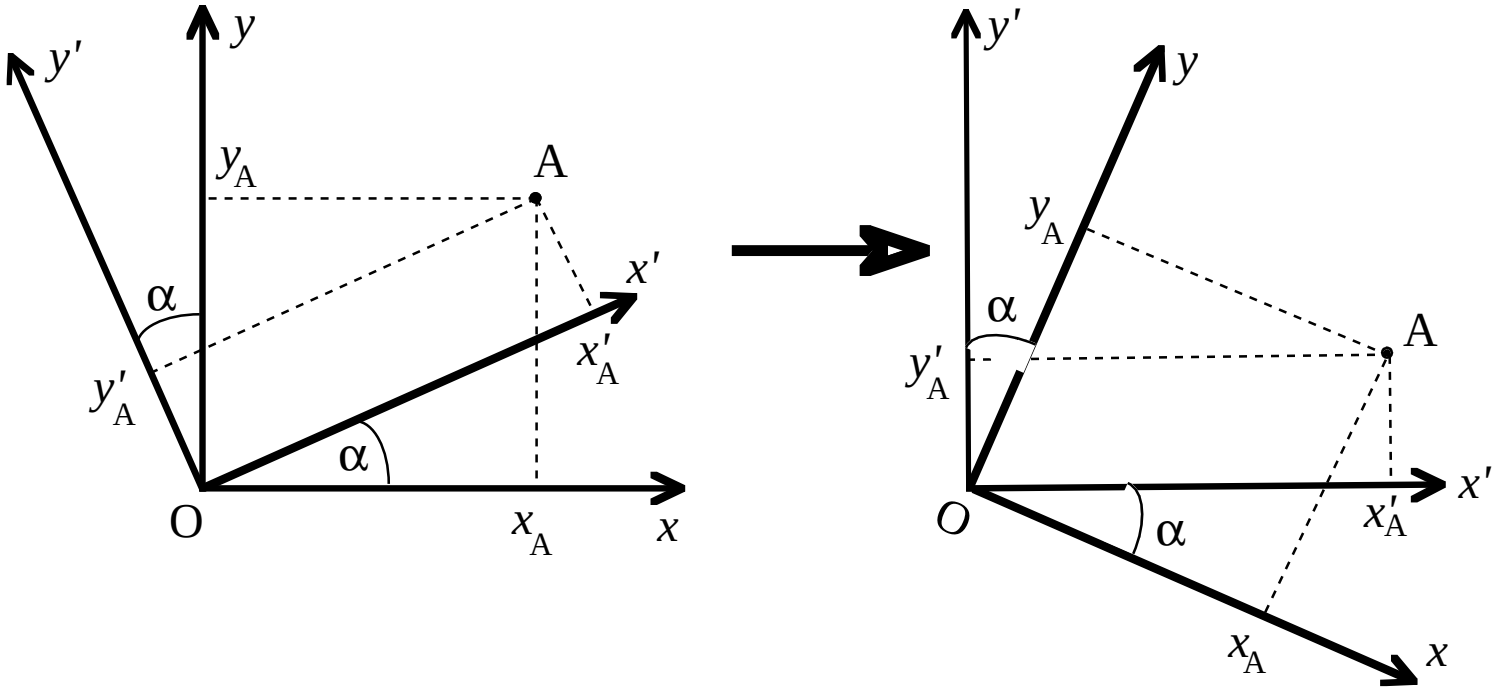
Coordinate Transformation in Space (on a Plane): “Rigid Rotation”

