

# Unification of Space and Time in SR

## The Reality of the Four- Dimensional World

“The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth *space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.*” [Emphasis added—yb]

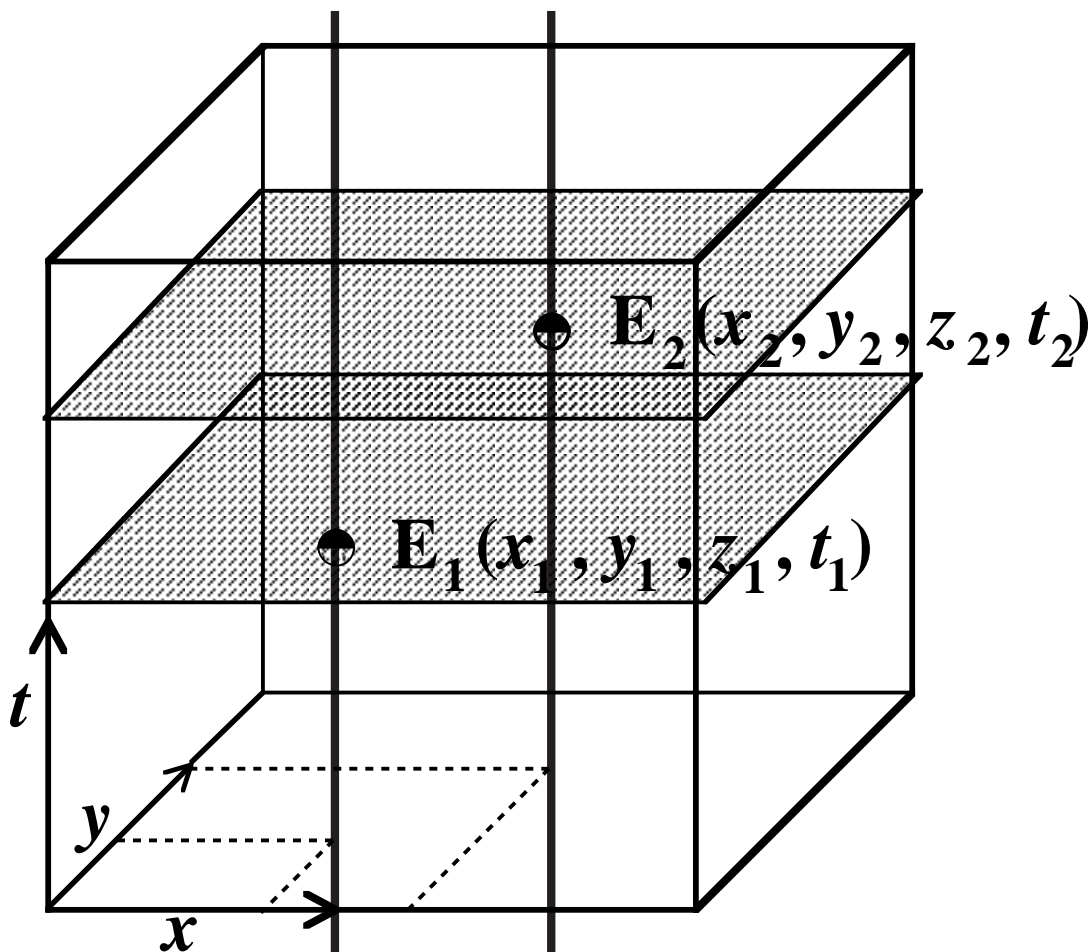
H. Minkowski, from the Address delivered at the 80<sup>th</sup> Assembly of German Natural Scientists and Physicians, at Cologne, 21 September 1908.

Skeptic:

- Four-dimensional space-time is no big deal. There is nothing more behind it than the observation that it takes four numbers to locate an event in the real world.
- After all, *any* physical theory (e.g., Aristotelian and Galilean) can be formulated in the 4D ST framework. How is SR different?

Aristotelian ST = Aristotelian (absolute) 3-space  $\times$   
Aristotelian (absolute) time

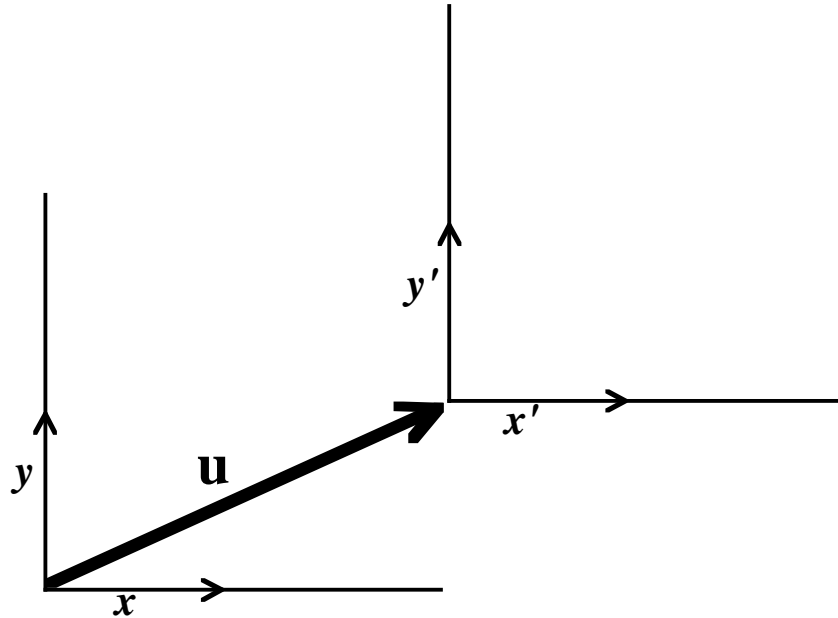
- The spatial location of any event and the time of its occurrence are *uniquely* determined in a single absolute reference frame (“the center of the universe”).
- Only *one* frame is legitimate.



