PHYS 534: Nanostructures and Nanotechnology II

- Logistical details
- What we’ve done so far
- What we’re going to do: outline of the course

Logistics

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Phone: x3214
E-mail: natelson@rice.edu

Lectures: MWF 13:00 - 13:50

Problem sets: mostly weekly, handed out Wednesday due following Wednesday. (45% of grade)

First paper (25%)

Second paper (25%)

Participation (5%)

Course webpage: http://www.owlnet.rice.edu/~phys534

More logistics

Text: no specific text. There will be some written lecture notes, as well as handouts based on a number of books. Also a large number of papers. I'm working on a textbook for this course and 533. If by some miracle I am able to get draft sections into workable form this semester, I'll hand them out accordingly.

Goal: to make you literate about the physics of nanostructures and their current and future roles in technology, to the point where you at least know what to consider and where to look for more information.

Smaller class size may mean that we can have more discussions and less pure lecturing.

Assignments will include readings of papers so that we can talk about them in class, as well as some very brief written assignments (paragraph-length).

What you’ve done so far

- Review of condensed matter physics
  - Band theory
  - Electronic properties of bulk solids
  - Finite size effects for quantum systems
- Physical electronics - industry demands
  - Semiclassical
  - Quantum effects: Tunneling, Landauer-Buttiker
- Nanoelectronics
  - Nanoscale FETs
  - Single electron devices
  - Molecular electronics

What you've done so far

- Magnetism and electronics - industry demands
  - Domains
  - Couplings between magnetism and current
- Nanomagnetism
  - GMR, TMR, MRAM

What we're going to do:

Physical optics

Photonics

Nanophotonics

Continuum mechanics

Microelectromechanical systems (MEMS)

Nanoelectromechanical systems (NEMS)

Fluid mechanics

Micro- and nanofluidic devices

Integrated nanosystems: nanobiotechnology, sensors

Overall theme: new developments in manipulating matter at the nanoscale lead to very exciting commercial and scientific possibilities.

Physical optics

Refresher about E&M field propagation and effects of interfaces:

- Bragg reflectors, dielectric mirrors, optical waveguides (fibers)
- Basics of lasers
- Evanescent waves, near-field effects, plasmons

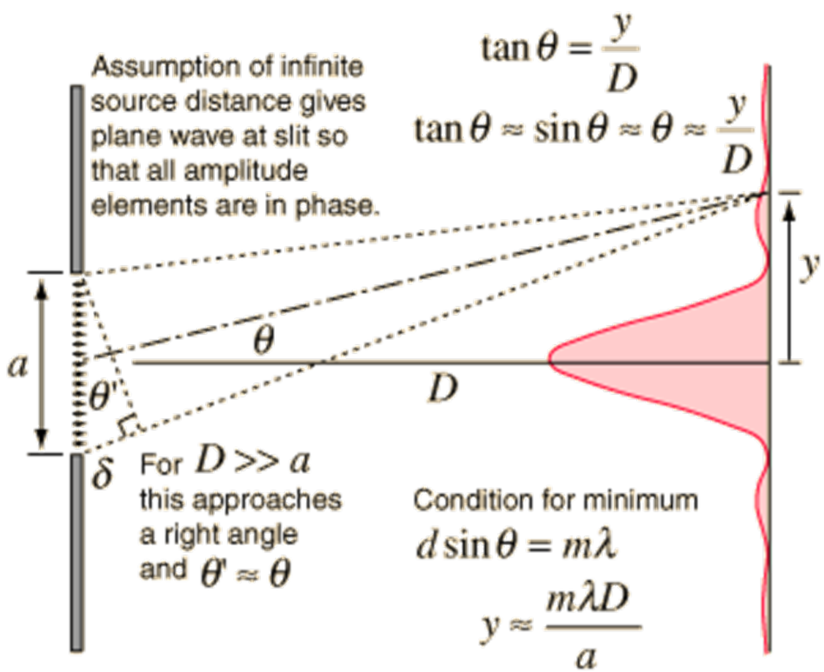


Image from Hyperphysics (GSU)

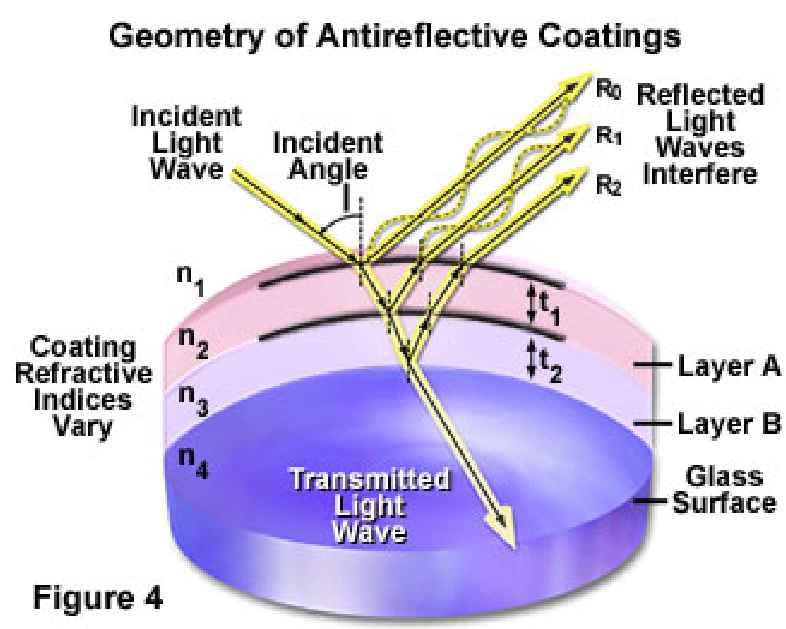
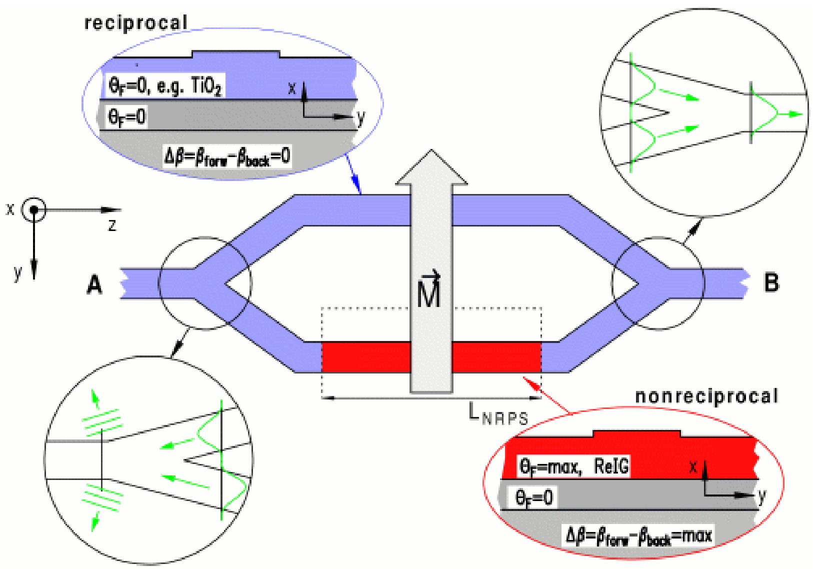


