# Charge decay in a conductor

What is the characteristic timescale for decay of a nonequilibrium charge distribution in a conductor?

Continuity: $$\nabla \cdot \mathbf{J} = \nabla \cdot \sigma\mathbf{E} = -\frac{\partial}{\partial t}\rho$$

Gauss' law: $$\nabla \cdot \varepsilon\mathbf{E} = \rho$$

Combining, $$\frac{\sigma}{\varepsilon}\rho = -\frac{\partial}{\partial t}\rho$$

$$\Rightarrow \frac{1}{\tau_c} \equiv \frac{\sigma}{\varepsilon}$$

Typical numbers for copper give $\tau_c \sim 10^{-19}$ s (!).

# EM waves and conductors

What happens to EM waves inside conductors? Electric field wave equations including currents becomes:

$$\nabla^2\mathbf{E} - \mu\frac{\partial}{\partial t}\mathbf{J} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

We know definition of conductivity:

$$\nabla^2\mathbf{E} - \mu\sigma\frac{\partial\mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

Plugging in our trial plane wave solution gives:

$$(-k^2 + i\omega\mu\sigma + \mu\varepsilon\omega^2)\mathbf{E}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = 0$$

Solve for the real and imaginary parts of $k = a + ib$, taking positive roots and assuming that all 3 components of $k$ have the same ratio of real to imaginary parts,

# EM waves and conductors

$$a = k_0 \frac{1}{\sqrt{2}}\left[1+\sqrt{1+\frac{1}{(\omega\tau_c)^2}}\right]^{1/2} \qquad k_0 \equiv \omega\sqrt{\varepsilon\mu}$$

$$b = k_0 \frac{1}{\sqrt{2}}\left[-1+\sqrt{1+\frac{1}{(\omega\tau_c)^2}}\right]^{1/2}$$

If $\omega\tau_c << 1$, the material is a "good" conductor;

If $\omega\tau_c >> 1$, the material is a "poor" conductor;

The poor conductor limit is boring; small corrections to what we already have seen, with very slow exponential decay of wave with propagation ($b \sim 0$).

# EM waves and conductors

In the good conductor limit, $a \approx b \approx k_0 \dfrac{1}{\sqrt{2\omega\tau_c}}$

This means the EM wave decays in the conductor on a length scale given by: $\delta = \sqrt{\dfrac{2}{\omega\mu\sigma}}$

This is the skin depth. In copper at optical frequencies, this is something like 3 nm.

- Nanoscale conductors can be on the same scale as the skin depth, and so can strongly interact with an electromagnetic wave.

So, in a good conductor the real and imaginary parts of *k* are of the same size.

# Waves and conductor interfaces

Consider TE case.

Apply cont. of tang. ***E*** at $x = 0$;

$$E_{0i} + E_{0r} = E_{0t}$$

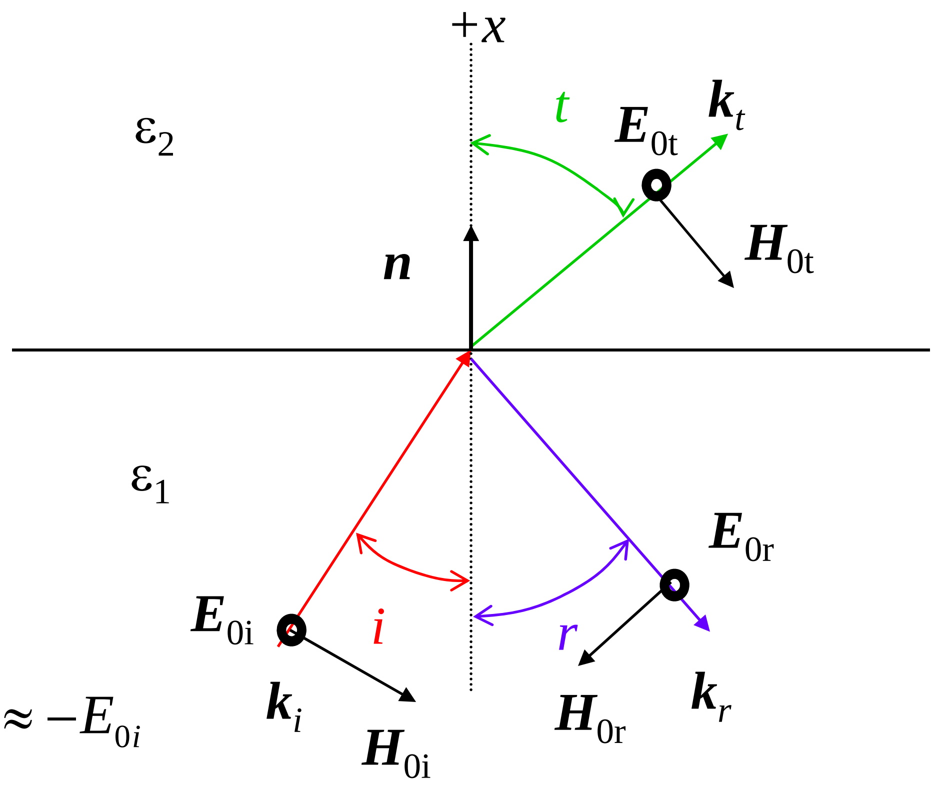
Also,

$$k_{tx} \approx \frac{1+i}{\delta}$$

Using the tang. ***H*** condition gives:

$$E_{0r} = -\left(1 - \frac{2n_1 \cos\theta_i}{n_2} \frac{1-i}{\sqrt{2}} \sqrt{\omega\tau_c}\right) E_{0i} \approx -E_{0i}$$

$$E_{0t} = \frac{2n_1 \cos\theta_i}{n_2} \frac{1-i}{\sqrt{2}} \sqrt{\omega\tau_c}\, E_{0i}$$

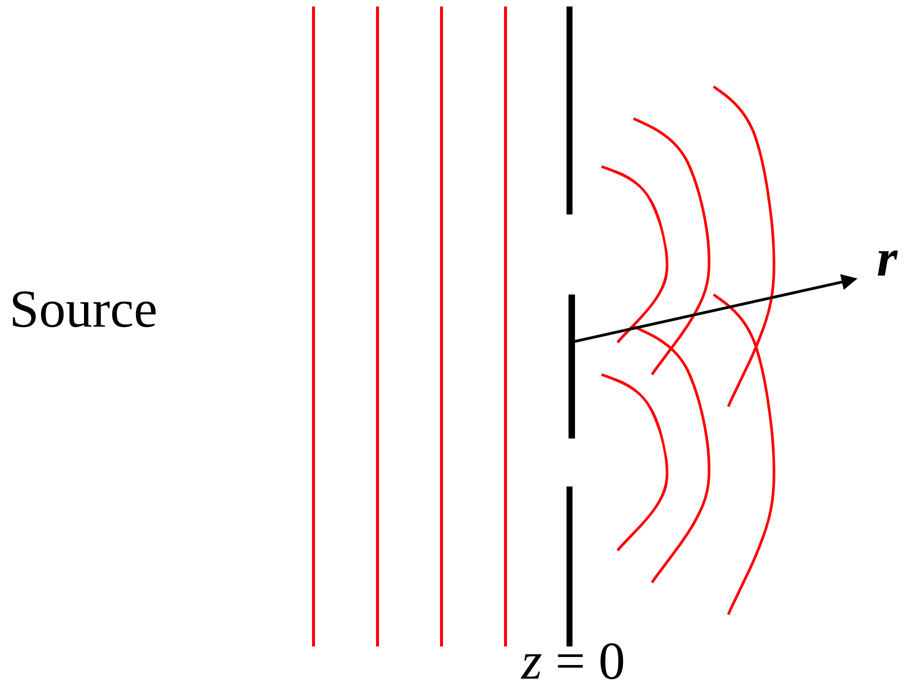



Can do energy flow and Poynting vector and so forth.

Note that a good conductor reflects very well….

# Diffraction

We want to solve the general problem of radiation impinging on a screen with holes of some size and shape in it, and ask what the resulting intensity pattern is at some distance away:




Can treat this as something like a boundary value problem.

Remember Huygen's principle?  Can apply that mathematically.

We'll do *scalar* diffraction theory (no info on polarization, fields).

# Scalar diffraction theory

Do theory for generic scalar variable ψ, and assume that's not a bad description for, say, the E-field amplitude of an EM wave.

Generic time-independent wave equation = Helmholtz eq.

$$\nabla^2\psi + k^2\psi = 0$$

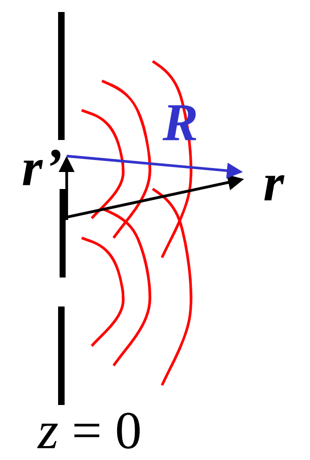
Green's fn for this: $\phi(\mathbf{r}) = \dfrac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$ satisfies: $(\nabla^2 + k^2)\phi(\mathbf{r}) = -4\pi\delta(\mathbf{r}-\mathbf{r}')$

Want to solve for region to right of screen, with boundary conditions at slits and at infinity. Solution is called the Kirchoff diffraction integral:

$$\psi_d(\mathbf{r}) = -\frac{1}{2\pi}\oiint_S da'\ \psi_i(\mathbf{r}')\ \mathbf{n}'\cdot\left(\frac{e^{ikR}}{R}\left(ik - \frac{1}{R}\right)\hat{\mathbf{e}}_{\mathbf{R}}\right)$$

$$\psi_d(\mathbf{r}) = -\frac{1}{2\pi}\oiint_S da'\ \frac{e^{ikR}}{R}\,\mathbf{n}'\cdot\nabla'\left(\psi_i(\mathbf{r}')\right)$$

$$\mathbf{R} \equiv \mathbf{r} - \mathbf{r}'$$



# Scalar diffraction theory

$$\psi_d(\mathbf{r}) = -\frac{1}{2\pi}\oiint_S da' \ \psi_i(\mathbf{r}') \ \mathbf{n}'\cdot\left(\frac{e^{ikR}}{R}\left(ik - \frac{1}{R}\right)\hat{\mathbf{e}}_{\mathbf{R}}\right)$$

$$\psi_d(\mathbf{r}) = -\frac{1}{2\pi}\oiint_S da' \ \frac{e^{ikR}}{R}\mathbf{n}'\cdot\nabla'\left(\psi_i(\mathbf{r}')\right)$$

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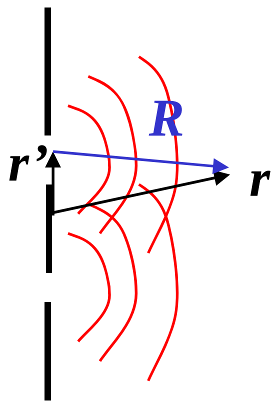



Typically assume either ψ or its gradient in the normal direction is zero on screen and undisturbed in the holes.

General case = *Fresnel* diffraction.

*Fraunhofer* diff. assumes incident plane wave, and very large *R*.

$R \approx r - \hat{\mathbf{e}}_{\mathbf{r}} \cdot \mathbf{r}'$ Defining $\mathbf{k} \equiv k\hat{\mathbf{e}}_{\mathbf{r}}$ leads to

$$\psi_d(\mathbf{r}) \approx -\frac{ik}{4\pi}\frac{e^{ikr}}{r}(\cos\theta_i + \cos\theta_d)\oiint_S da' \ \psi_i(\mathbf{r}') \ e^{-i\mathbf{k}\cdot\mathbf{r}'}$$

# Fraunhofer limit

$$\psi_d(\mathbf{r}) \approx -\frac{ik}{4\pi}\frac{e^{ikr}}{r}(\cos\theta_i + \cos\theta_d)\oiint_S da'\ \psi_i(\mathbf{r}')\ e^{-i\mathbf{k}\cdot\mathbf{r}'}$$

For a normally incident planewave, this calculation looks like a 2d Fourier transform of a function that is 1 in the holes and 0 outside the holes.