Galilean ST = Absolute Newtonian time  $\times$  3-space in all inertial reference frames

- Spatial position becomes relative to the state of motion of a given Aristotelian observer.
- Different observers assign different spatial positions to the *same* events. Such assignments conflict. But they are equally legitimate (democracy).
- To stop the “bickering” among the observers, we *declare* that their conflicting assignments of spatial positions to events are *not* part of the *intrinsic structure* of ST.
- These assignments are “merely” *perspectival*: they are induced by individual perspectives on a *common reality*: the Galilean ST.
- But all observers agree on the *time* of occurrence of events: time remains absolute. It can be completely “detached” from space.
- The structure of time 3-planes is invariant, hence part of the intrinsic structure of the Galilean ST.

# Inertial (Galilean) Coordinate Transformations



$$x' = x - u_x t$$

$$y' = y - u_y t$$

$$z' = z$$

$$t' = t$$

$$v'_x = v_x - u_x$$

$$v'_y = v_y - u_y$$

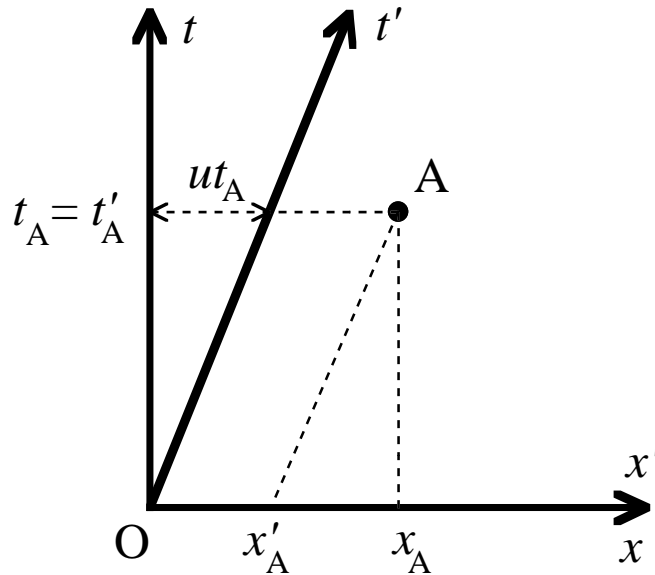
$$v'_z = v_z$$

$$a'_x = \frac{dv'_x}{dt'} = \frac{d(v_x - u_x)}{dt} = \frac{dv_x}{dt} - \frac{du_x}{dt} = a_x - 0 = a_x$$

$$a'_y = a_y$$

$$a'_z = a_z$$

# Galilean Transformations in Space-Time



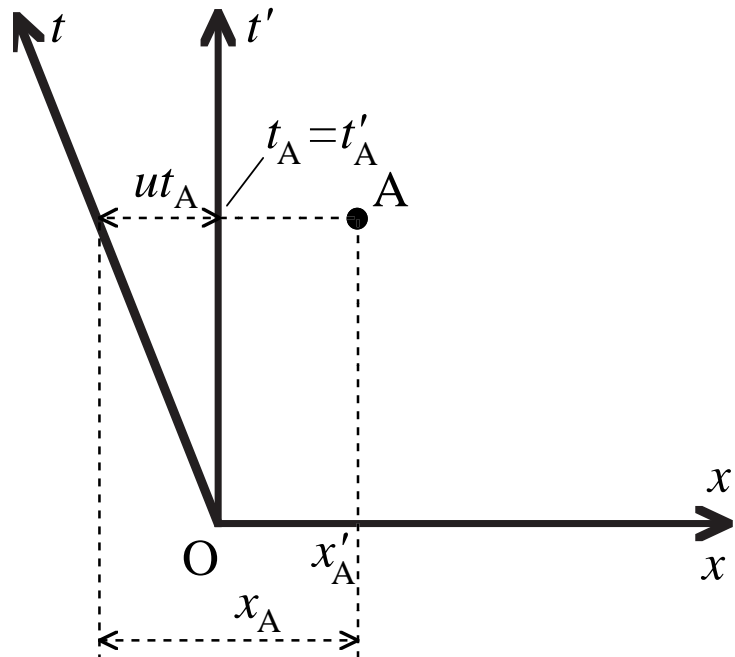
From  $(x, t)$  to  $(x', t')$ :