Image from FSU

Photonics

Photonics and optoelectronics - industrial needs

- Semiconductor lasers
- Resonators
- Interferometers and optical switching



http://www.physik.uni-osnabrueck.de

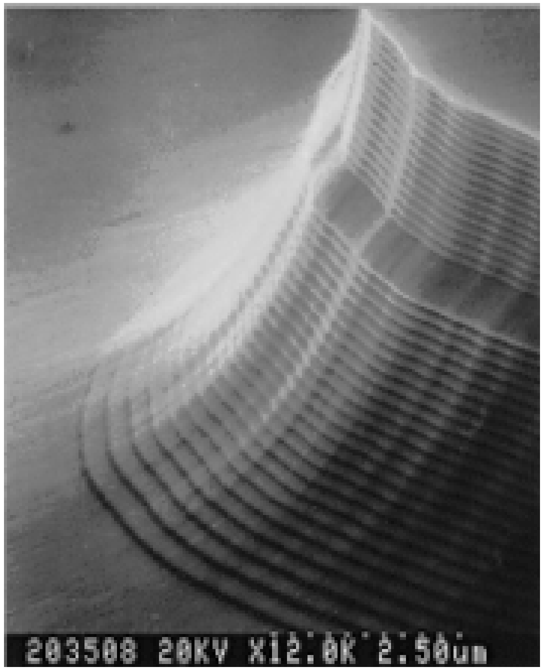
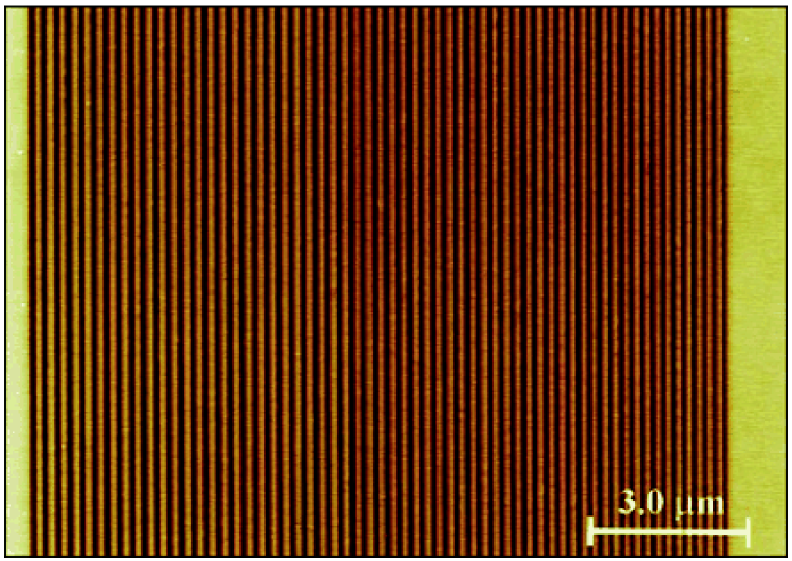


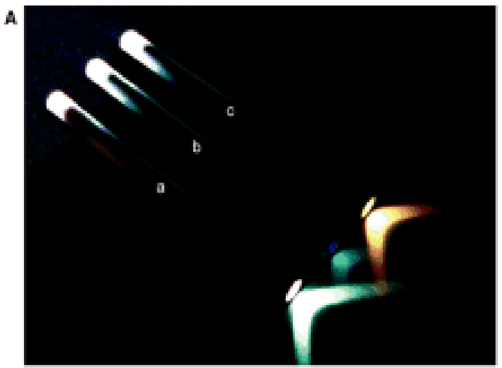
Figure from Parry and Khan, Oxford University Mat. Sci.

Photonics

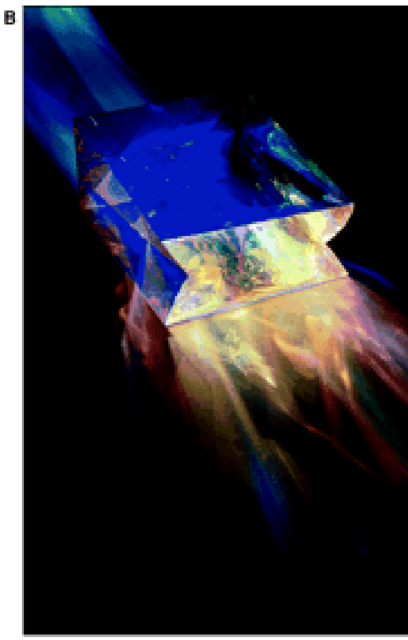
Nanostructured photonic materials: polymer-based mirrors and modulators