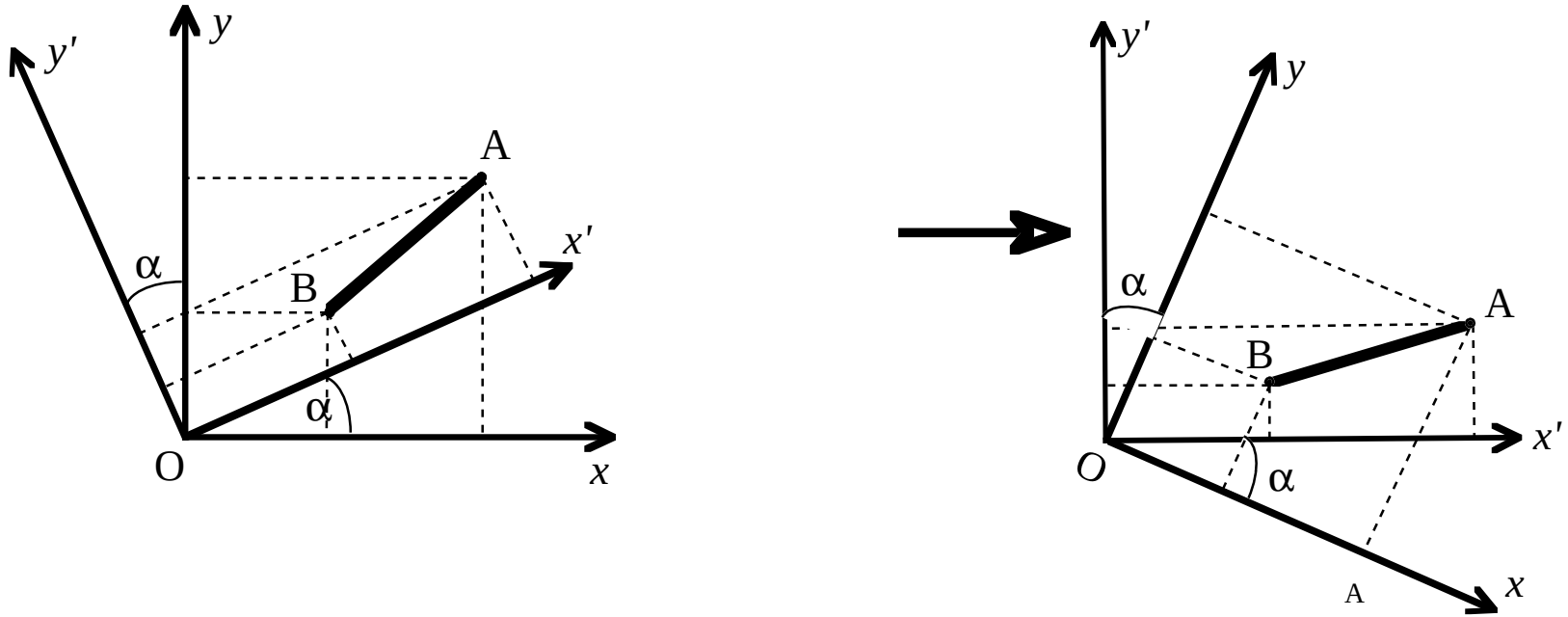


(x_A,y_A) to (x'_A,y'_A):

$$\boxed{\begin{aligned} x'_A &= x_A\cos\alpha + y_A\sin\alpha \\ y'_A &= -x_A\sin\alpha + y_A\cos\alpha \end{aligned}} \qquad \begin{pmatrix} x'_A \\ y'_A \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x_A \\ y_A \end{pmatrix}$$

(x'_A,y'_A) to (x_A,y_A):

$$\boxed{\begin{aligned} x_A &= x'_A\cos\alpha - y'_A\sin\alpha \\ y_A &= x'_A\sin\alpha + y'_A\cos\alpha \end{aligned}} \qquad \begin{pmatrix} x_A \\ y_A \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x'_A \\ y'_A \end{pmatrix}$$