A tilted incident wave gives the same result, but offset.

Classic example: single slit diffraction, for rectangular slit of width 2*a* and height 2*b*. For normal incidence, $\psi_i(z=0) = \psi_0$

$$\psi_d(\mathbf{r}) \approx -\frac{ik}{4\pi}\frac{e^{ikr}}{r}(1+\cos\theta_d)\psi_0 \underbrace{\int_{-a}^{a} dx' \int_{-b}^{b} dy'\, e^{-i(k_x x' + k_y y')}}$$

$$= \frac{4}{k_x k_y}\sin k_x a \sin k_y b$$



$z = 0$

# Single slit diffraction

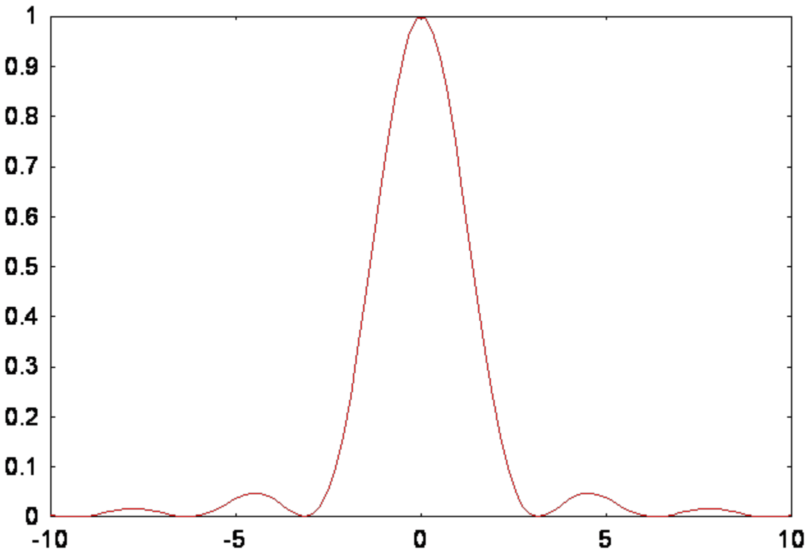
$$\psi_d(\mathbf{r}) \approx -\frac{ik}{4\pi}\frac{e^{ikr}}{r}(1+\cos\theta_d)\psi_0 \underbrace{\int_{-a}^{a} dx' \int_{-b}^{b} dy' e^{-i(k_x x' + k_y y')}}_{= \frac{4}{k_x k_y}\sin k_x a \sin k_y b}$$

In sensible spherical coordinates,

$$k_x = k \sin\theta_d \cos\phi \qquad k_y = k \sin\theta_d \sin\phi$$

Diffracted intensity proportional to square of wave:

$$I = I_0 \frac{k^2(1+\cos\theta_d)^2}{\pi^2 r^2}\frac{\sin^2 k_x a \sin^2 k_y b}{k_x^2 k_y^2}$$




Narrower the slit, the wider the diffraction pattern….

# Circular aperture

For a circular aperture of radius *a*, we can rewrite the integral:

$$\psi_d(\mathbf{r}) \approx -\frac{ik}{4\pi}\frac{e^{ikr}}{r}(1+\cos\theta_d)\psi_0 \int_0^a \rho \; d\rho \int_0^{2\pi} d\phi \, e^{-ik_x \rho \cos\phi}$$

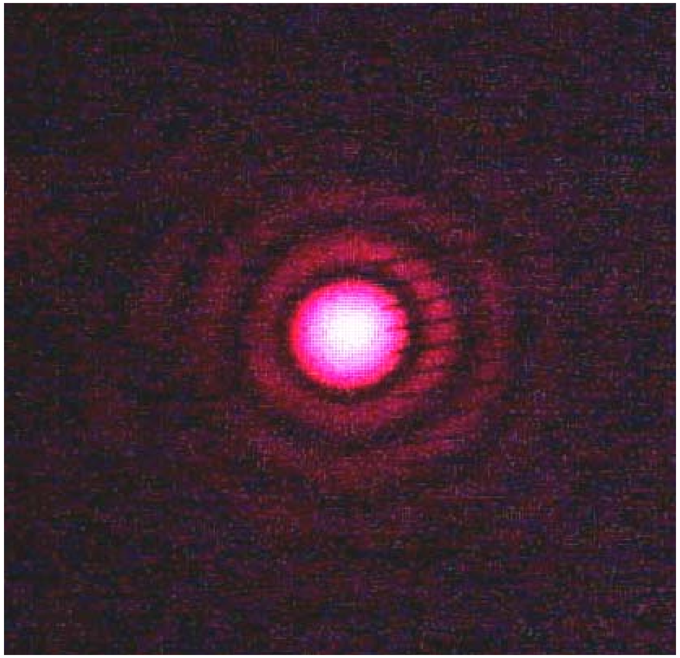
Turns out this can be done using special functions to give:

$$I = I_0 \frac{k^2 a^4}{4r^2}\left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right)^2$$ assuming $\theta_d$ is small.

$J_1$ is the first order Bessel fn of the first kind, and has its first zero when its argument = 3.832.

The result of this diffraction is an Airy pattern, with a central spot surrounded by rings. Playing with numbers, that first zero hits at

$$\theta = 1.22\frac{\lambda}{2a}$$



from Indiana Univ.

# Diffraction

- Scalar diffraction theory is just an approximation. To keep track of polarization, boundary conditions at a conducting screen, etc. requires *vector* diffraction theory.

- Because of form of Frauenhofer integral, diffracted bright spots in this (far-field) limit always have a minimum size on the order of the wavelength of the incident light.

We will revisit diffraction shortly, to consider the Fresnel case and get a feel for what near-field optics is about.

# Antireflection coatings

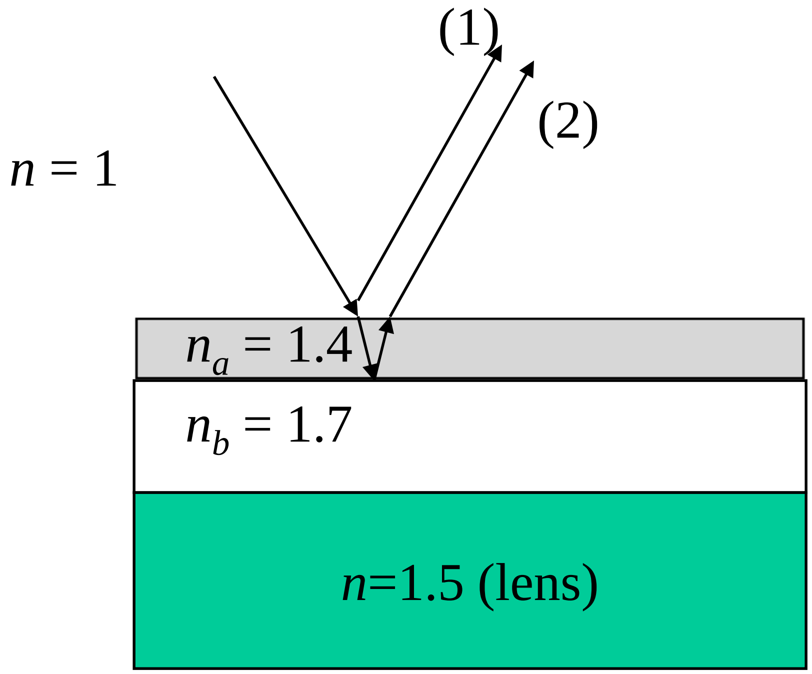
Basic idea is straightforward.

Some of incident wave reflects off first interface, and picks up a π phase shift.

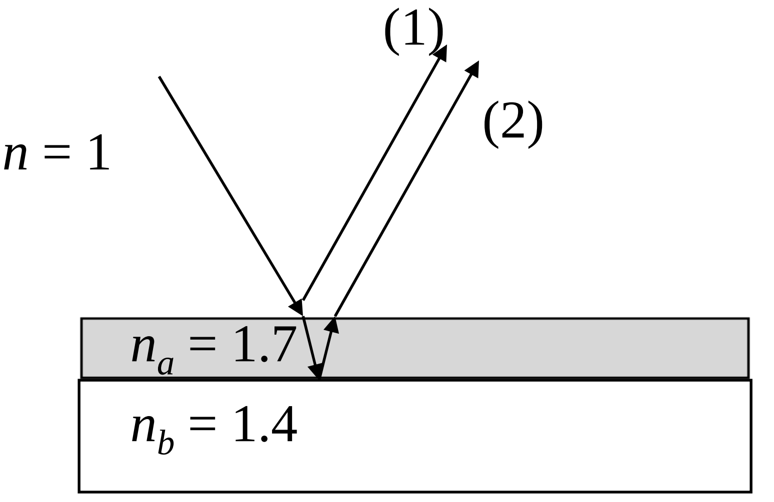
The transmitted light racks up additional phase while propagating through the high index medium.

Some of that light is then reflected off the second interface (another π phase shift), racks up more phase, and (some) reenters the original medium.

If the optical path lengths are chosen correctly, (1) and (2) can be chosen to interfere *destructively*.



# Dielectric mirrors




The basic idea here is the same, only this time the interfaces are chosen to give *constructive* interference, enhancing reflected intensity.

We'll do an example of this here, assuming normal incidence and TE wave.

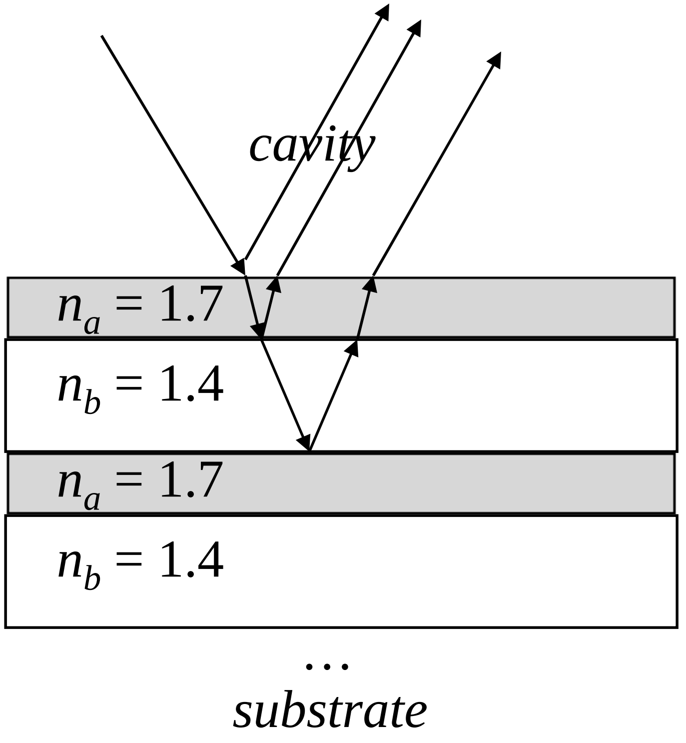
Going from $n$ to $n_a$, the reflection coefficient $r$ is: $r \equiv \frac{E_{0r}}{E_{0i}} = \frac{n - n_a}{n + n_a}$

Note that $r < 0$ for $n_a > n$, so there's a $\pi$ phase shift for this reflection.

