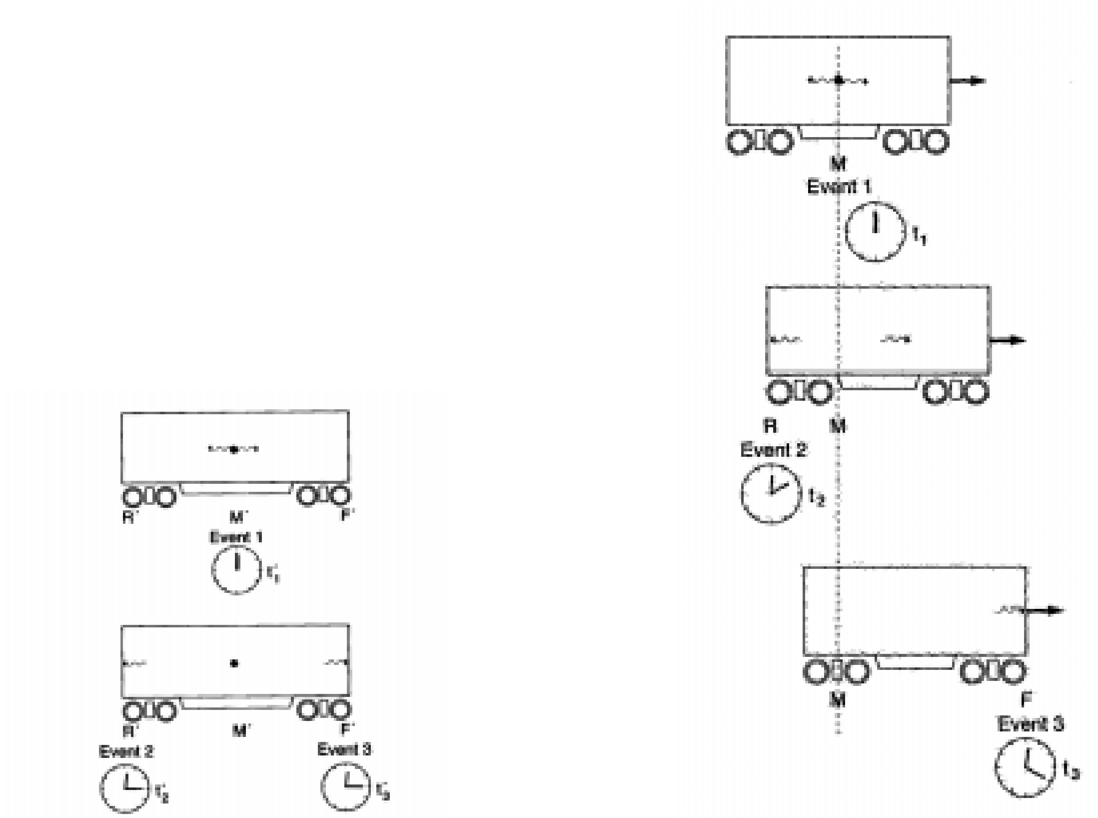
- Light travels with the same speed in all directions in all (inertial) reference frames (Postulate 2).

Question:

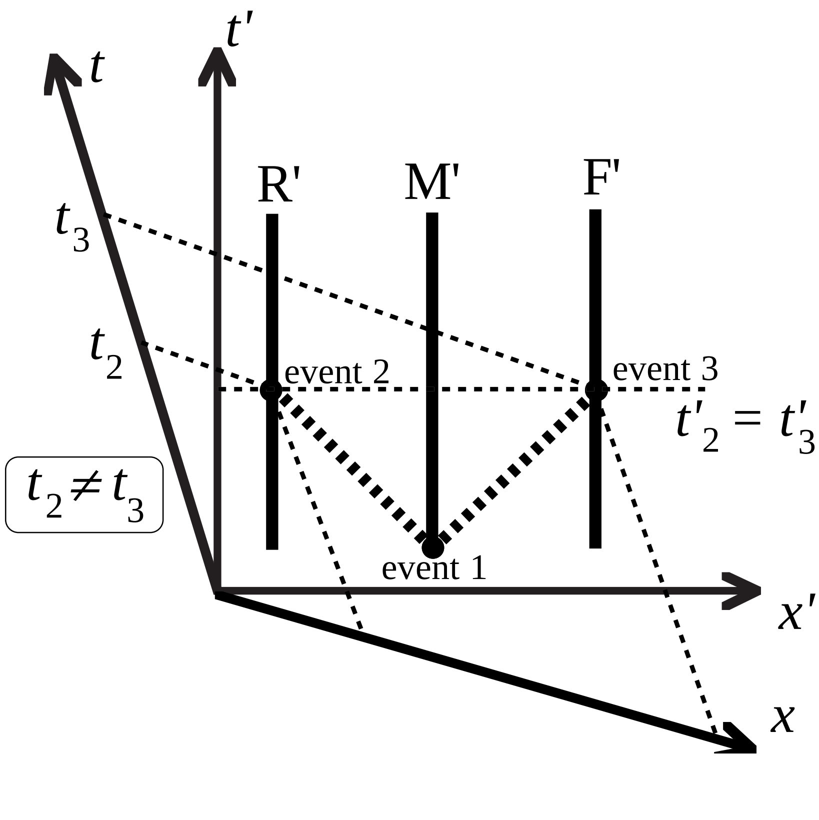
- How to do we know that the notion of simultaneity grounded in the propagation of light is the *right* one?