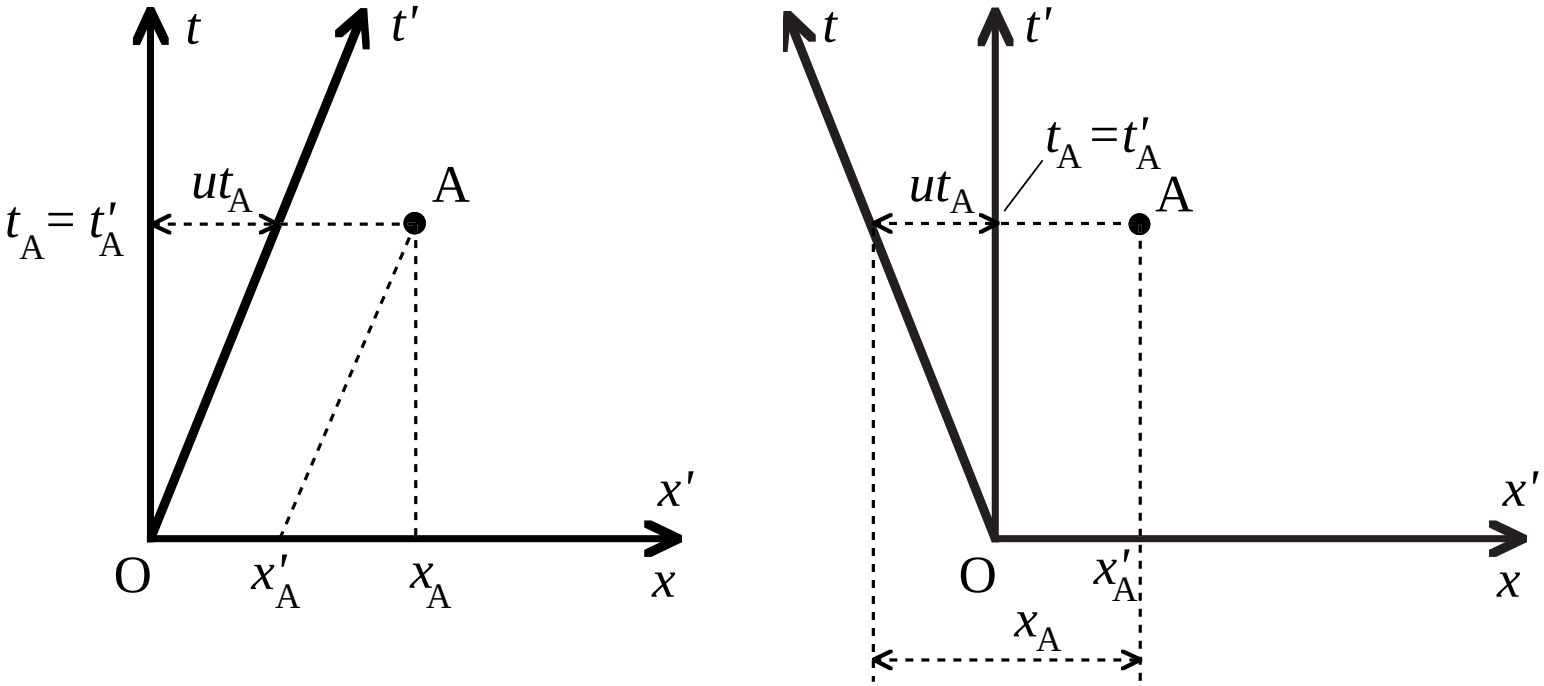


Features preserved under rigid spatial rotations:

- straight lines in space
- distances between points
- angles between lines.

They are *invariant*, hence *objective*, as opposed, e.g., to spatial locations of single points, which are *relative* or *perspectival* features restricted to particular coordinate systems.