Choose thickness $t_a$ to be the right thickness that light reflected from the a-b interface comes back to the surface with exactly a $\pi$ phase shift with respect to the incident light:

$$k_a = n_a \frac{\omega}{c} \qquad \text{Want} \quad 2 \times k_a t_a = \pi \longrightarrow t_a = \frac{\pi}{2} \frac{c}{2\pi\nu} \frac{1}{n_a} = \frac{\lambda_0}{4} \frac{1}{n_a}$$

# Dielectric mirrors




Now suppose you continued the stack, and wanted constructive interference between the reflected waves from each succeeding interface. Assume $n_a > n_b$.

Going from $n_a$ to $n_b$, the reflection coefficient $r$ is:

$$r = \frac{n_a - n_b}{n_a + n_b}$$

Since $n_b < n_a$, another $\pi$ phase shift at this reflection.

Choose thickness $t_b$ to be the right thickness; eventually need a $\pi$ phase shift wrt the incident light. If we pick

$$t_b = \frac{\pi}{2}\frac{c}{2\pi\nu}\frac{1}{n_b} = \frac{\lambda_0}{4}\frac{1}{n_b},$$

total phase shift is then $$\frac{\pi}{2} + \frac{\pi}{2} + \pi + \frac{\pi}{2} + \frac{\pi}{2} = 2\pi + \pi$$

That's why these structures are sometimes called "quarter wave stacks".

# Dielectric mirrors

- Each bilayer called a *period*.
- For normal incidence and a limited wavelength range, dielectric mirrors are often superior to metal mirrors – higher reflectance, lower loss. Why?
- We know the reflection coefficient for each interface; we can therefore sum the series of reflection coefficients for a stack of *m* periods, and find that the total reflection coefficient for the chosen wavelength is:

$$r_m = -\frac{1-\alpha}{1+\alpha} \qquad \alpha \equiv \frac{n_s n_c}{n_a^2}\left(\frac{n_b}{n_a}\right)^{2m}$$

Here $n_s$ and $n_c$ correspond to the substrate and "cavity" indices, respectively.

- Can get higher reflectivity (smaller $\alpha$) by having larger index contrast ($n_b/n_a << 1$) or more periods.

# Dielectric mirrors

- Can solve for number of periods needed to reach a given reflectance:

$$m \geq 0.5 \ln\left[\frac{n_a^2(1-\sqrt{R})}{n_s n_c(1+\sqrt{R})}\right] / \ln(n_b / n_a)$$

- What happens a little away from the design-optimized wavelength?

The half-maximum reflectance happens at:

$$\pm\Delta(\lambda_0 / \lambda) = \frac{2}{\pi}\sin^{-1}\left(\frac{n_a - n_b}{n_a + n_b}\right)$$

# General interface problems: transfer matrices

Also called "matching matrices" and "propagation matrices".

Recall:

$$\mathbf{H}_{0\mathbf{k}} = \sqrt{\frac{\varepsilon}{\mu}}\hat{\mathbf{e}}_{\mathbf{k}} \times \mathbf{E}_{0\mathbf{k}} \equiv \frac{1}{\eta}\hat{\mathbf{e}}_{\mathbf{k}} \times \mathbf{E}_{0\mathbf{k}}$$

Can write local (transverse) electric field at a position *z* as a sum of components propagating in the + and –*z* directions:

$$E(z) = E_{0+}e^{-ikz} + E_{0-}e^{+ikz} = E_+(z) + E_-(z)$$

$$H(z) = \frac{1}{\eta}\left[E_{0+}e^{-ikz} - E_{0-}e^{+ikz}\right] = \frac{1}{\eta}\left[E_+(z) - E_-(z)\right]$$

Rewriting,

$$E_+(z) = \frac{1}{2}\left[E(z) + \eta H(z)\right]$$

$$E_-(z) = \frac{1}{2}\left[E(z) - \eta H(z)\right]$$

http://www.ece.rutgers.edu/~orfanidi/ewa/

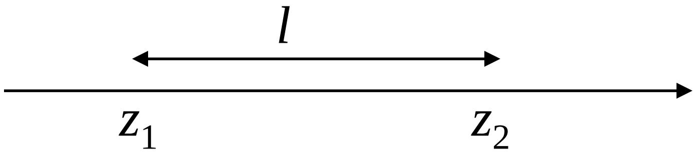
# Transfer matrices

$$\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \eta^{-1} & -\eta^{-1} \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix} \qquad \begin{pmatrix} E_+ \\ E_- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \eta \\ 1 & -\eta \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

Can define wave impedance $\equiv E(z)/H(z)$

And reflection coefficient $\equiv E_-(z)/E_+(z)$

Propagation matrix:




$$E_+(z_2) = E_{0+}e^{-ikz_2} = E_{0+}e^{-ikz_1} \cdot e^{ik(z_1 - z_2)} = E_+(z_1)e^{-ikl}$$

$$E_-(z_2) = E_-(z_1)e^{+ikl}$$

$$\begin{pmatrix} E_+(z_1) \\ E_-(z_1) \end{pmatrix} = \begin{pmatrix} e^{ikl} & 0 \\ 0 & e^{-ikl} \end{pmatrix} \begin{pmatrix} E_+(z_2) \\ E_-(z_2) \end{pmatrix}$$

Can then find $E(z_2)$, $H(z_2)$ as a function of $E(z_1)$, $H(z_1)$ by matrix multiplication.

# Transfer matrices

## Boundary conditions: matching matrices

Remember, $E$ and $H$ here must be continuous across interfaces.

Expressing in terms of $E_+$ and $E_-$,

$$E_+ + E_- = E'_+ + E'_-$$

$$\frac{1}{\eta}\left(E_+ - E_-\right) = \frac{1}{\eta'}\left(E'_+ - E'_-\right)$$

Rewriting,

$$\begin{pmatrix} E'_+ \\ E'_- \end{pmatrix} = \frac{1}{\tau'}\begin{pmatrix} 1 & \rho' \\ \rho' & 1 \end{pmatrix}\begin{pmatrix} E_+ \\ E_- \end{pmatrix}$$

where

$$\rho' \equiv \frac{\eta - \eta'}{\eta + \eta'} = \frac{n' - n}{n' + n} \qquad \tau' \equiv \frac{2\eta}{\eta + \eta'} = \frac{2n'}{n' + n}$$

# Transfer matrices

$$E'_+ = \rho' E'_- \qquad E_- = \rho E_+$$

$$E_- = \tau' E'_- \qquad E'_+ = \tau E_+$$

Can write a scattering matrix

$$\begin{pmatrix} E'_+ \\ E_- \end{pmatrix} = \begin{pmatrix} \tau & \rho' \\ \rho & \tau' \end{pmatrix} \begin{pmatrix} E_+ \\ E'_- \end{pmatrix}$$

The point is, one can continue in this vein and treat multiple interfaces easily by multiplying matrices.

See, for example, Ch. 4 of the online textbook on the course page.

This is actually the best way to treat problems like antireflection coatings.

# Application: Distributed Bragg reflectors

This type of dielectric mirror is also known as a distributed Bragg reflector, and one particular application is in solid state lasers:

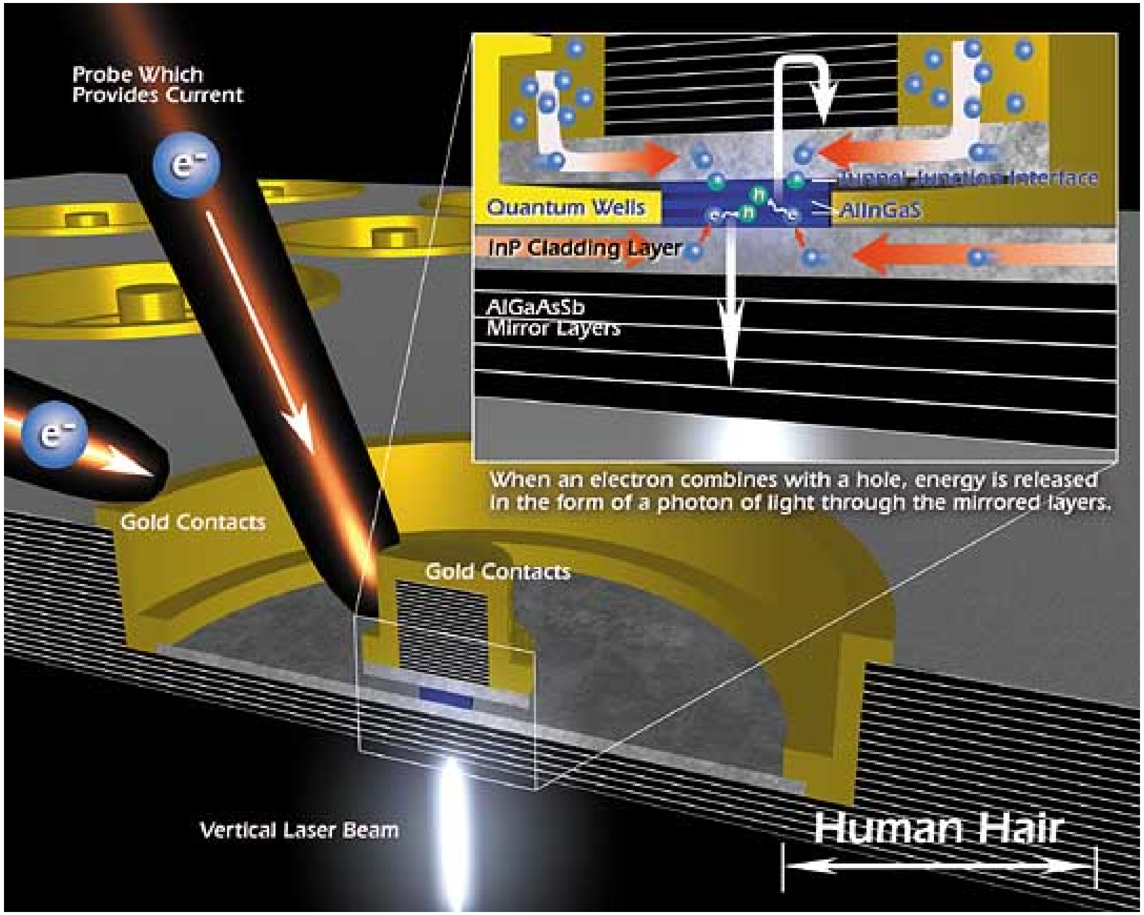



image from UCSB

# Application: Distributed Bragg reflectors

It's clear that by MBE growth of semiconductor layers, it's possible to achieve very large reflectances, and thus very high quality optical cavities.

These cavities can be used for VCSELs, and also for manipulations of optically active nanoparticles, cavity QED, etc.

Materials limitations are tricky: how do you make a solid state DBR that works at very high frequencies?

# Application: fiber gratings

Another version of the DBR appears in fiber optic communications: the fiber grating:

By varying the index along the length of an optical fiber, can result in very high reflectance of particular wavelengths; makes possible several switching technologies with less pain (fewer fiber splices / transitions to electrical components).

Even better: Chirped gratings to compensate for dispersion.

Different wavelengths propagate different distances before being reflected back. Can make up for fiber dispersion….

# Complications

We’ve talked a lot about normal incidence, largely because the algebra is less messy.

Things get complicated at nonnormal incidence.  As you might imagine, polarization effects (which come in at every interface) are severe in structures with dozens or hundreds of interfaces.