$$x'_A = x_A - ut_A$$

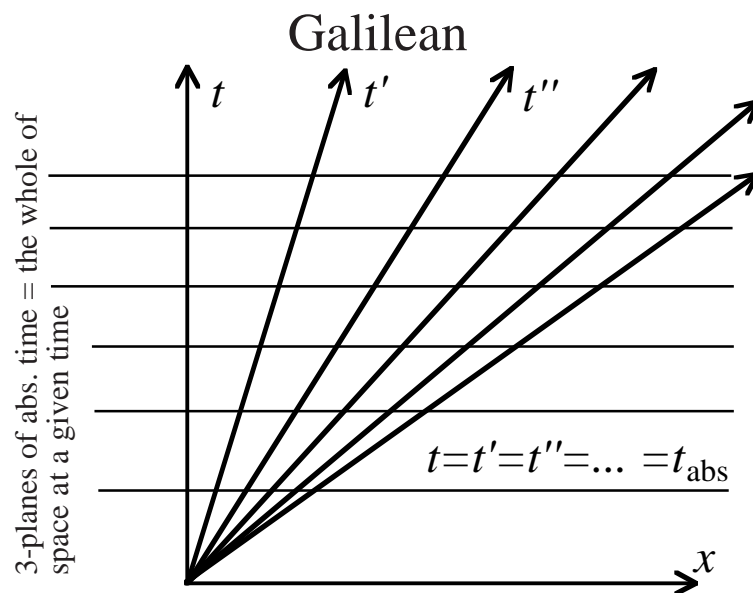
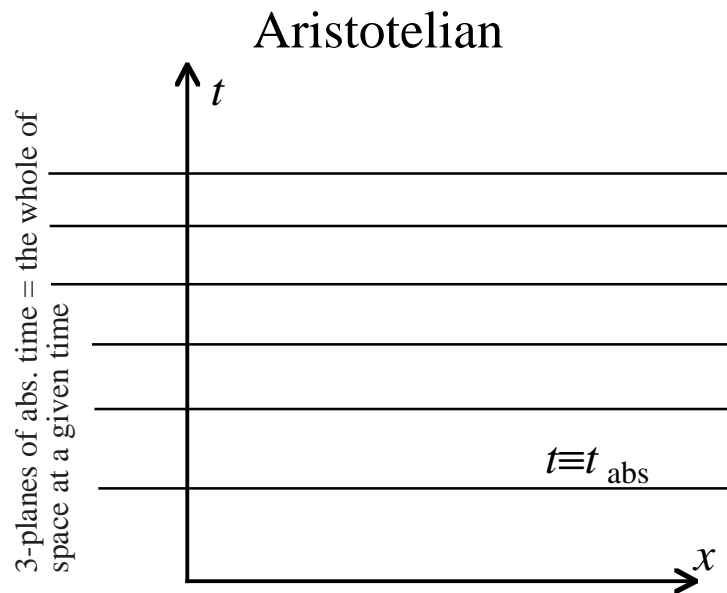
$$v' = v - u$$

$$a' = a$$

$$t'_A = t_A$$



- Both the Aristotelian and the Galilean ST's can be *uniquely decomposed* into time 3-planes, each such 3-plane representing the *whole of space* at a certain moment of absolute time.



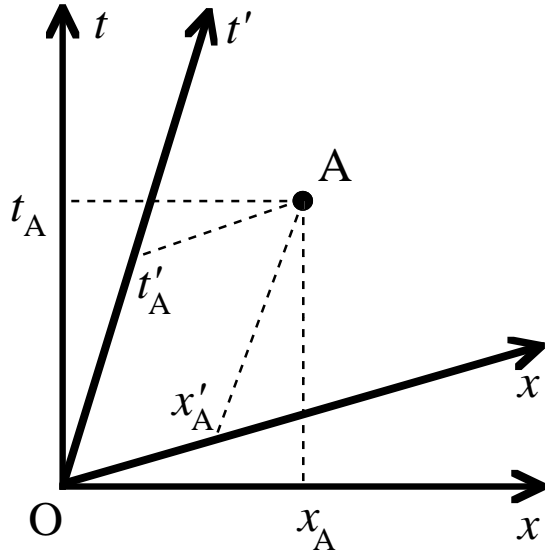
- Combining time with space in building the structure of the Aristotelian and Galilean ST's does *not* involve a genuine *unification*.
- In the Aristotelian ST, time and space remain completely separate.
- In the Galilean ST, time “affects” space in the following sense:
  - What counts as the *same* position in space at *different* times depends on the choice of a frame = Trans-temporal identification of positions in space depends on the choice of the time axis.
- But space does *not* “affect” time:
  - What counts as the same time does not depend on the choice of a frame = Trans-spatial identification of moments of time does not depend on the choice of the coordinate axes.
- This explains why we could introduce and conceptualize the intrinsic structure of the Aristotelian and Galilean ST's “from top down,” starting with common sense intuitions about space and combining them with such intuitions about time.
  - Aristotelian: Take 3-space and add time.
  - Galilean: Take 3-space and add time (often “at an angle”).
  - The notion of the 4D ST manifold is useful in, but not indispensable to, such conceptualizations.

- In contrast, building the structure of the relativistic ST requires *starting* with the notion of the 4D ST manifold (“rubber sheet”) and *imposing* various perspective-dependent coordinatizations on that manifold.
- Time “affects” space:
  - What counts as the *same position in space* at *different times* depends on the choice of a frame = Trans-temporal identification of positions in space depends on the choice of the coordinate axes.
- But space equally “affects” time:
  - What counts as the *same time* at *different places* depends on the choice of a frame = Trans-spatial identification of moments of time depends on the choice of the coordinate axes.