Answer:

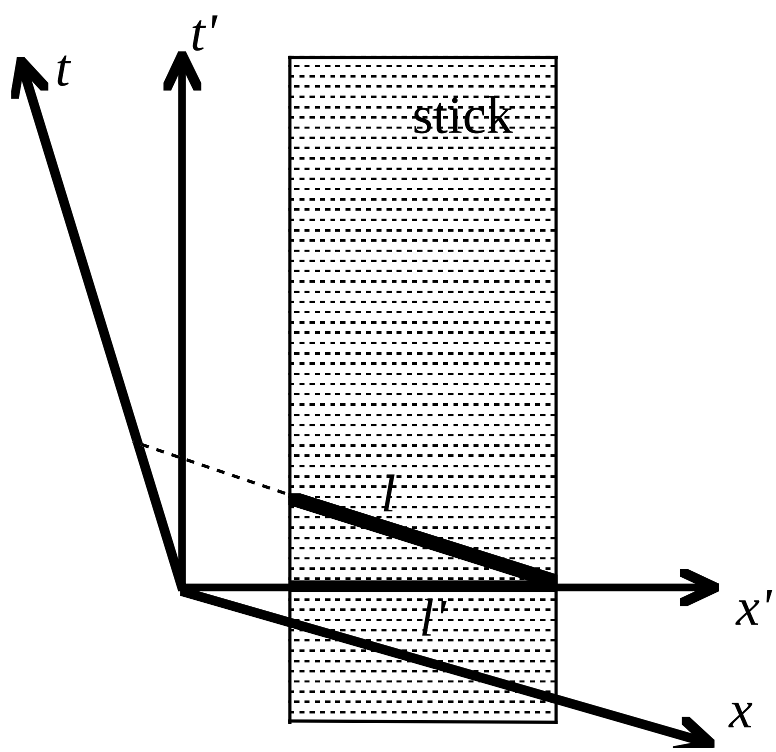
- This is a *wrong question*. We are suggesting a *definition* of simultaneity, one that might prove workable in a new relativistic setting.

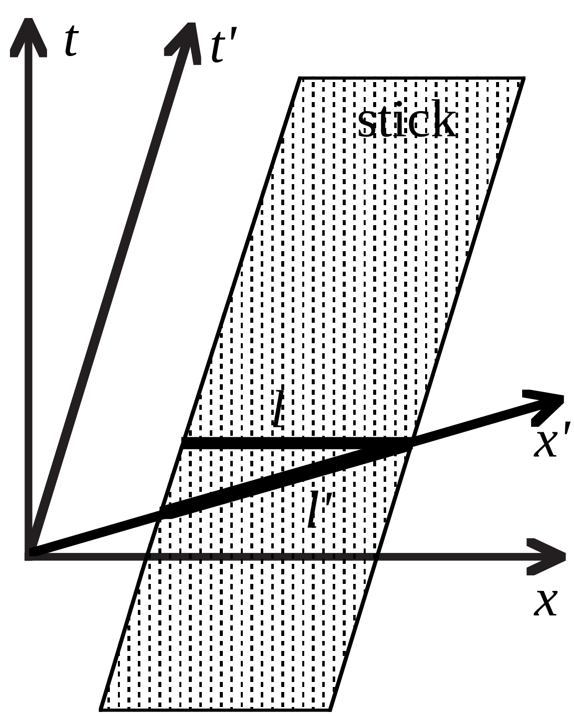


In the train frame, events 2 and 3 are simultaneous;
In the rest frame, they are not.



Length Contraction





The Lorentz Transformations

(x,t) to (x',t'):

$$x'_A = \gamma(x_A - ut_A)$$

$$t'_A = \gamma(t_A - \frac{u}{c^2}x_A)$$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

(x',t') to (x,t):

$$x_A = \gamma(x'_A + ut'_A)$$

$$t_A = \gamma(t'_A + \frac{u}{c^2}x'_A)$$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

3)Relativity of simultaneity:

Two events (x_A,t_A) and (x_B,t_B) are simultaneous in (x,t): $t_A = t_B$

$$t'_A = \gamma(t_A - \frac{u}{c^2}x_A),\ t'_B = \gamma(t_B - \frac{u}{c^2}x_B);$$

$$t'_B - t'_A = \gamma(t_B - t_A) + \gamma\frac{u}{c^2}(x_A - x_B) = \gamma\frac{u}{c^2}(x_A - x_B)$$

$$\neq 0 \qquad \text{(unless } x_A = x_B\text{)}$$

Length contraction:

Length of a stick at rest in (x',t') = $l_{proper} = x'_{right} - x'_{left}$

Length measured in (x,t) = $l = x_{right} - x_{left}$, provided $t_{right} = t_{left}$

$l_{proper} = x'_{right} - x'_{left} = \gamma(x_{right} - ut_{right}) - \gamma(x_{left} - ut_{left}) = \gamma(x_{right} - x_{left}) + \gamma u\,(t_{left} - t_{right}) = \gamma l$

$$l = l_{proper}/\gamma < l_{proper} \qquad (\gamma < 1)$$