A



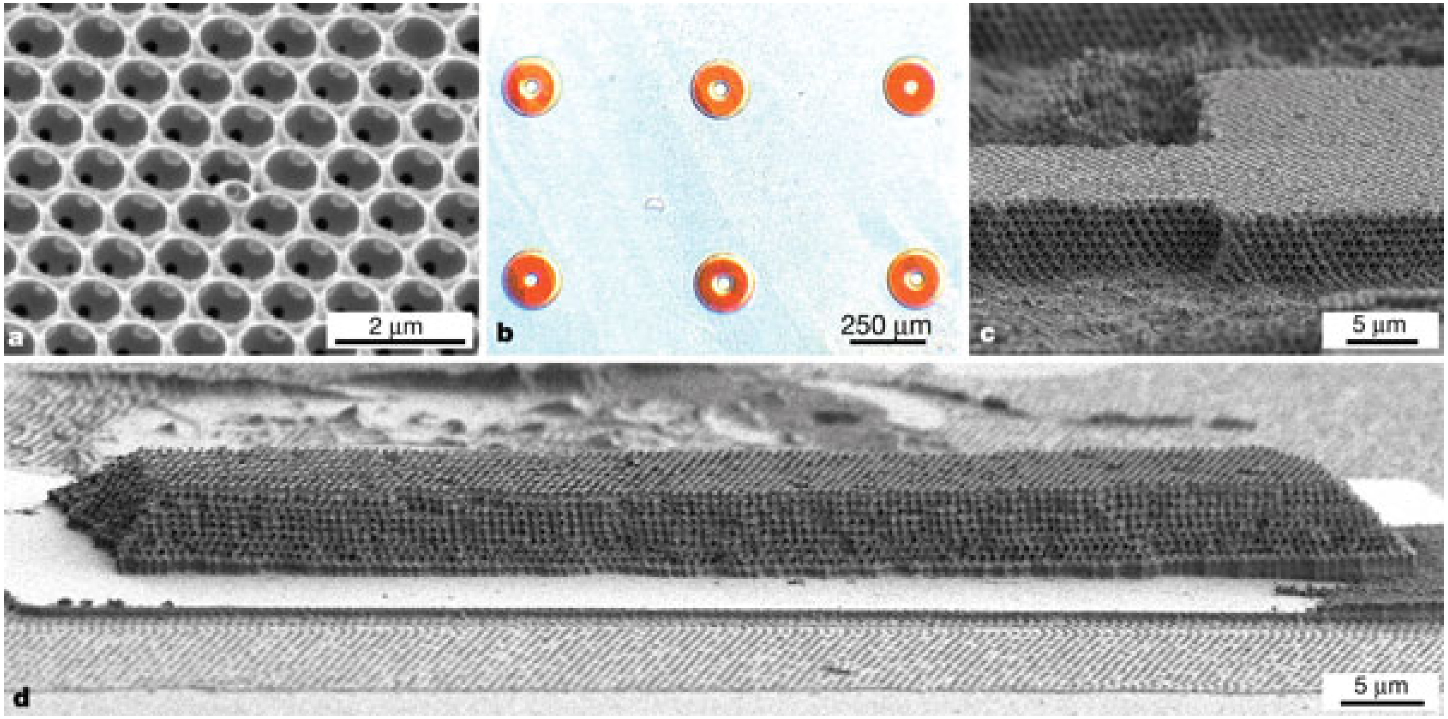
B



Figures from Weber *et al.*, Science **287**, 2451 (2000).

Photonics

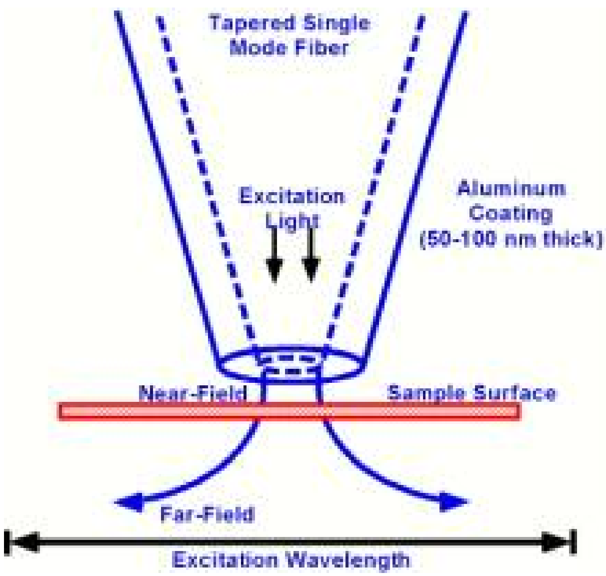
Photonic band gap materials for integrated photonics



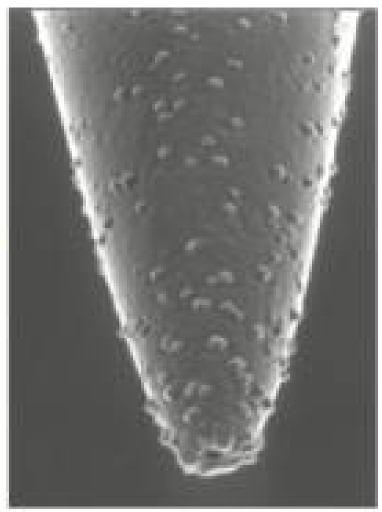
Figures from Vlasov *et al.*, Nature **414**, 289 (2001).

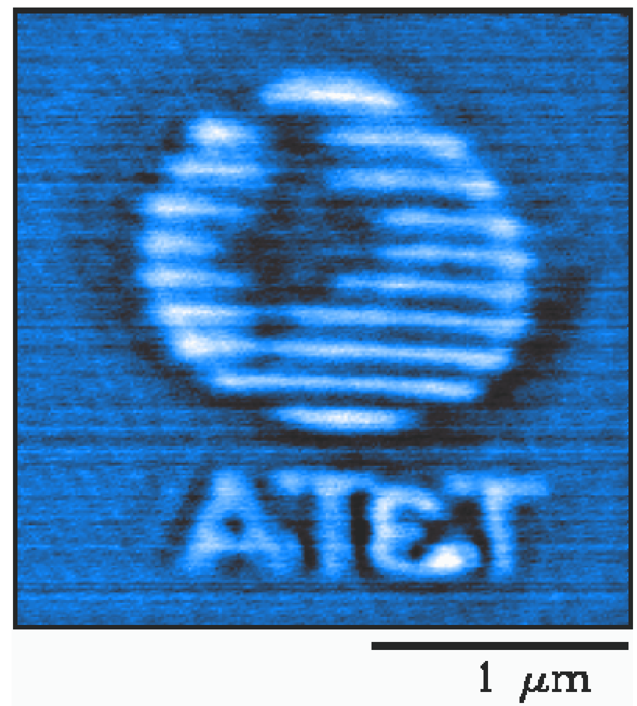
Photonics: near-field

Near-field + evanescent fields for patterning and imaging well below the diffraction limit.