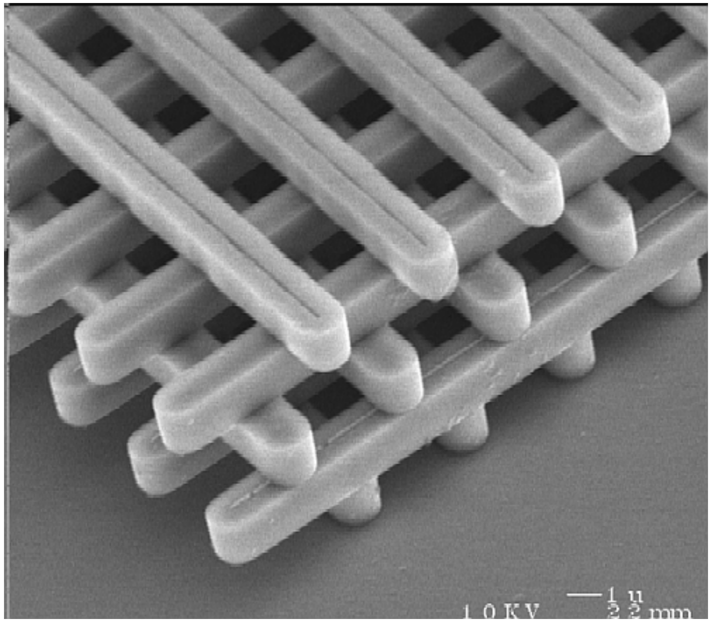
On the other hand, these effects can be turned to our advantage - see the paper you’re supposed to read for problem 3.

# Next time:

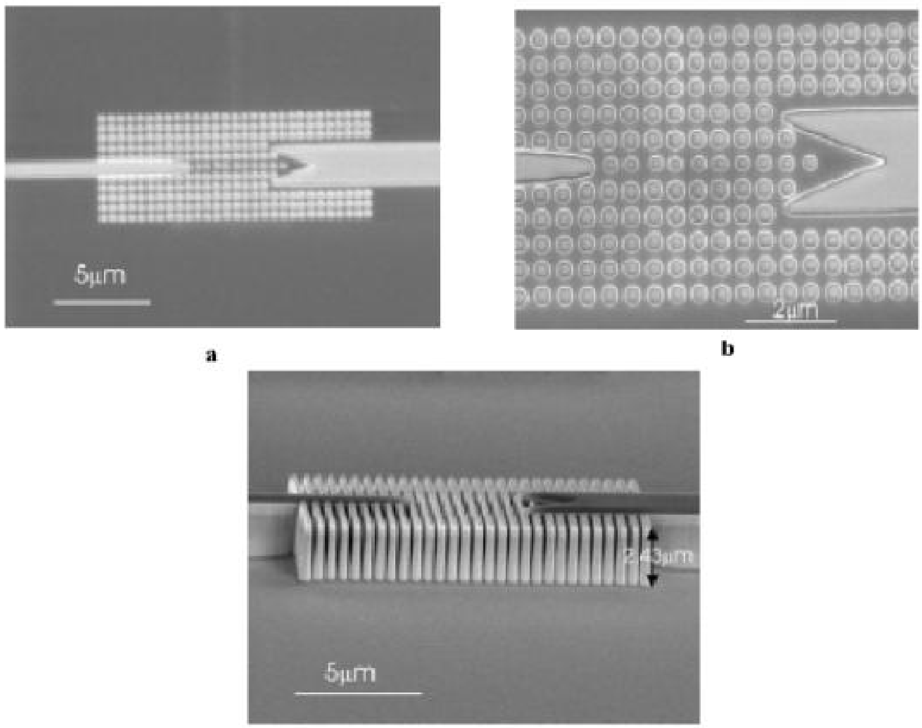
Photonic bandgap systems

# Photonic bandgap materials

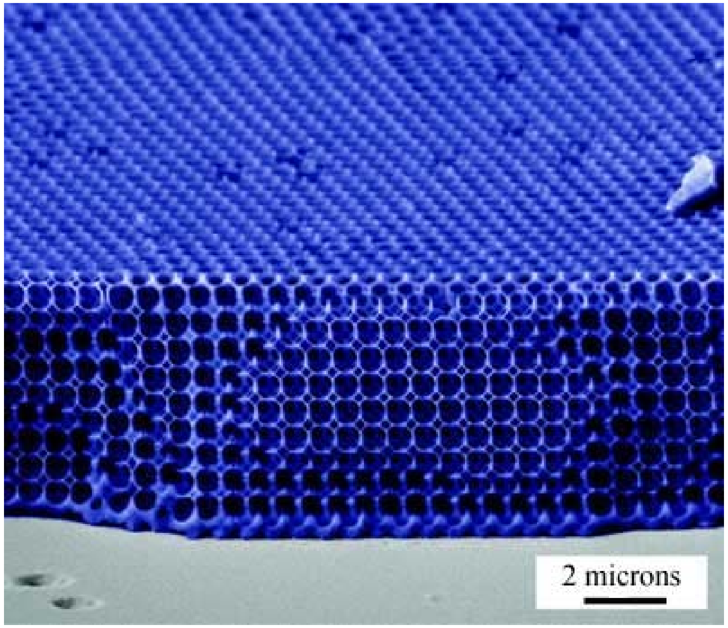
Sandia



- What is a photonic bandgap?
- Why are these metamaterials important and such a hot topic of research?
- How are photonic bandgap systems made?




MIT photonics group




NEC

# Idea of the photonic band gap

We've seen that by carefully choosing layer thicknesses and indices of refraction, it's possible to create extremely high reflectance dielectric mirrors around a specific wavelength.

This works for large numbers of periods, even if the dielectric contrast between layers is relatively small.

It turns out that the physics here (essentially no available propagating optical modes compatible with boundary conditions imposed by the periodic material) is exactly analogous to the formation of electronic band gaps in solids.

To see this in 1d, start from the wave equation:

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \mu_0 \varepsilon(x) \frac{\partial^2 E(x,t)}{\partial t^2}$$

$$= \frac{\kappa(x)}{c^2} \frac{\partial^2 E(x,t)}{\partial t^2}$$

Assume the usual time dependence:

$$E(x,t) = E(x)\exp(-i\omega t)$$

# Idea of the photonic band gap

Rearranging, $$-\frac{\partial^2}{\partial x^2}E(x)-\frac{\kappa(x)-1}{c^2}\omega^2 E(x)=\frac{\omega^2}{c^2}E(x)$$

Remember, here κ($x$) is a periodic function in space: $\kappa(x+d)=\kappa(x)$

As a result, this equation for the electric field looks remarkably like the Schroedinger equation for a particle in a 1d periodic potential.

This means solutions are in the form of Bloch waves!

$$E(x+d)=\exp(i\gamma d)E(x)$$

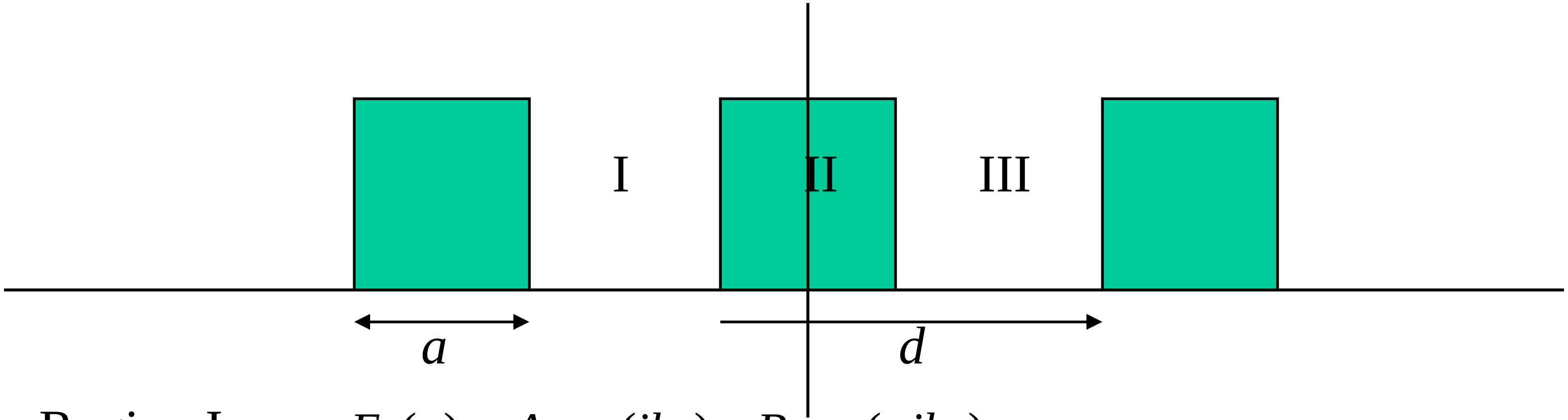
Now γ takes the role of the Bloch wavevector (or crystal momentum).

Just as in the electronic structure case,

- We can label allowed EM modes by a Bloch wavevector and a band index.
- For values of γ that coincide with a reciprocal lattice vector of the periodic dielectric array, the Bragg condition is satisfied, and the EM wave is strongly diffracted off the medium: no propagation.

# Example in 1d

We can do a case in 1d, analogous to the Kronig-Penney model from the Schroedinger equation version.




Region I: $E_I(x) = A\exp(ikx) + B\exp(-ikx)$

Region II: $E_{II}(x) = f\exp(iqx) + g\exp(-iqx)$

Region III: $E_{III}(x) = C\exp(ikx) + D\exp(-ikx)$

$k = \omega / c, \qquad q = \sqrt{\kappa}\omega / c$

Usual boundary conditions: $E$ and its derivative must be continuous at the interfaces between the regions.

# Example in 1d

As in Kronig-Penney case, can write as a matrix equation: $\begin{pmatrix} C \\ D \end{pmatrix} = \mathbf{M} \begin{pmatrix} A \\ B \end{pmatrix}$

where

$$\mathbf{M} = \begin{bmatrix} \exp(-ika)\left[\cos(qa) + i\left(\frac{k}{q} + \frac{q}{k}\right)\frac{\sin(qa)}{2}\right] & i\left(\frac{q}{k} - \frac{k}{q}\right)\frac{\sin(qa)}{2} \\ -i\left(\frac{q}{k} - \frac{k}{q}\right)\frac{\sin(qa)}{2} & \exp(ika)\left[\cos(qa) - i\left(\frac{k}{q} + \frac{q}{k}\right)\frac{\sin(qa)}{2}\right] \end{bmatrix}$$

From $E(x+d) = \exp(i\gamma d)E(x)$ we know $\begin{pmatrix} C\exp(ikd) \\ D\exp(-ikd) \end{pmatrix} = \exp(i\gamma d)\begin{pmatrix} A \\ B \end{pmatrix}$

Defining: $\mathbf{T} \equiv \begin{bmatrix} \exp(ikd) & 0 \\ 0 & \exp(-ikd) \end{bmatrix}$

We can write: $(\mathbf{TM} - \exp(i\gamma d)\mathbf{I})\begin{pmatrix} A \\ B \end{pmatrix} = 0$

# Example in 1d

$$(\mathbf{TM} - \exp(i\gamma d)\mathbf{I})\begin{pmatrix} A \\ B \end{pmatrix} = 0$$

For this to have a solution requires that the determinant of the left hand side be equal to zero:

$$\cos(\gamma d) = \cos(k(d-a))\cos(qa) - \frac{1}{2}\left(\frac{k}{q} + \frac{q}{k}\right)\sin(k(d-a))\sin(qa)$$

Just like the K-P model: solutions can only exist if the right hand side is between –1 and 1; if ω is chosen so that the rhs is outside this domain, there are no allowed solutions, and we are in the photonic band gap.

What happens when there are defects?

As you might imagine, defects (and surfaces) can introduce states in the gap.

Those states are *localized* (!), and correspond to local resonant modes or standing waves.