### In Special Relativity:

- Time and space become “on a par.”
- Time is “mixed” with space.
- Time and space “interpenetrate.”
- Etc.

# Lorentz Transformations in Relativistic Space-Time

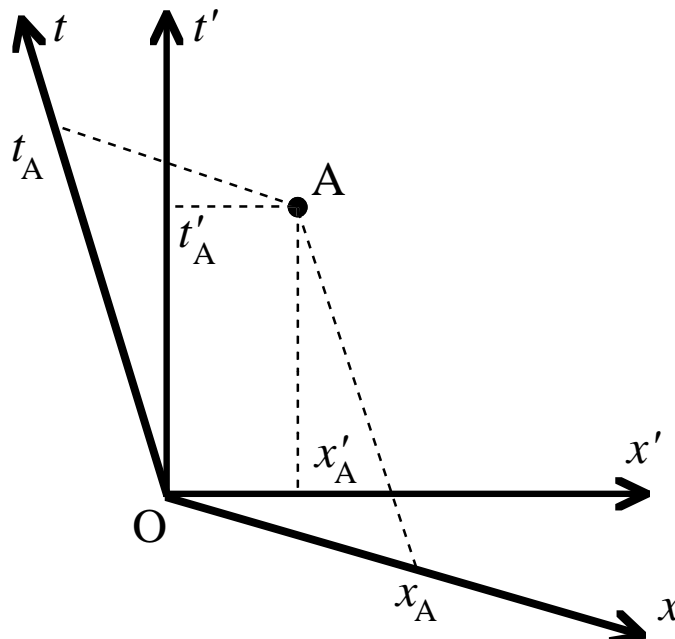


From  $(x, t)$  to  $(x', t')$ :

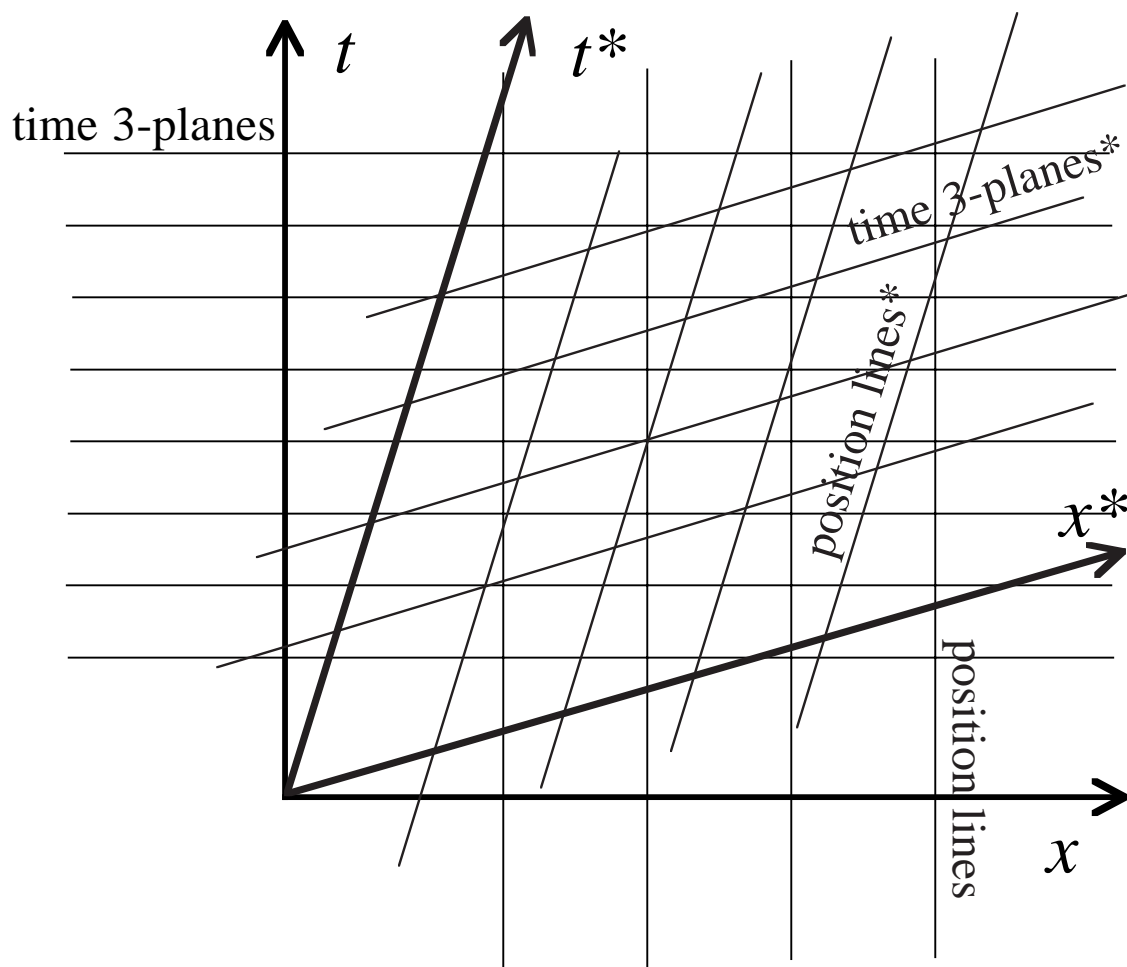
$$x'_A = \gamma(x_A - ut_A)$$

$$t'_A = \gamma\left(t_A - \frac{u}{c^2}x_A\right)$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$