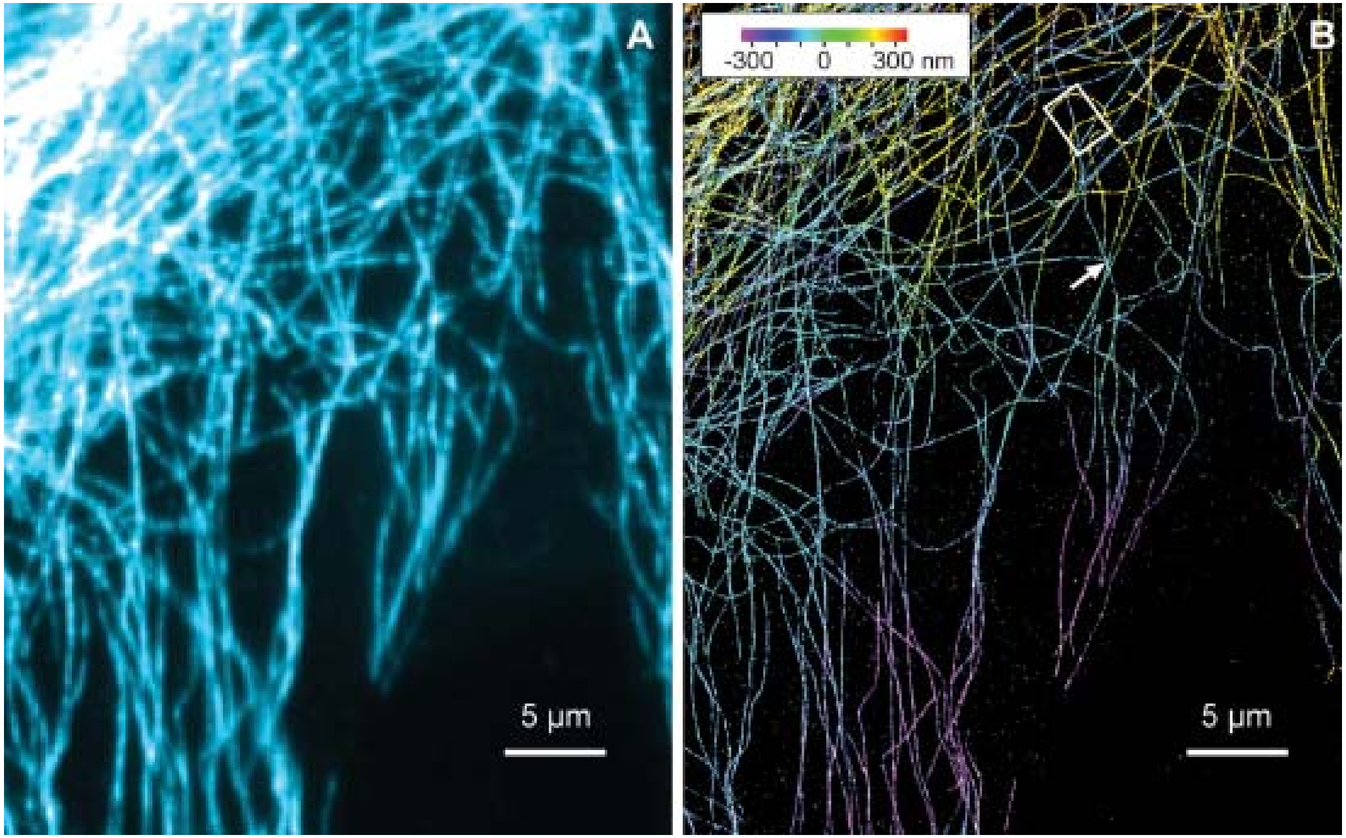
Univ. of Kansas





Bell Labs

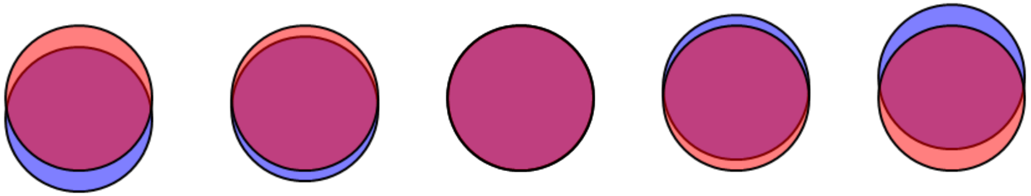
Photonics: “superresolution”



Huang *et al*., Science (2008), in press.

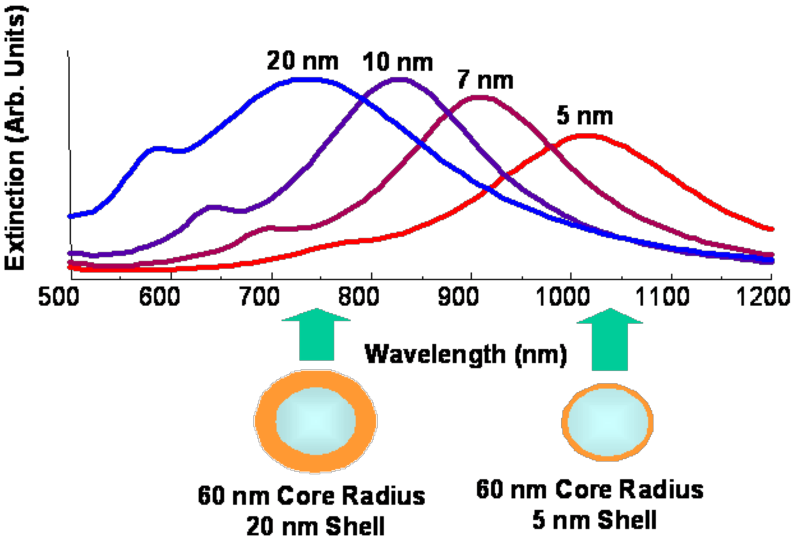
Photonics: plasmons

Sub-wavelength metal nanoparticles or metal surface patterns exhibit *plasmon* resonances.



These lead to:

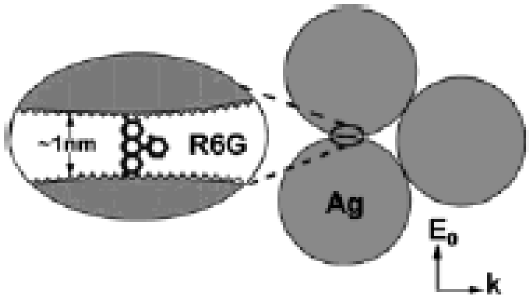
- remarkable optical properties
- local concentrations of electric field intensity
- possibility for controlled manipulation of optical energy at small scales.



Halas lab

Photonics: novel spectroscopies

Jiang *et al.*, J. Phys. Chem. B **107** 9964 (2003)



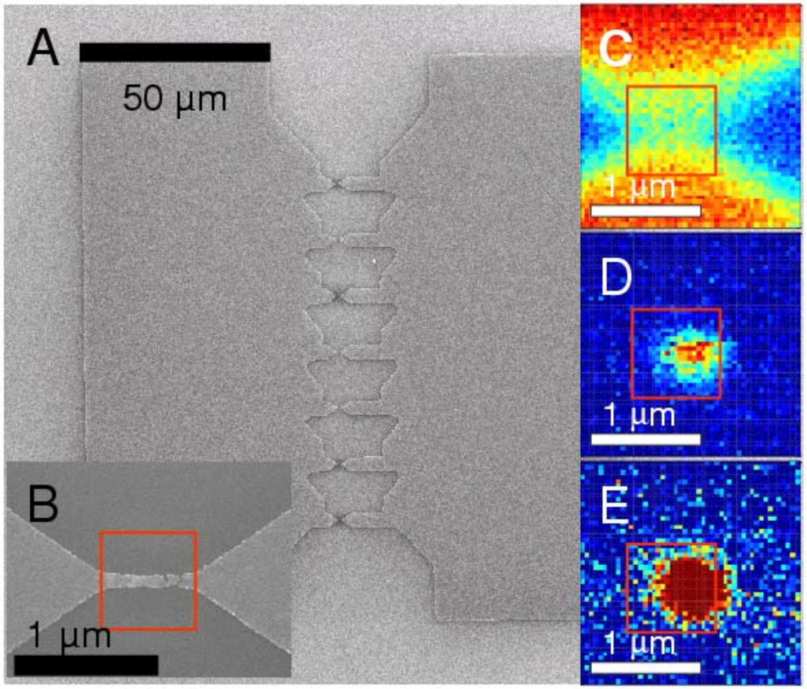
Nanostructured metals

- Colloids
- Nearly-complete nanoshells
- Nanoscale electrodes

Small gaps in metal surfaces -> dramatically enhanced local electric fields when illuminated.

Raman intensity ~ $|\boldsymbol{E}|^4$

Result: enhancements of Raman scattering by up to 10^{14} (!)