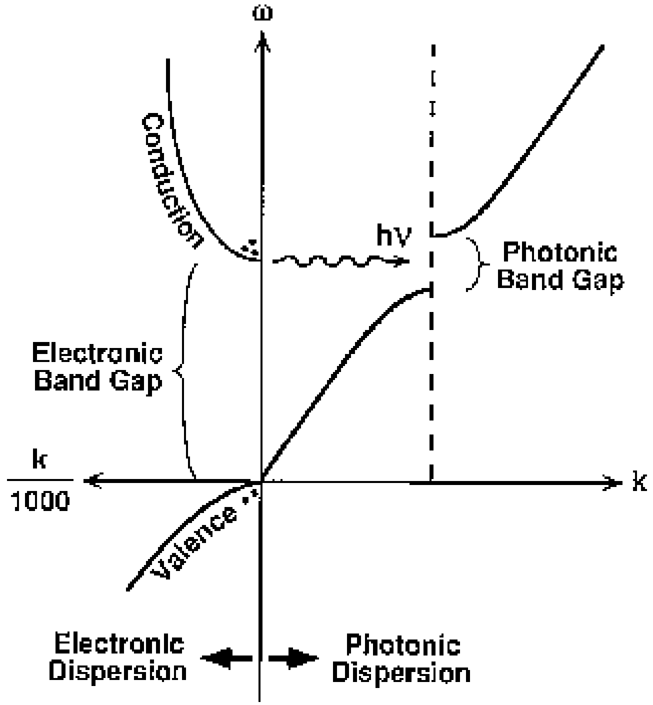
# What are the requirements to do this in higher dimensions?

Nothing too special; we just need a spatially periodic array of dielectric contrast in all three dimensions.

Remember the electronic structure case, and the “spaghetti diagrams”:  the size of the photonic band gap depends in a nontrivial way on the direction of $\gamma$ in reciprocal space.

One other restriction:  clearly spatial modulations on a scale vastly shorter than the wavelength of light we care about are not useful (basically our trial plane wave solutions for the periodic potential problem break down).  Optimal size again corresponds to typical features ~ $\lambda/4$.

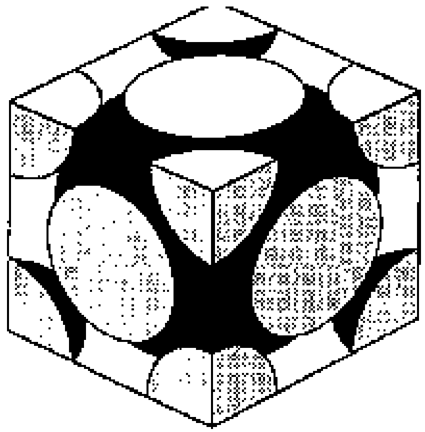
Polarization is also a complication that only really matters in higher dimensions (nonnormal incidence).




Yablanovitch, JOSA B **10** 283 (1993).

# Typical difficulty in early experimental work

Yablanovitch, JOSA B **10** 283 (1993).



8000 atoms!

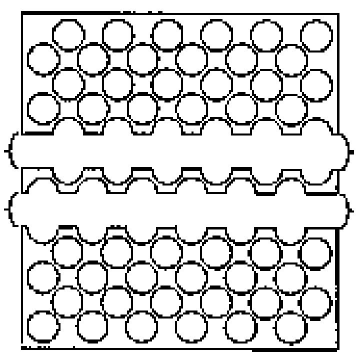
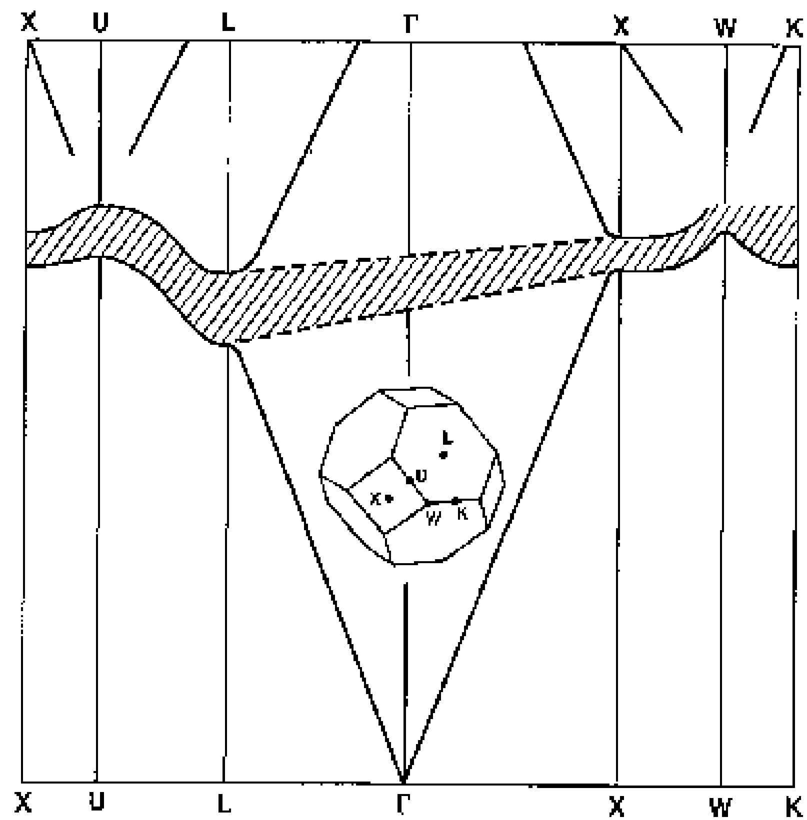


Fig. 12. Construction of fcc crystals, consisting of spherical voids. Hemispherical holes are drilled on both faces of a dielectric sheet. When the sheets are stacked up, the hemispheres meet, producing a fcc crystal.

50% VOLUME FRACTION fcc AIR SPHERES




$\frac{n_1}{n_0} = 1.6$

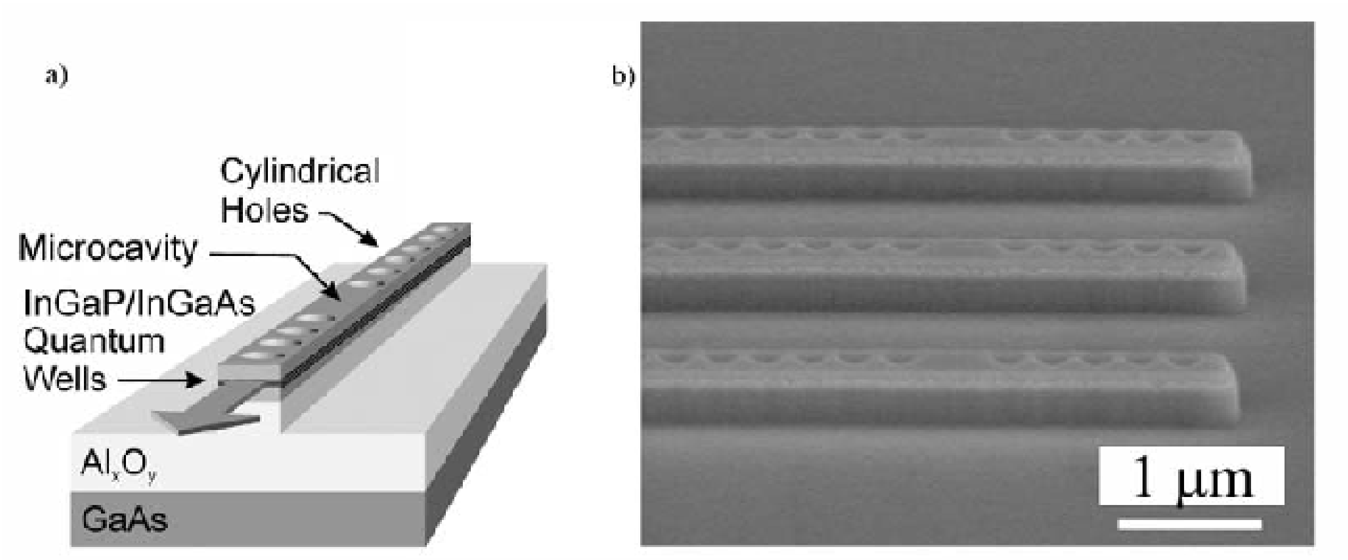
PREDOMINANTLY P POLARIZED

# Possible applications of photonic bandgap materials

- Extremely small (“microcavity”) lasers
- Superefficient semiconductor optoelectronic devices
- “Infinitely single-mode” optical fibers
- Unconventional optical waveguides and routing
- Controlled optical switching

# Microcavity lasers

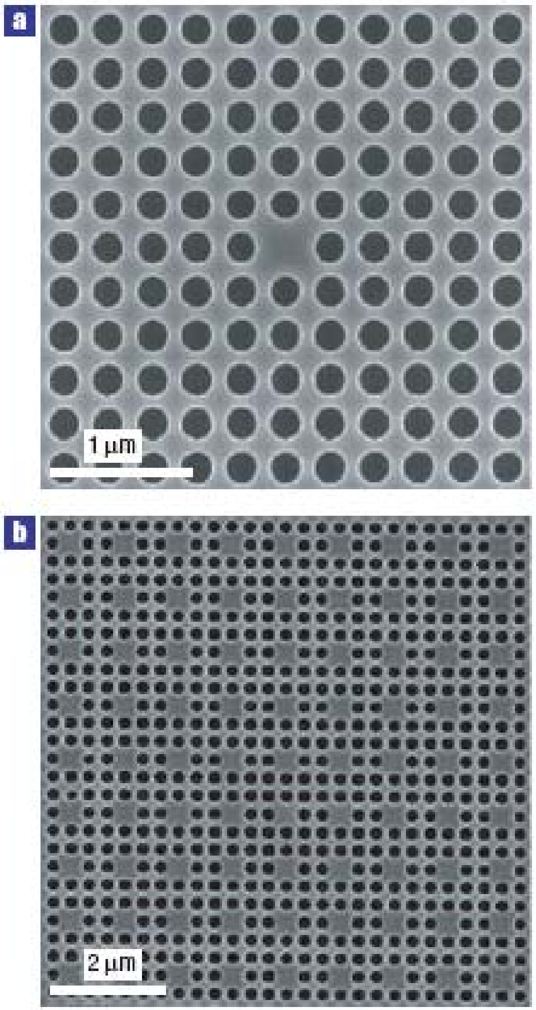
MIT

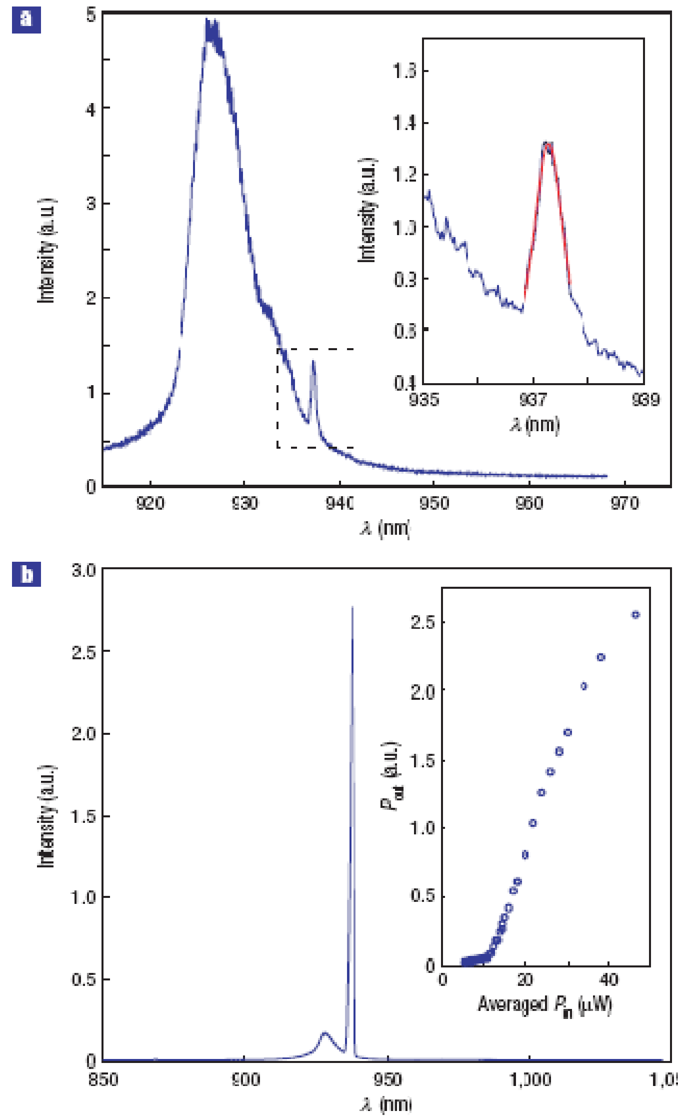



Usually many allowed modes in an optical cavity that is pumped, and only one is the lasing mode of interest. Result: much pumping energy is wasted, + other modes lead to noise in laser.