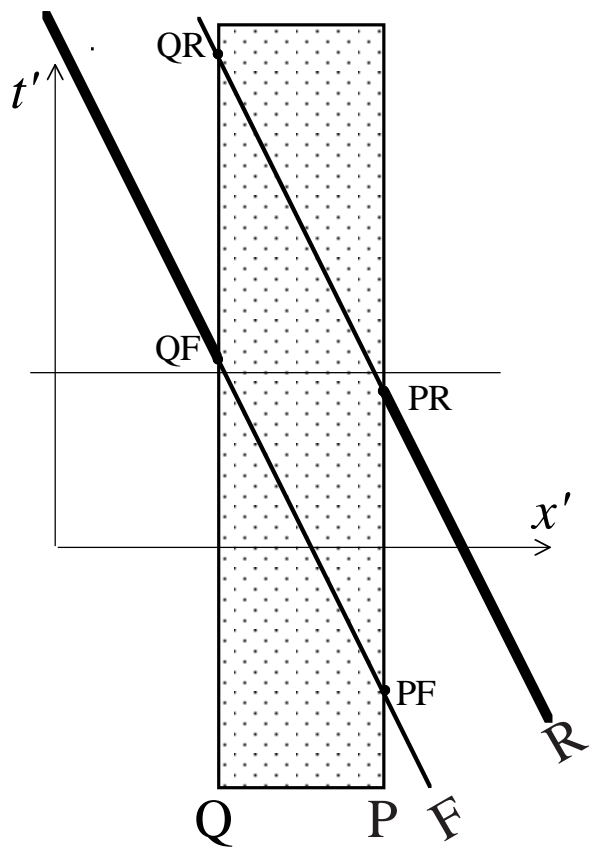
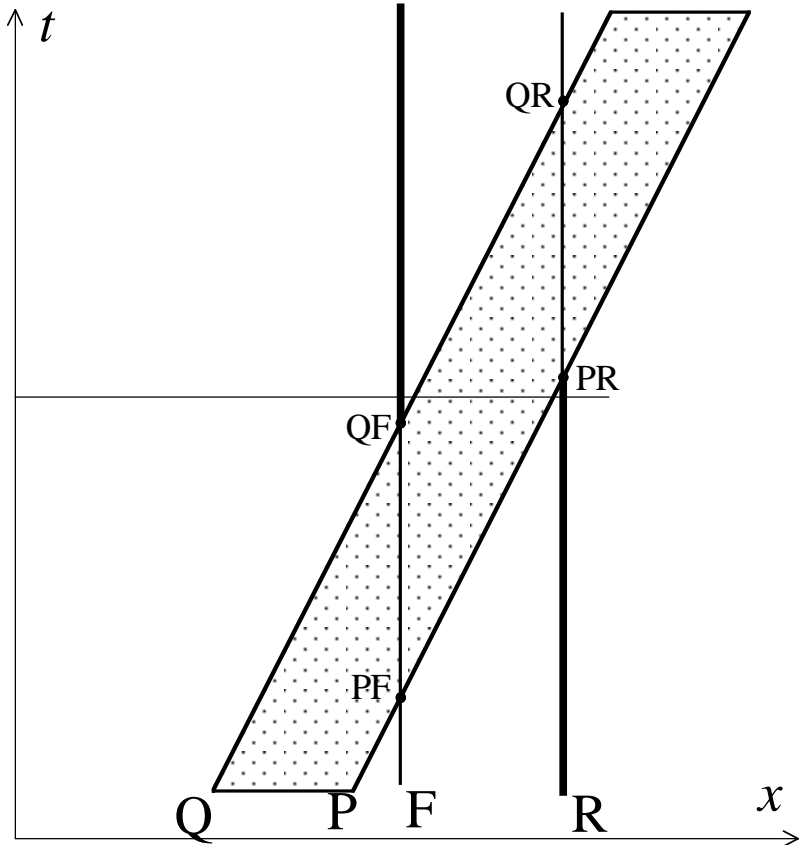
- It's not possible to build the structure of the relativistic ST just by "adding" time to space.
- Time and space need to be "extracted" from the 4D manifold by means of various coordinatizations (i.e., space and time coordinate assignments in various inertial frames).
- Time and space are not given "in advance" as independent coordinates. We start with a single 4D manifold and "slice" it into different space+time perspectives.