Ward *et al.*, Nano Lett. **7**, 1396 (2007)

Photonics: superlenses and metamaterials

Ozbay *et al.*, Science **311**, 189 (2006)

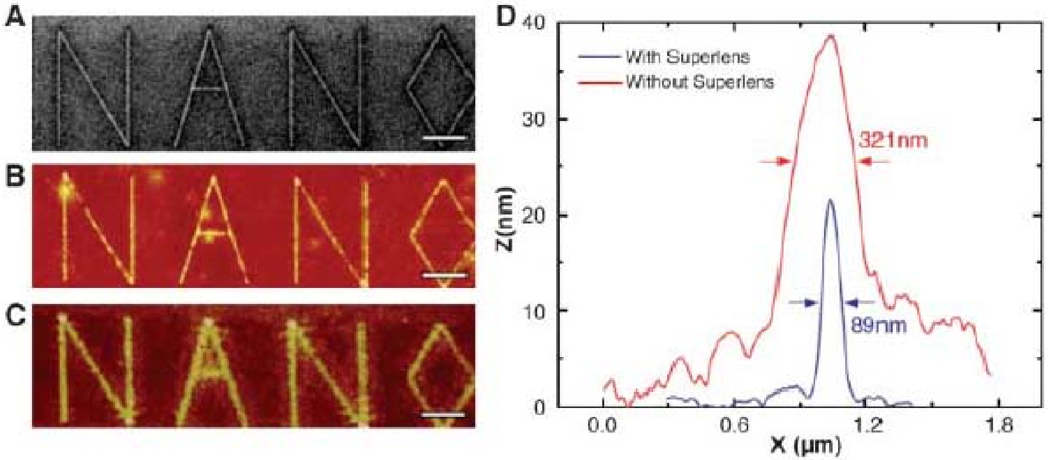
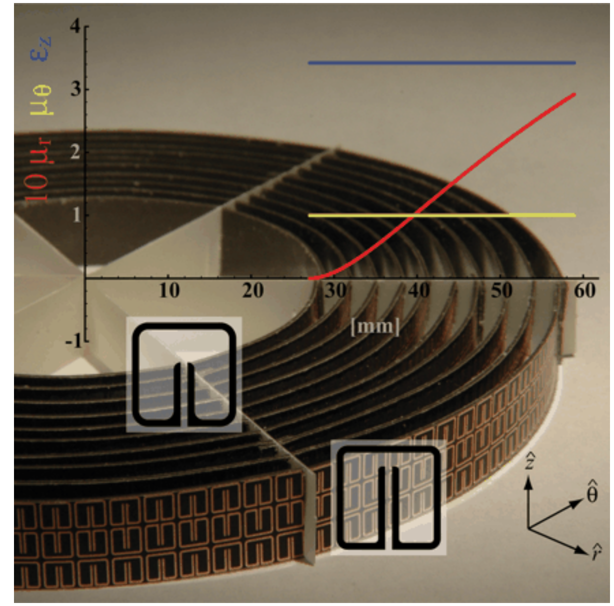
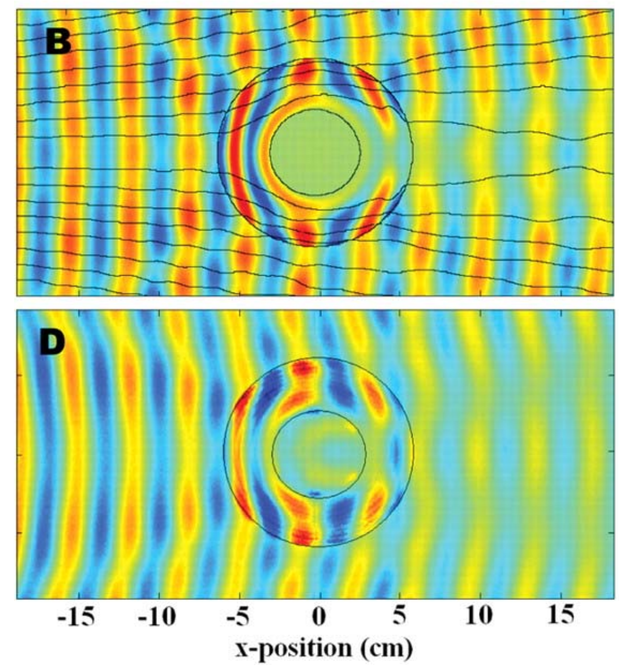


Fig. 4. The images of an arbitrary object obtained by different methods. (**A**) FIB image of the object. (**B**) The image obtained on photoresist with a silver superlens. (**C**) The image obtained on photoresist with conventional lithography. (**D**) Comparison of both methods. [Adapted from (*40*)]

- Beating the diffraction limit.
- “Invisibility cloaks”.

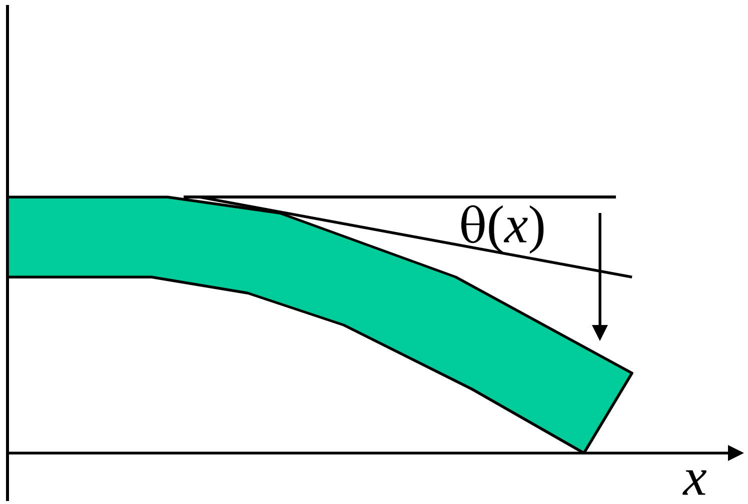


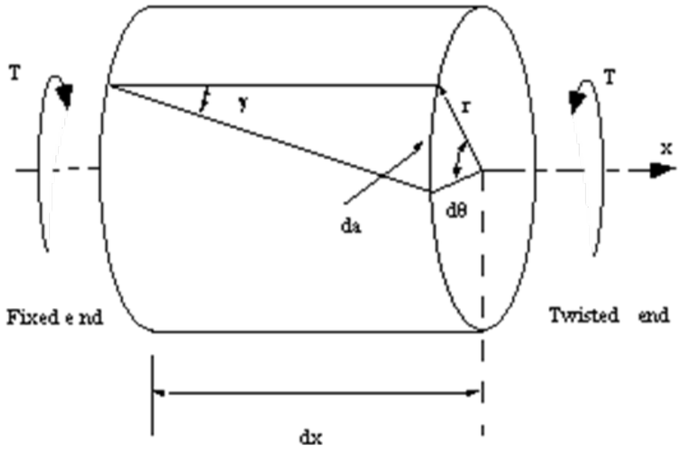


Schurig *et al.*, Science **314**, 977 (2006)

Continuum mechanics

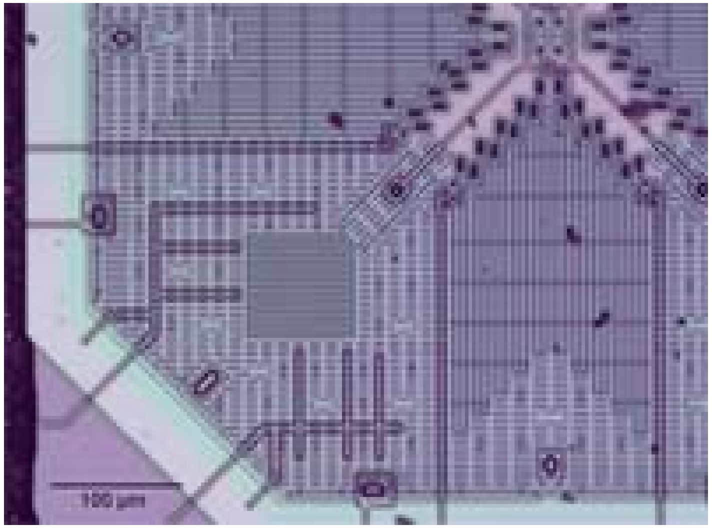
- Basics of continuum elasticity theory
- Bending of beams; torsion of rods
- When should continuum theory break down?