PBG solution: controlled placement of defect in PBG material leads simultaneously to a *single* well defined mode for lasing (lower threshold) and a minimum size cavity.

# Microcavity lasers



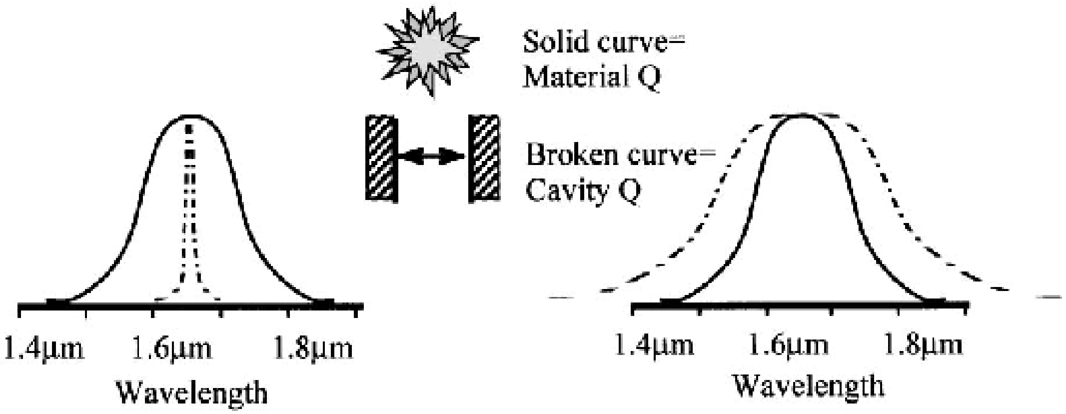

Altug *et al*., Nature Phys. **2**, 484 (2006)

# Superefficient optoelectronic devices

How does this work, a little more rigorously?

Typically a laser requires an optical cavity (the photons need to stick around long enough to cause stimulated emission). This cavity has some *Q*.

The material doing the radiating also has some effective *Q* - that is, there is a linewidth to the radiation.




Usual laser desired for LED

Krause *et al.*, Prog. Quant. Elect. **23**, 51 (1999).

# Superefficient optoelectronic devices

How to get a better match between material and cavity properties?

Sharpen up material emission by changing the available density of final states for the radiation: a cavity enhancement effect.

Free space density of states for photons: $g(\omega) = \frac{\omega^2}{\pi^2 c^3}$

Case for cavity of quality $Q$: $g_c(\omega) = \frac{2}{\pi} \frac{Q}{\omega V}$

Can get an additional factor of three enhancement if cavity is right shape to couple to emitting system well. Total enhancement is therefore:

$$f_P \equiv \frac{g_c(\omega)}{g(\omega)} = \frac{3}{4\pi^2} \frac{Q\lambda^3}{V}$$

"Purcell factor". So, a higher $Q$ and a smaller size for the cavity can greatly improve efficiency, essentially by restricting the available phase space for photon emission.

# Superefficient optoelectronic devices

## What are the physical limits on the Purcell factor?

**Ultrasmall Mode Volumes in Dielectric Optical Microcavities**

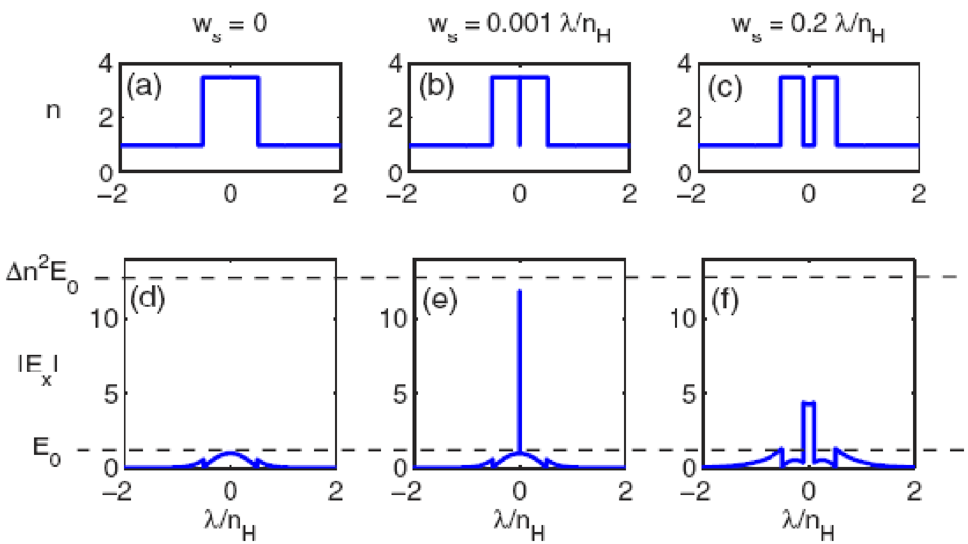
Jacob T. Robinson, Christina Manolatou, Long Chen, and Michal Lipson

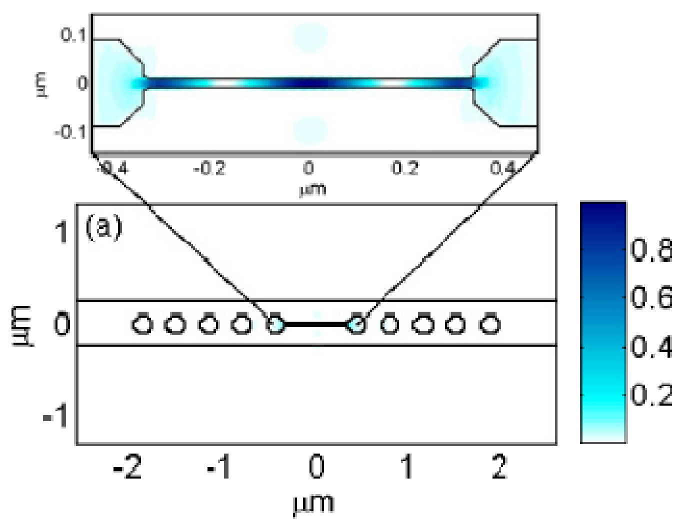
*Department of Electrical and Computer Engineering, Cornell University, Ithaca, New York 14853, USA*

(Received 3 May 2005; published 27 September 2005)

We theoretically demonstrate a mechanism for reduction of mode volume in high index contrast optical microcavities to below a cubic half wavelength. We show that by using dielectric discontinuities with subwavelength dimensions as a means of local field enhancement, the effective mode volume ($V_{\rm eff}$) becomes wavelength independent. Cavities with $V_{\rm eff}$ on the order of $10^{-2}(\lambda/2n)^{-3}$ can be achieved using such discontinuities, with a corresponding increase in the Purcell factor of nearly 2 orders of magnitude relative to previously demonstrated high index photonic crystal cavities.

## How should this work? ***Slot waveguides***




Robinson *et al*., Phys Rev. Lett. **95**, 143901 (2005)

# Superefficient optoelectronic devices

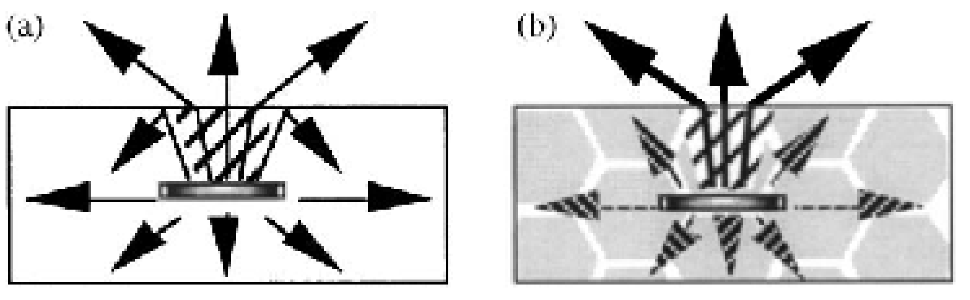
Consider a typical GaAs/AlGaAs light emitting diode.

Quantum efficiency (# of photons produced per e-h pair) can be exceedingly high (~ 99%). Photons are typically emitted isotropically.

However, because of high index of refraction, only those intersecting the wafer surface within 16 degrees of normal are able to be transmitted at all; the rest experience total internal reflection.

Only 3-15% of the light generated is therefore useful.

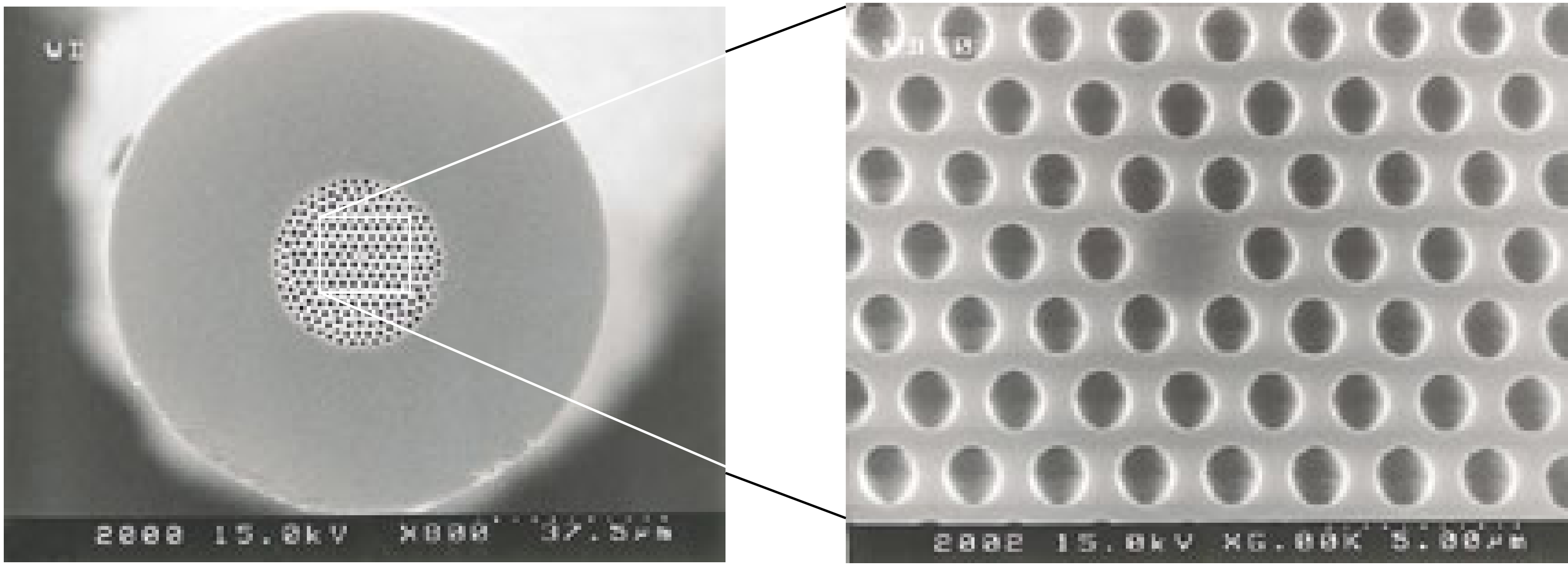
With a correctly designed PBG LED, though, one can stack the deck so that the LED active region only couples to a single propagating mode!