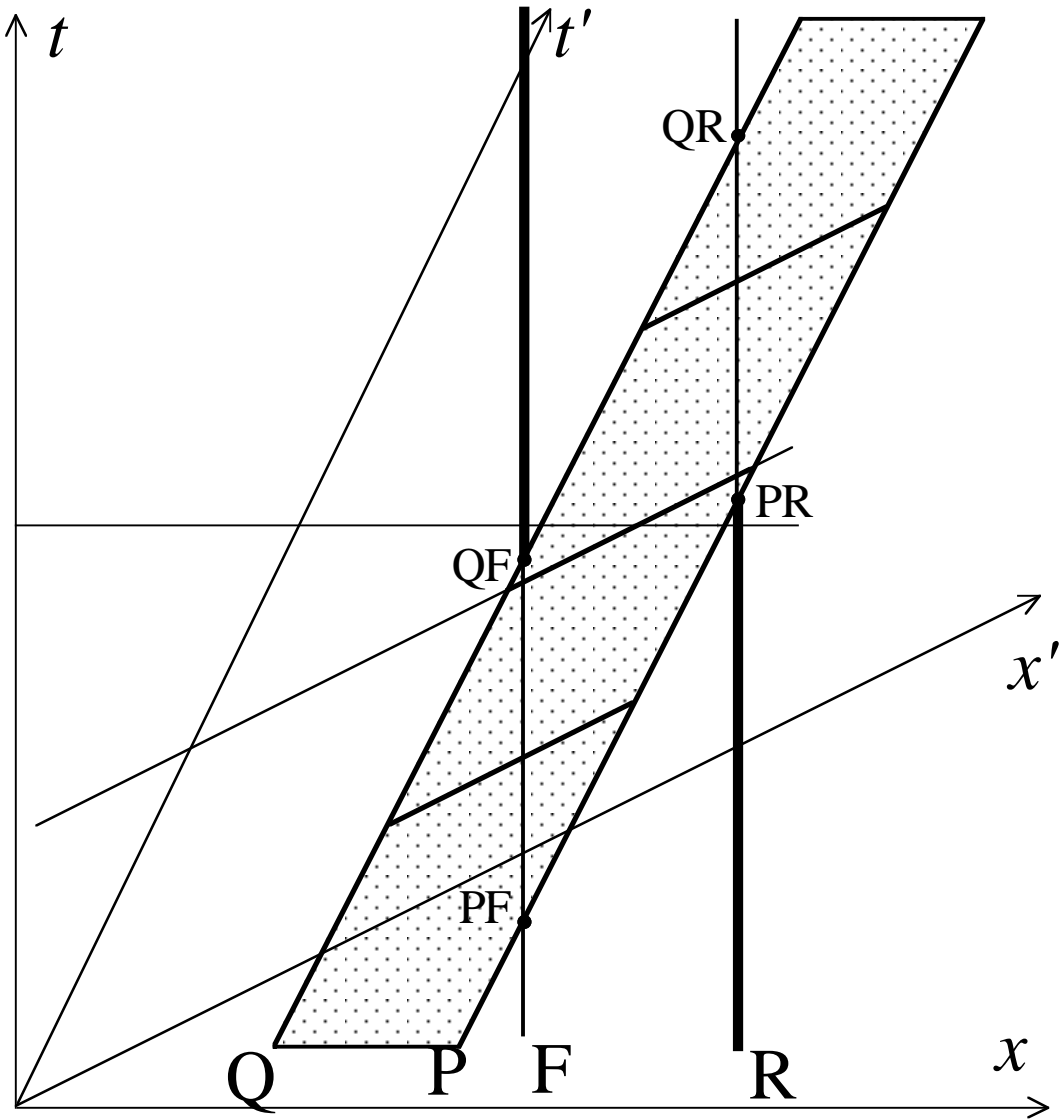


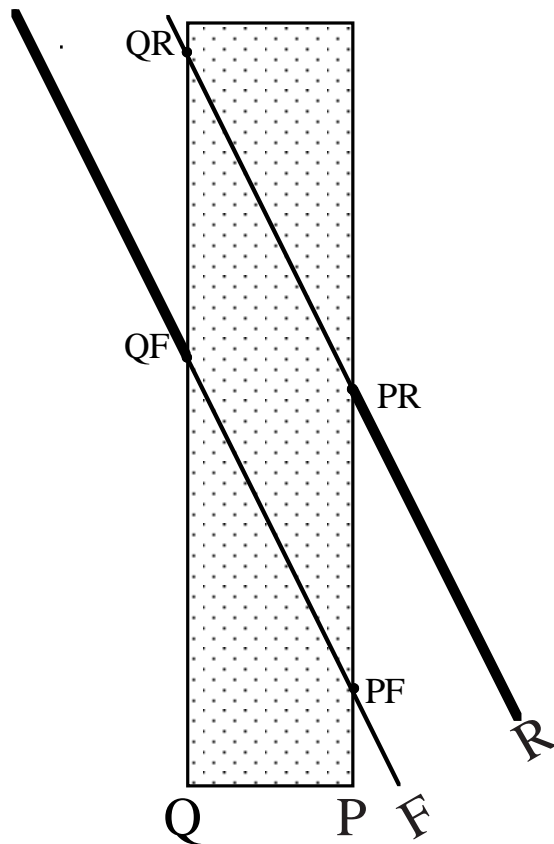
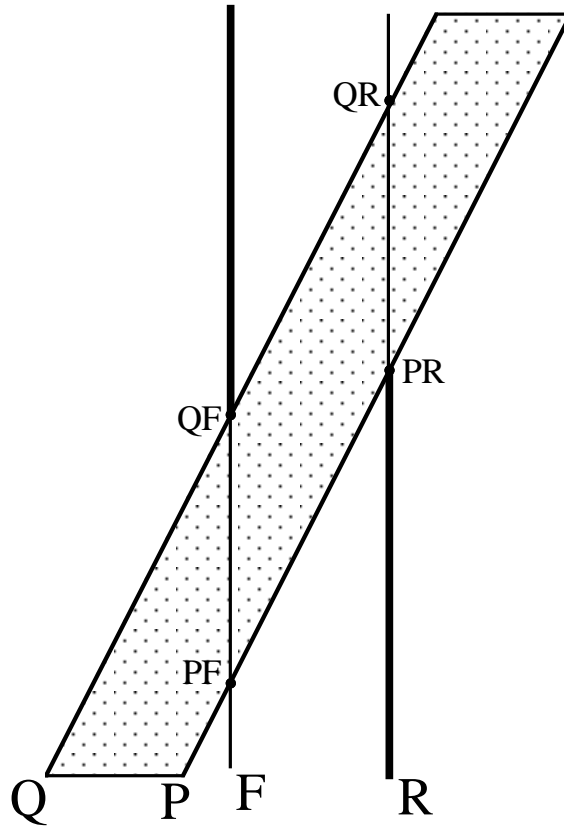
# The Reality of the Four-Dimensional World