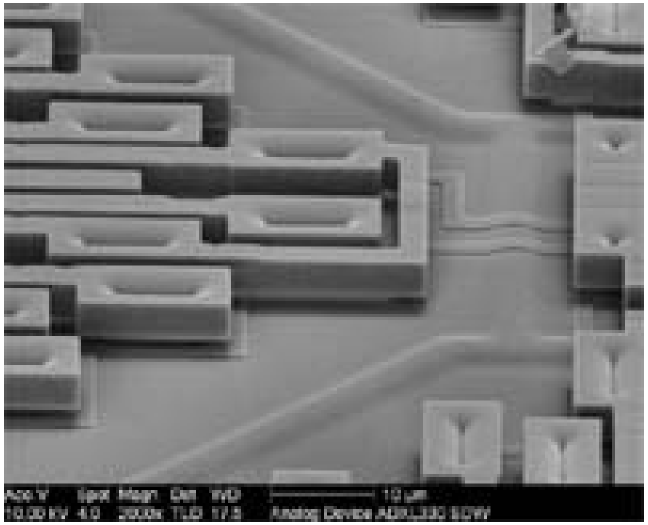




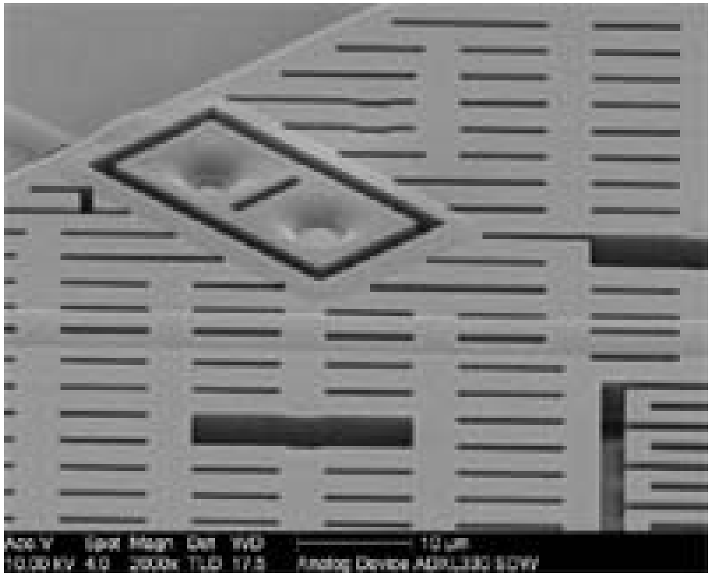
MEMS / NEMS

Commercial application: accelerometers



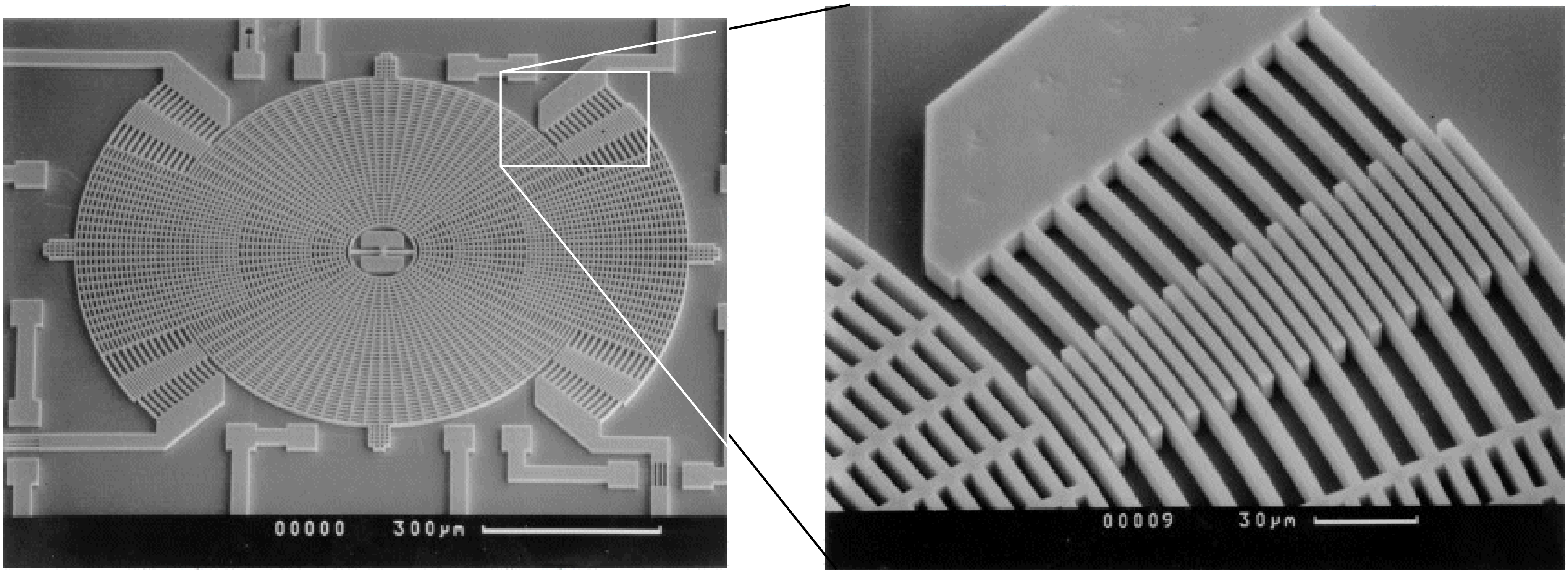


Analog Devices **ADXL330**



MEMS / NEMS

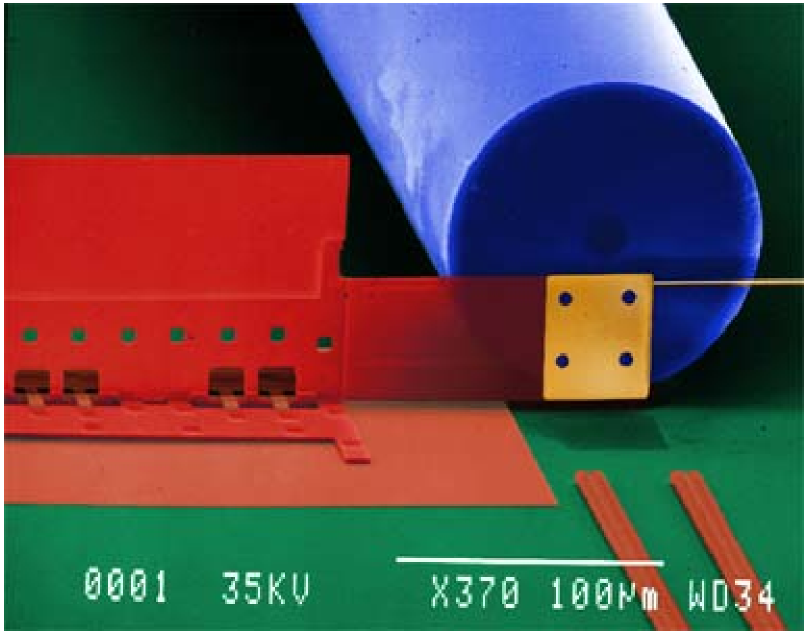
Commercial application: gyroscopes

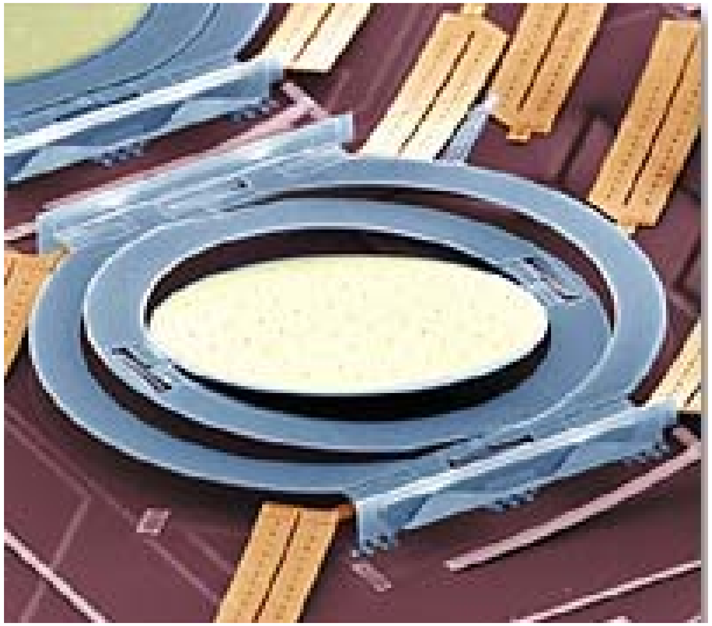


Physics of operation, detection, and limitations of state of the art micromechanical devices.

MEMS / NEMS

Commercial application: optical switching

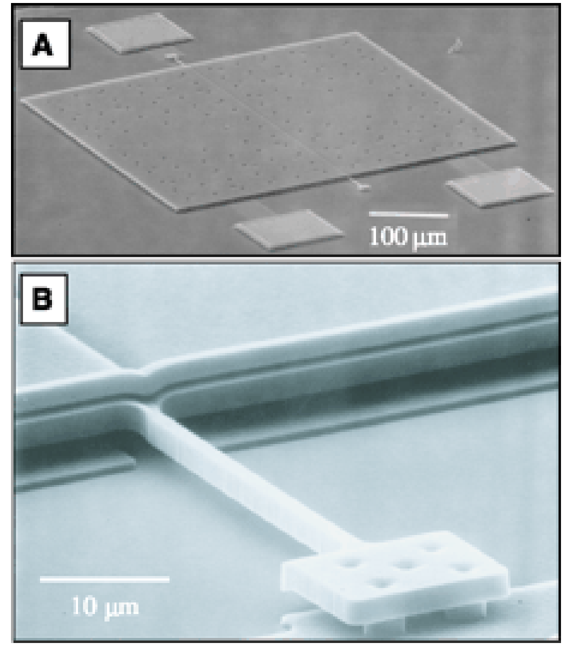




MEMS / NEMS

Fundamental science:

- Quantum forces