Galilean Transformations in Space-Time

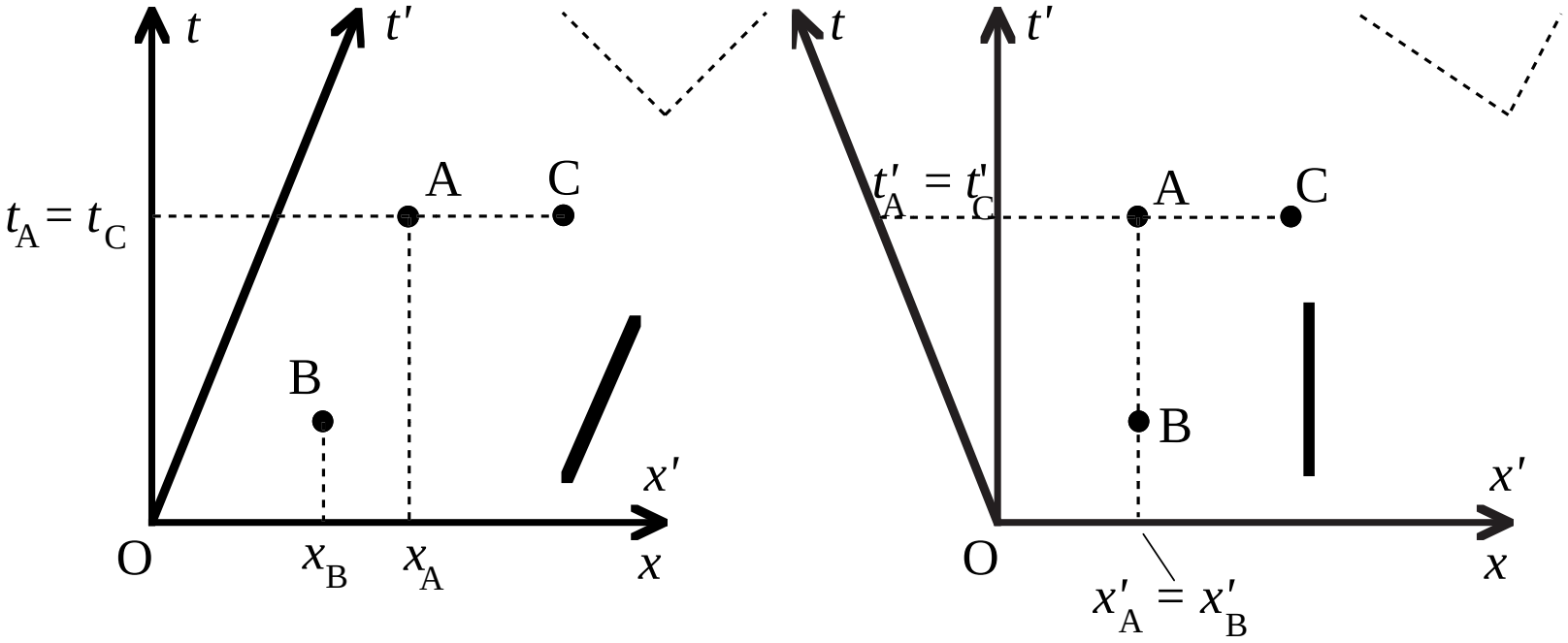


From (x,t) to (x',t'):

$x'_A = x_A - ut_A$ $\quad t'_A = t_A$

$v' = v - u$

$a' = a$

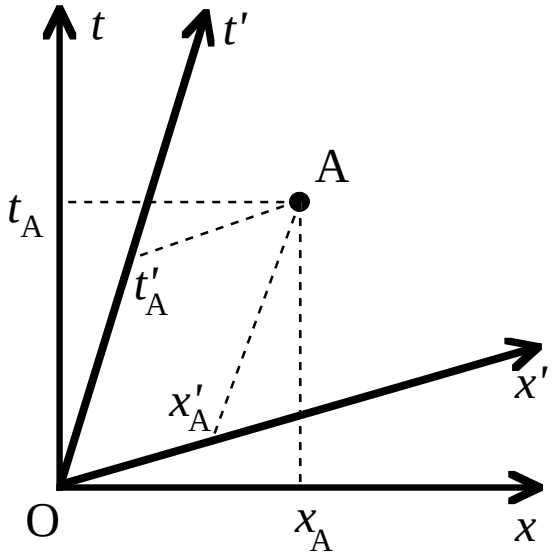


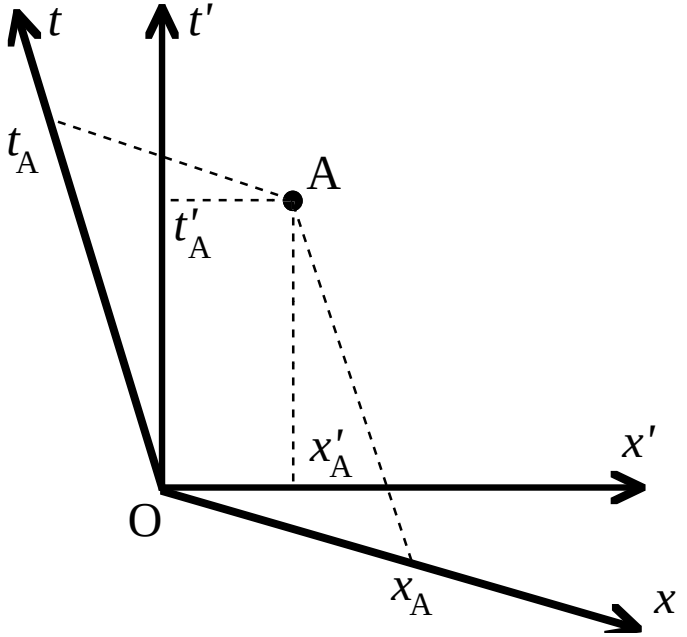
Features preserved under Galilean transformations:

- Times of events (time 3-planes)
- Distances between simultaneous events
- Straight world-lines of particles
- Relative velocities

They are *invariant*, hence *objective*, as opposed, e.g., to spatial locations of events, distances between non-simultaneous events, absolute velocities of objects, and light-cones, which are *relative* or *perspectival* features restricted to particular Aristotelian perspectives.