Krause *et al.*, Prog. Quant. Elect. **23**, 51 (1999).

# Photonic crystal fibers

Nakazawa group, Tohoku Univ.



Apply same basic idea of restricted density of states available for loss mechanisms to optical fibers.

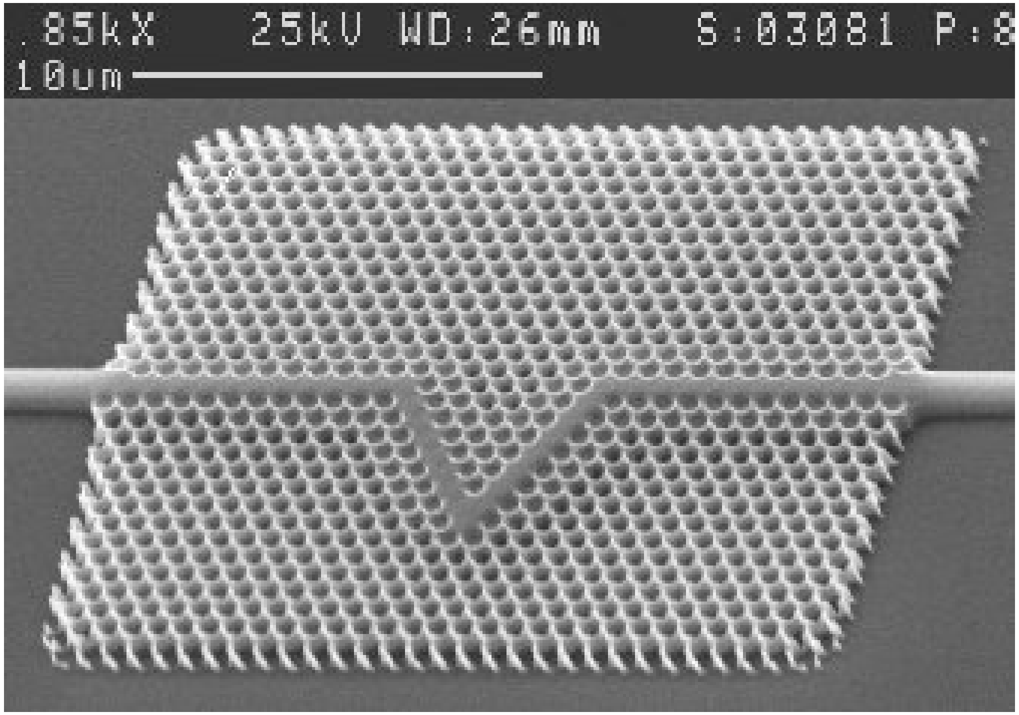
Can cut down dramatically on couplings between modes, and possibly open up new switching technologies (ex.:  fluid in voids that can be moved to allow positioning of lossy segment of fiber).

# Waveguides and optical routing

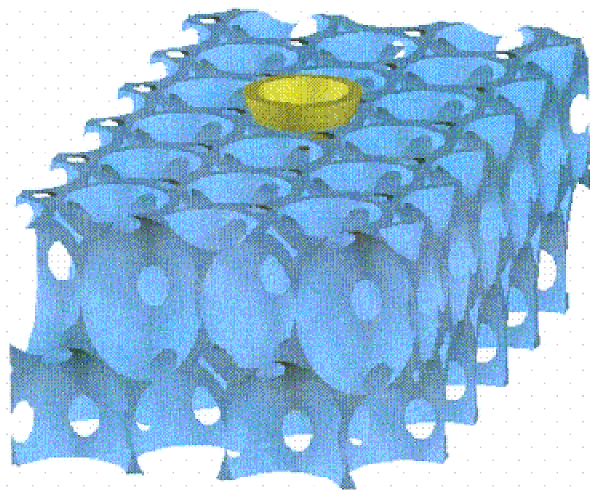
University of Jena, Germany

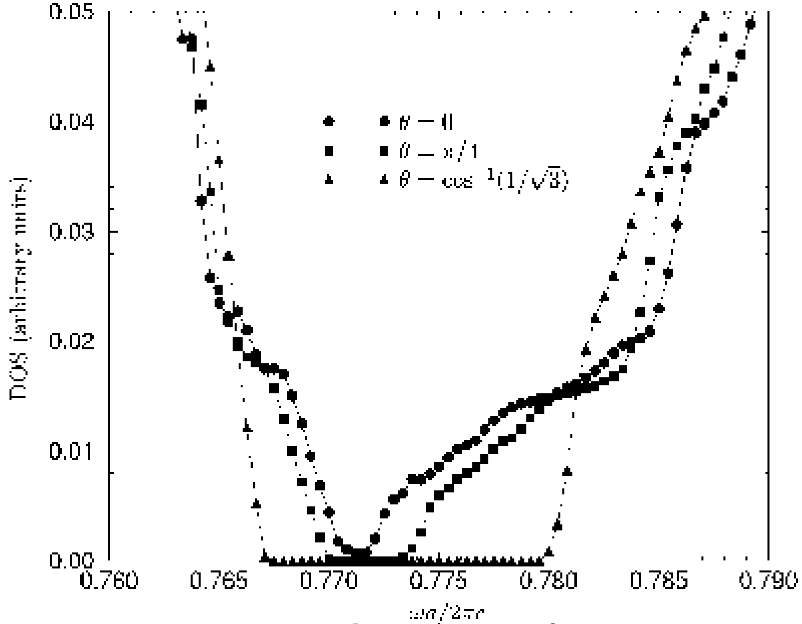
We've already said that an isolated defect in a PBG material can act as a resonant cavity.

Extended series of defects can be used as waveguides, and it's possible (because of the very special properties of the surrounding PBG medium) to steer light in unusual ways - around sharp corners, for example.



# Controlled optical switching






Busch, University of Toronto

Infiltrate a PBG material with a liquid crystal, which you already know is birefringent and tunable.

Reorienting LC molecules can open and close the PBG reversibly!

# Controlled optical switching

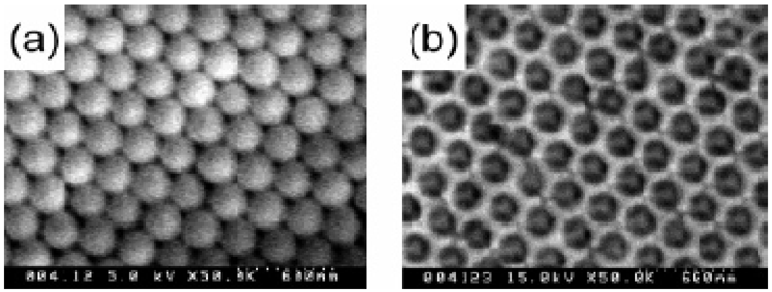
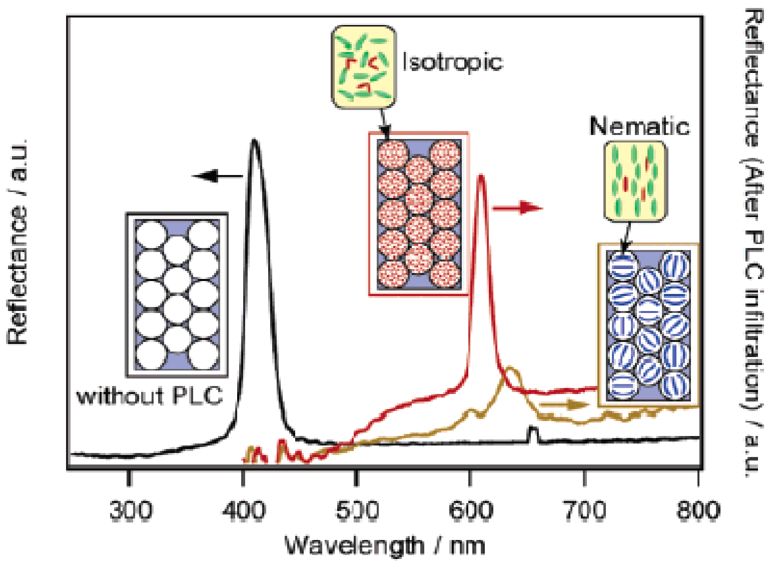
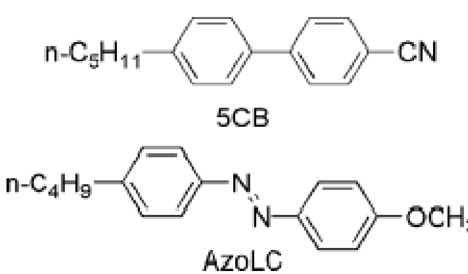
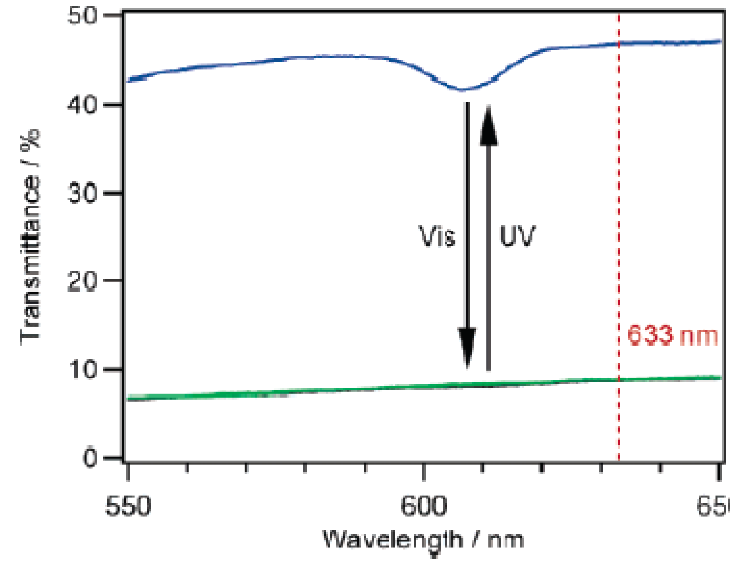



**Figure 1.** SEM images of polystyrene opal film (a) and $SiO_2$ inverse opa film (b) prepared from polystyrene spheres with a diameter of 260 nm.





Kubo *et al*., Chem Mat. **17**, 2298 (2005).