- Quantum limits on resonators + damping
- True quantum *mechanics*.



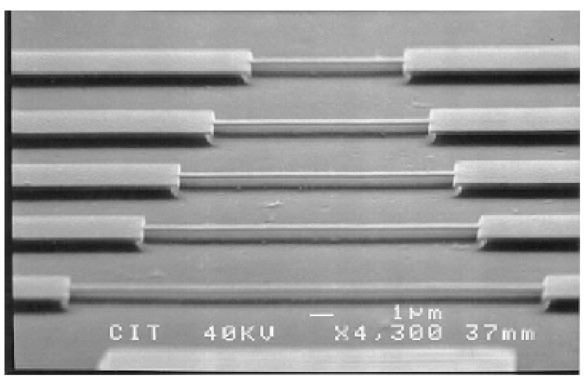


FIG. 2. A series of silicon beams with transverse dimensions in the submicron scale.

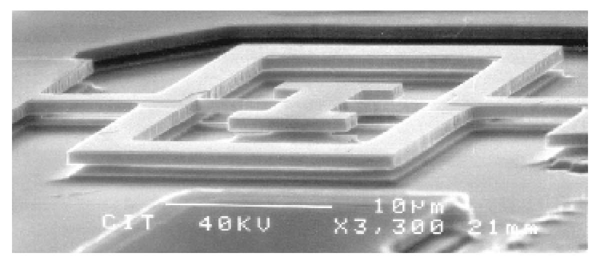


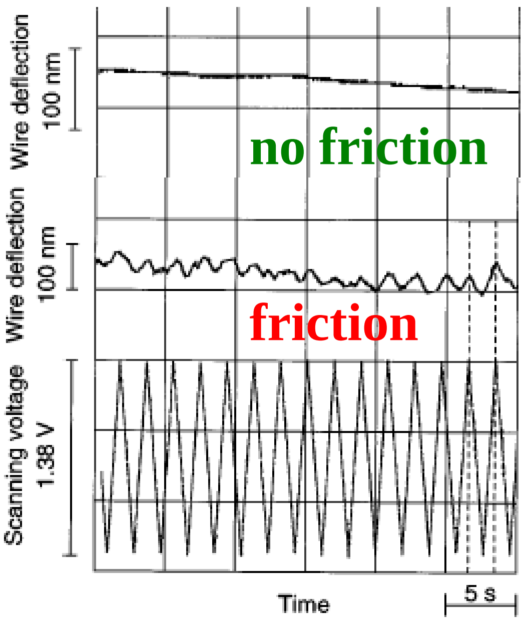
FIG. 4. SEM image of the suspended GaAs resonator. The submicron-sized thin rod connecting the outer and inner torsion elements supports the strain for the antisymmetric torsion, mode C.

mode A, symmetric torsion: mode B, antisymmetric torsion: mode C and the second flexure: mode D. Antisymmetric tor-