- What is invariant (i.e., frame-independent) is *real* (objective).
- What is frame-dependent is “merely” perspectival (“subjective”)  $\approx$  “unreal,” not part of the objective matter of fact.
- Invariant structures of the relativistic ST:
  - Straight world-lines (i.e., the fact that they are straight)
  - Light cones
  - The Interval between events
- In an important sense, the notions of invariance (objectivity, reality) vs. frame-dependence (perspectivalism, “non-reality”) can be applied to *complete ST diagrams* representing physical phenomena and processes in different frames.
- Different accounts of the same process in different 3+1 frames may seem to “conflict.” But we can reconcile them by looking at them as partial, frame-restricted representations of a certain *underlying reality*.

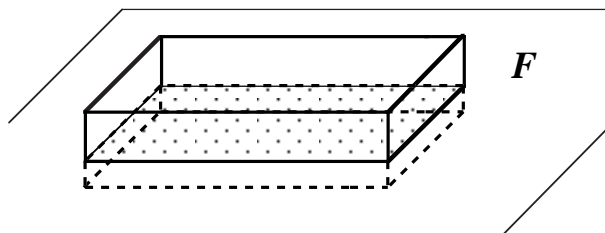
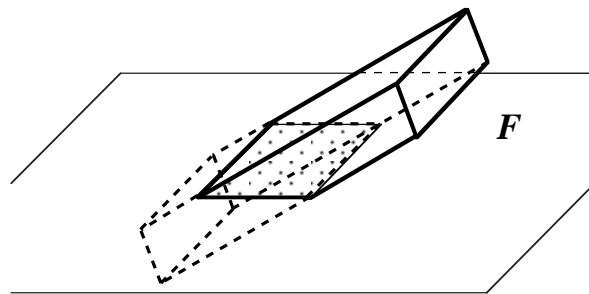
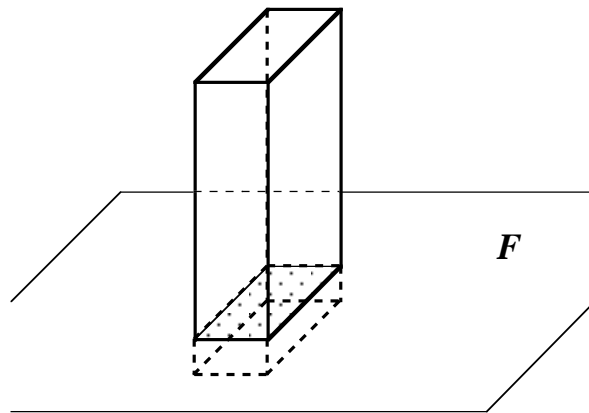
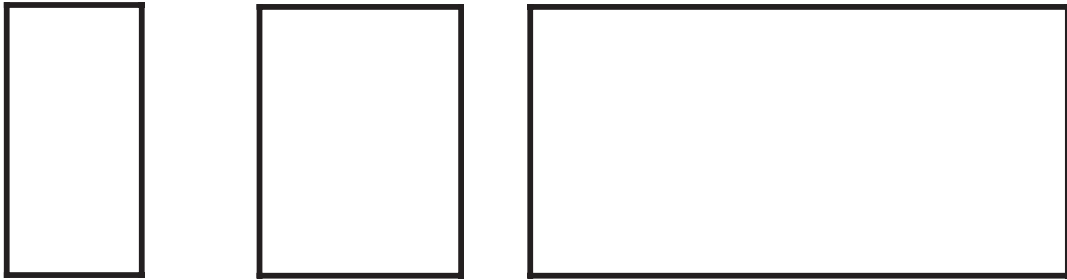
What is this “underlying reality”?