Lorentz Transformations in Relativistic Space-Time





(x,t) to (x',t'):

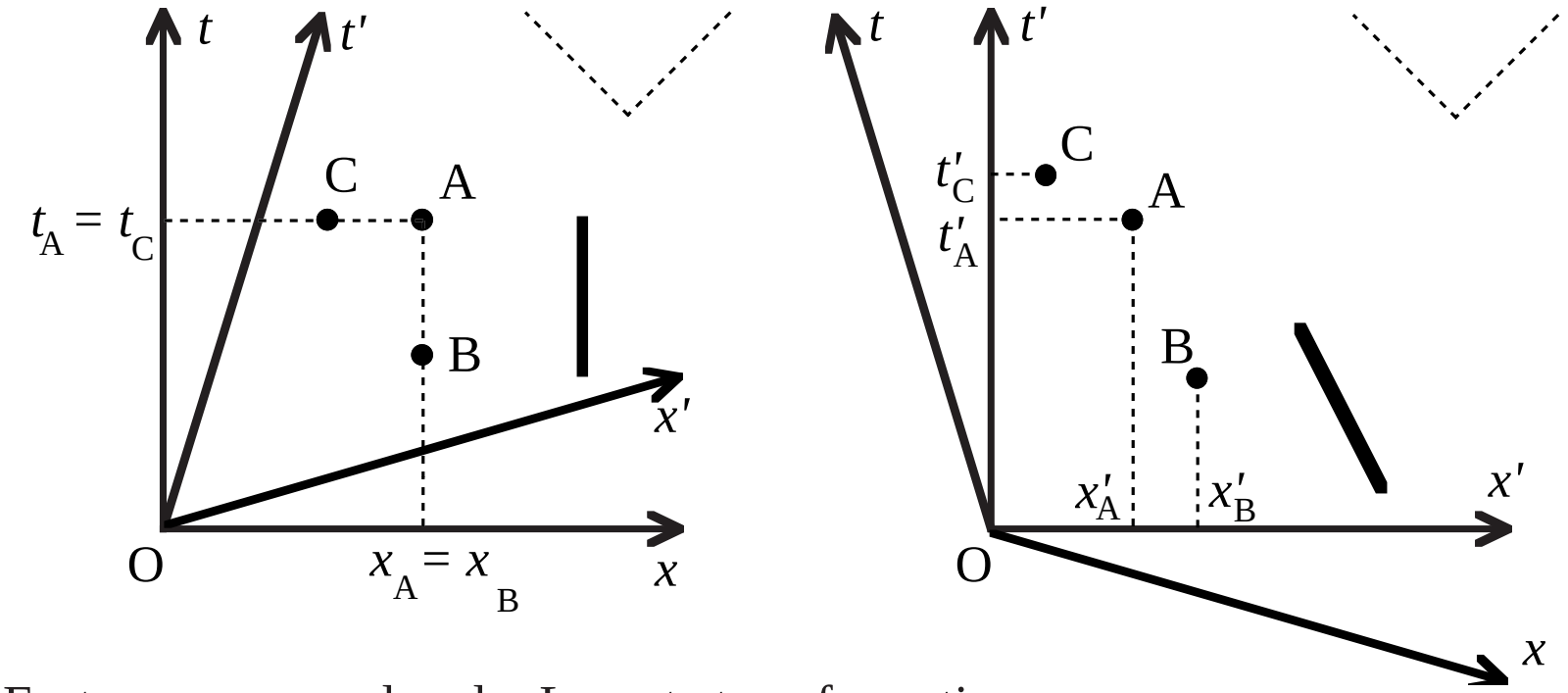
$$x'_A = \gamma(x_A - ut_A)$$

$$t'_A = \gamma\left(t_A - \frac{u}{c^2}x_A\right)$$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$v = \frac{v-u}{1-\frac{uv}{c^2}}$$

$$\begin{pmatrix} x'_A \\ t'_A \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\frac{u}{c} \\ -\gamma u & \gamma \end{pmatrix} \begin{pmatrix} x_A \\ t_A \end{pmatrix}$$



Features preserved under Lorentz transformations:

- Straight world-lines of particles
- Light cones
- The Interval

They are *invariant*, hence *objective*, as opposed, e.g., to spatial distances between events (hence, “length contraction”) and times elapsed between events (hence “time dilation”), which are *relative* or *perspectival* features restricted to particular spatio-temporal “perspectives” (i.e. inertial reference frames).