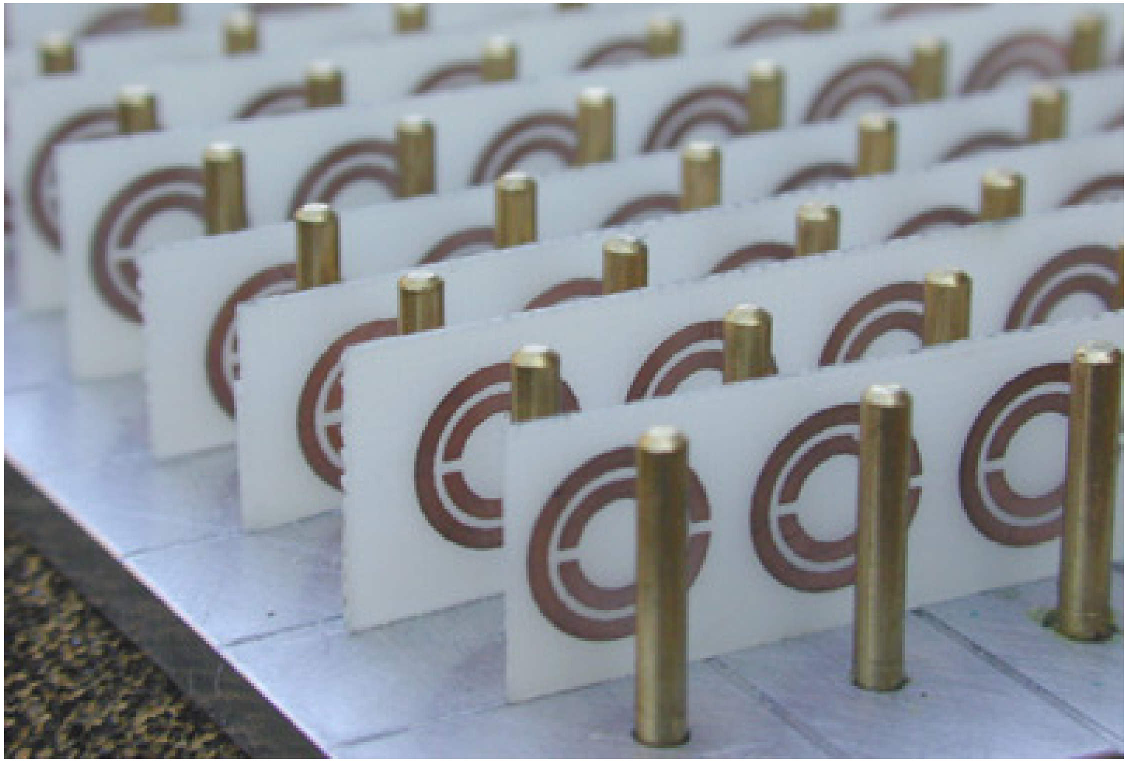
# How are these metamaterials made?

- Early implementations
- Lithography + etching:  holes
- Lithography + etching:  posts
- Lithography + etching:  complicated 3d shapes
- Self-assembly:  opals
- Self-assembly:  inverted opals

# Early implementations

Much early work (late 1980s) was done using microwaves rather than IR or visible light.

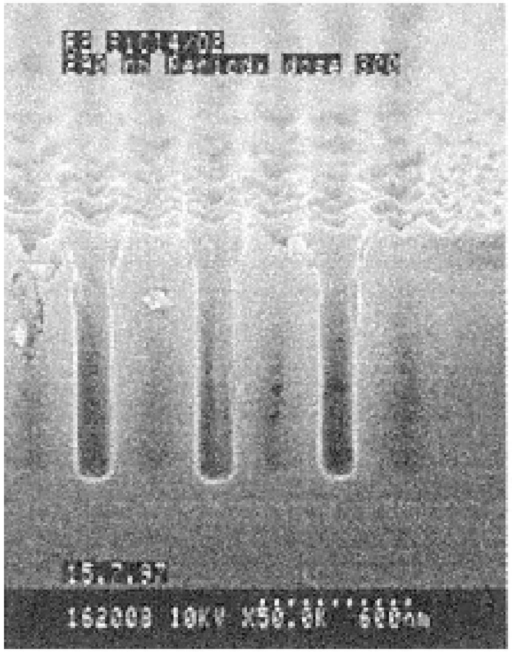
Vastly easier to assemble materials - can be done by hand with simple tools!

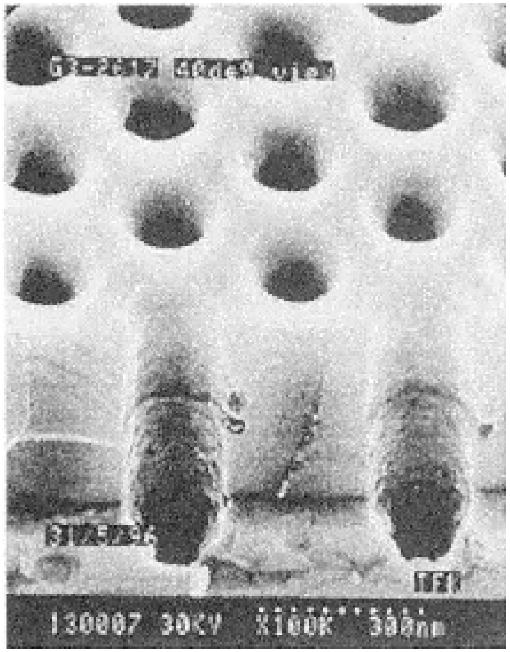


D. Smith, UCSD

# Holes and posts

For 1d and 2d systems, basic lithography and etching are typically the way to go:



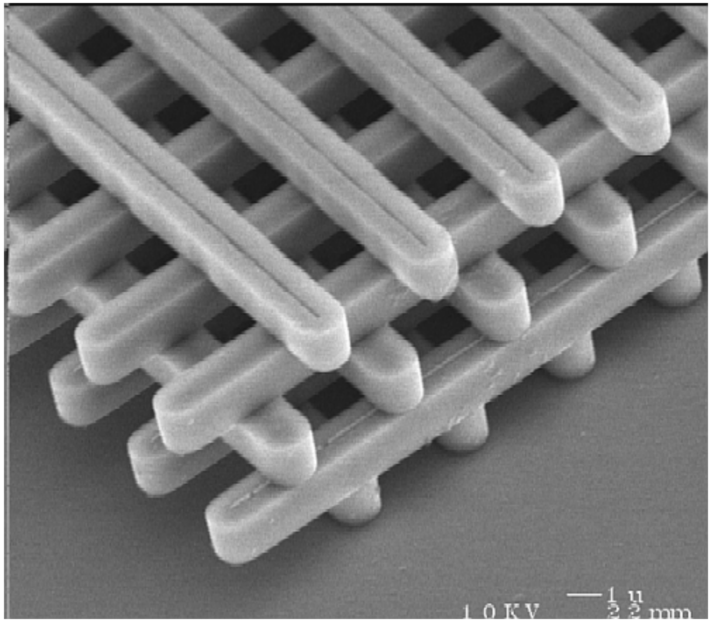

Krause *et al.*, Prog. Quant. Elect. **23**, 51 (1999).

# Complicated 3d shapes

Clearly executing 3d periodic architectures many wavelengths deep by doing purely surface processing (like previous slide) is incredibly challenging.

Best approach lithographically is to use either multilayered starting substrates (as we'll see in micromachining), or *interference lithography* and photopolymerization.

Sandia



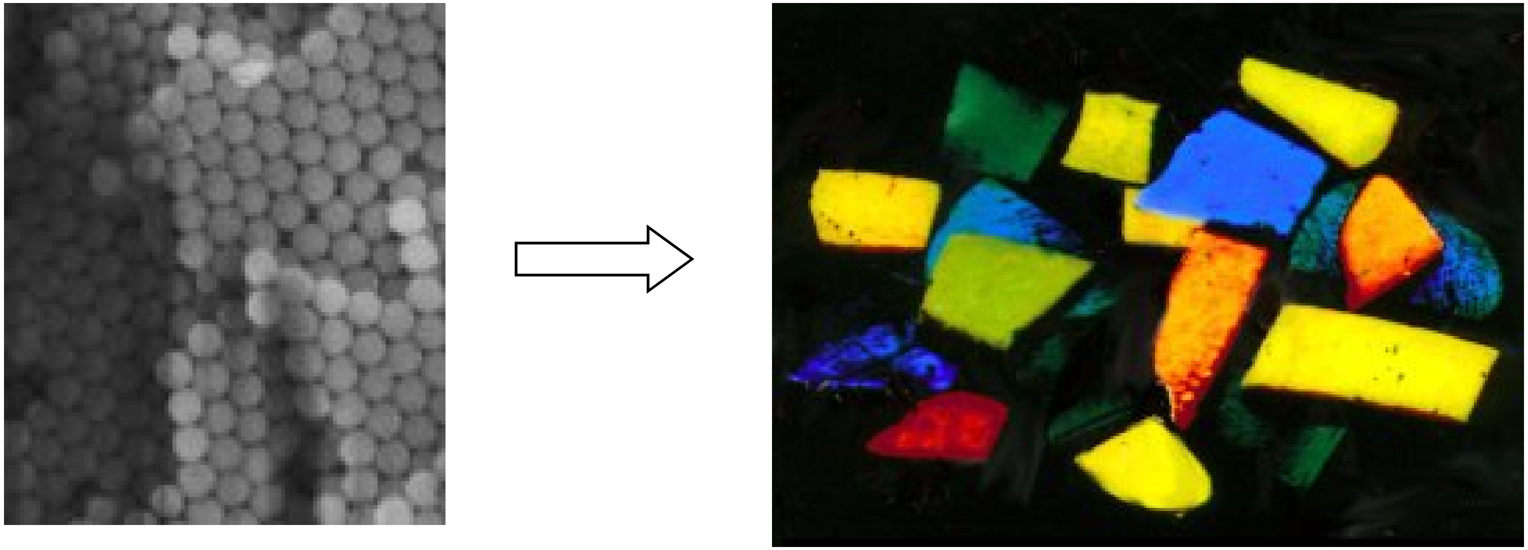
**Big advantage of lithography**: controlled fabrication of defects (cavities, waveguides, etc.)

# Opals

Natural opal looks opalescent because of intrinsic photonic band gap properties due to its structure.

Opals consist of hydrated silica spheres ~ hundreds of nm in diameter stacked in regular patterns.

Through work with colloids, it is possible to create artificial opal structures *by self assembly*:

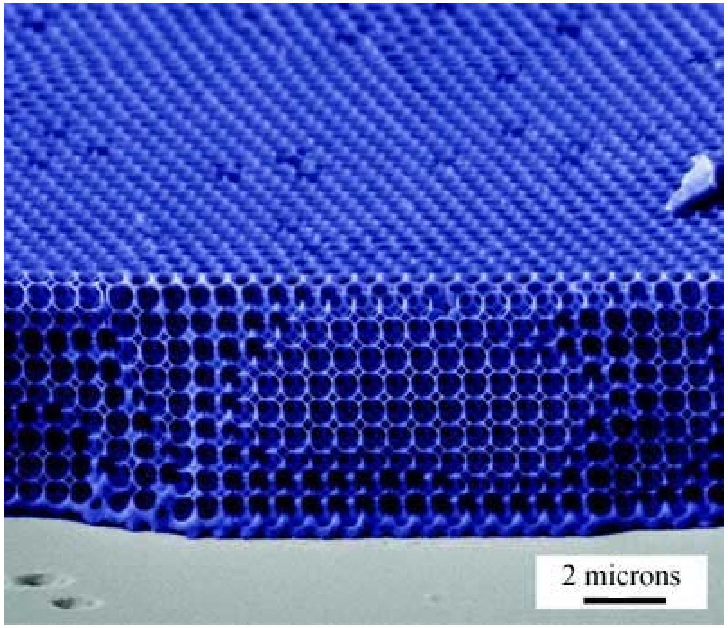


University of Sheffield, UK

# Inverted opals

Another exciting variant on this is to use self-assembly to create an opal structure, infiltrate a high index material (Si?  Se?  $TiO_2$?), and remove the original opal spheres, to get the highest possible dielectric contrast.

At right, an inverted opal photonic crystal made by infiltration growth of amorphous Si by low pressure chemical vapor deposition.  The original spheres were then etched away.




NEC

**Big advantage of self-assembly**:  large volume 3d periodic structures are naturally produced!

# Conclusions:

- 3d structuring of dielectics on the nano scale makes possible an enormous variety of optically interesting structures.

- Tremendous technological potential here, with top-down and bottom-up assembly techniques both likely to play a role.

- Eventual goal: all-optical processing of information.

- Moral: possible hidden opportunities out there even when the essential physics is “well understood”.