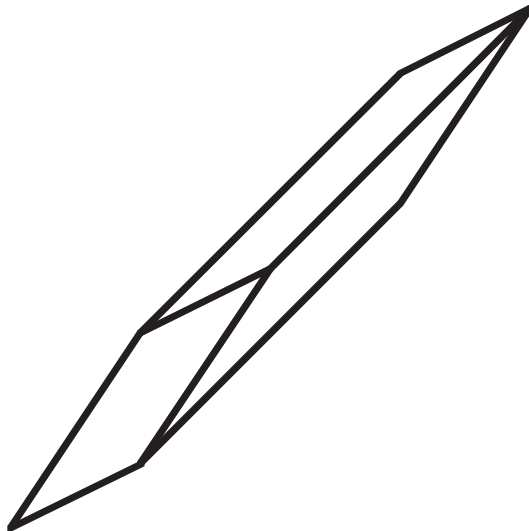
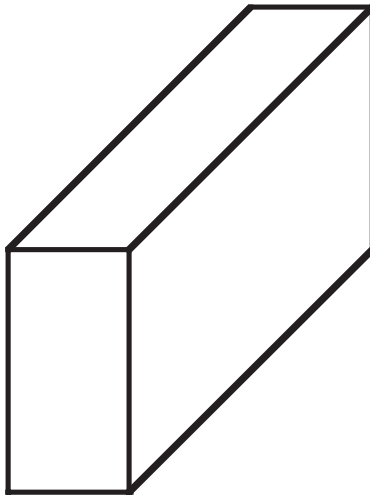
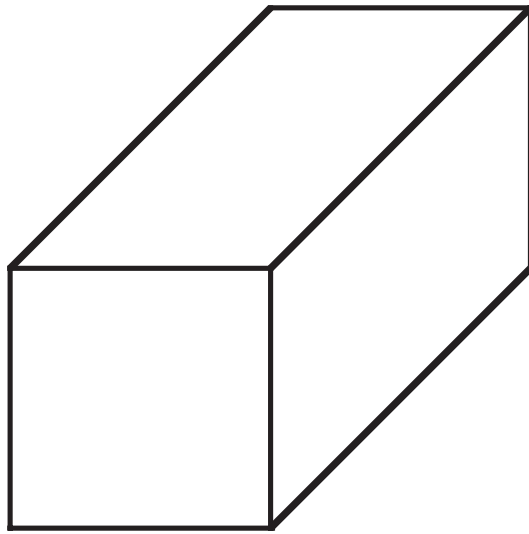


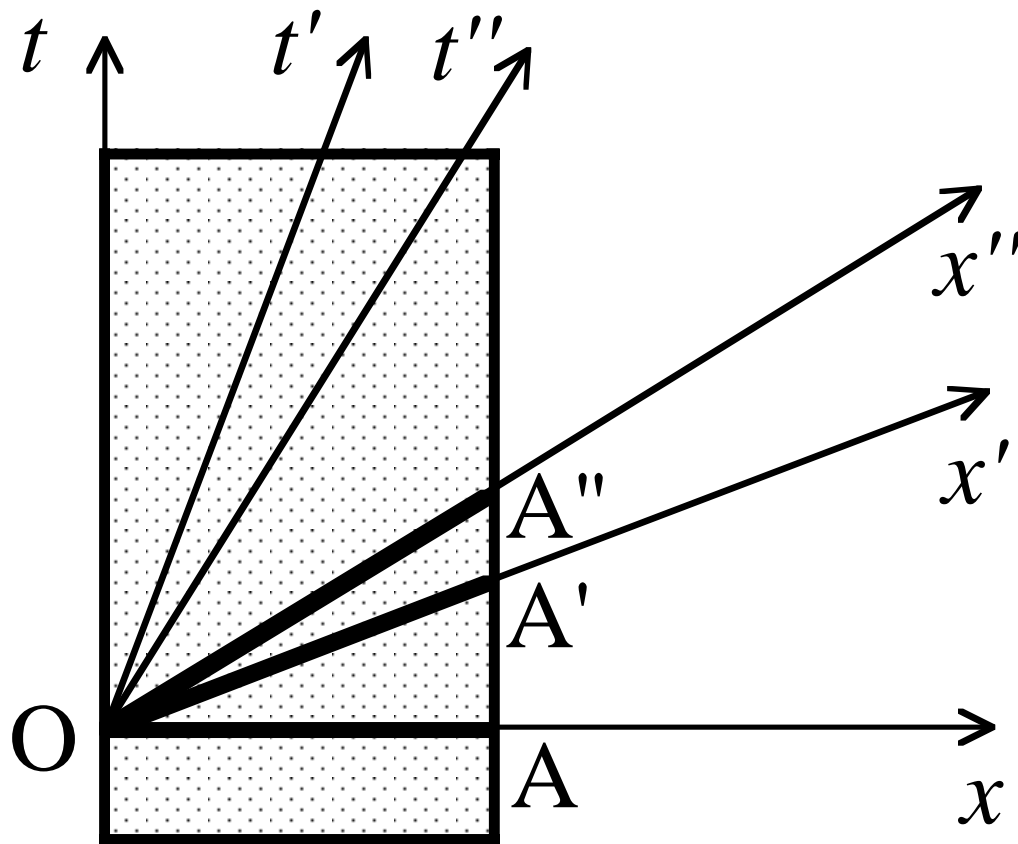


# The Flatland Analogy









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“We should then have in the world no longer *space*, but an infinite number of spaces, analogously as there are in three-dimensional space an infinite number of planes. Three-dimensional geometry becomes a chapter in four-dimensional physics. Now you know why I said at the outset that space and time are to fade away into shadows, and only a world in itself will subsist.” [Hermann Minkowski, 1908]