MEMS / NEMS

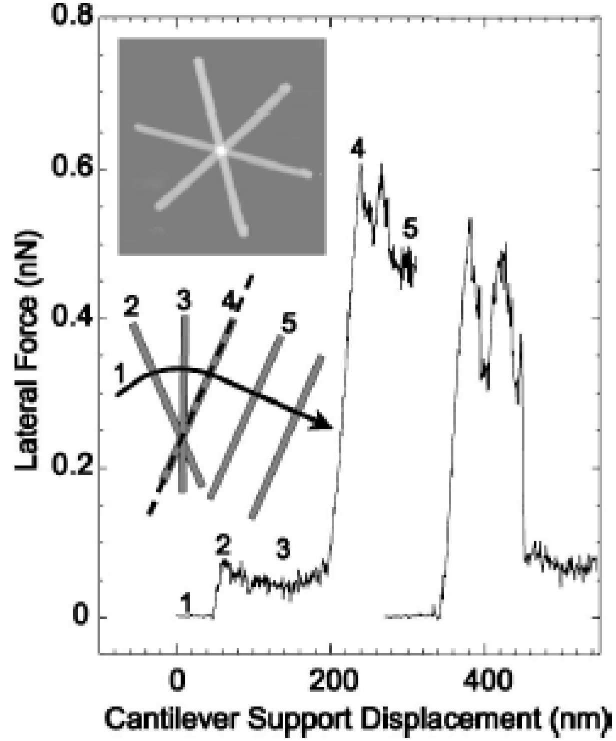
Fundamental science:

- Origins of friction
- Superlubricity (!)



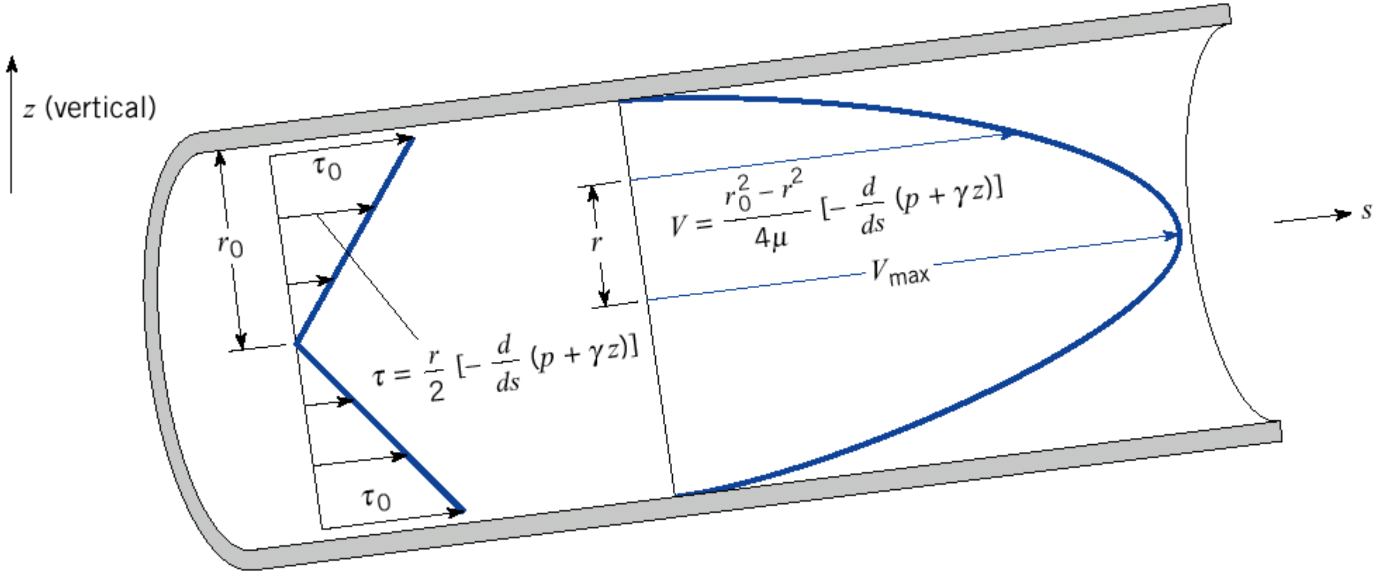
Hirano *et al.*, Phys. Rev. Lett. **78** 1448 (1997)

Falvo *et al.*, Phys. Rev. B **62** R10665 (2000)



Fluid mechanics

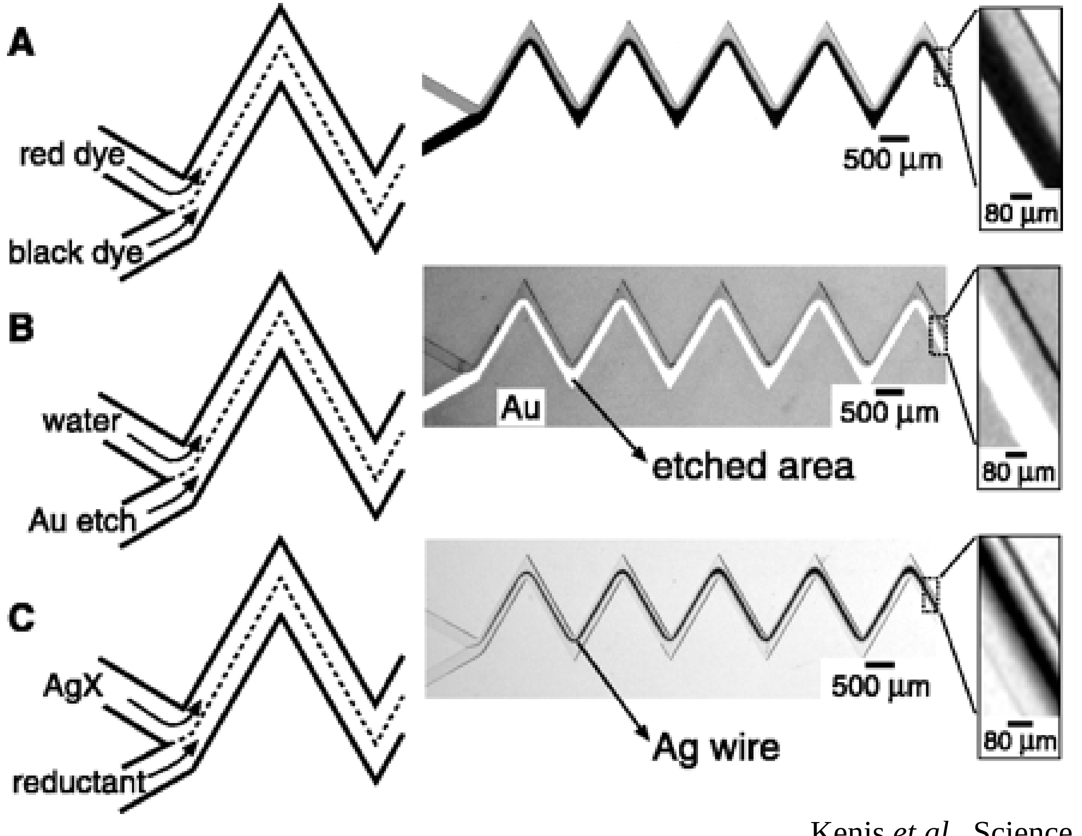
- Dimensional analysis and scaling
- Basic fluid mechanics
- Viscosity and laminar flow: “Life at low Reynolds number”



Microfluidic devices

Combine micro / nanofabrication capabilities and fluids to contrl fluid flows on very small length scales.

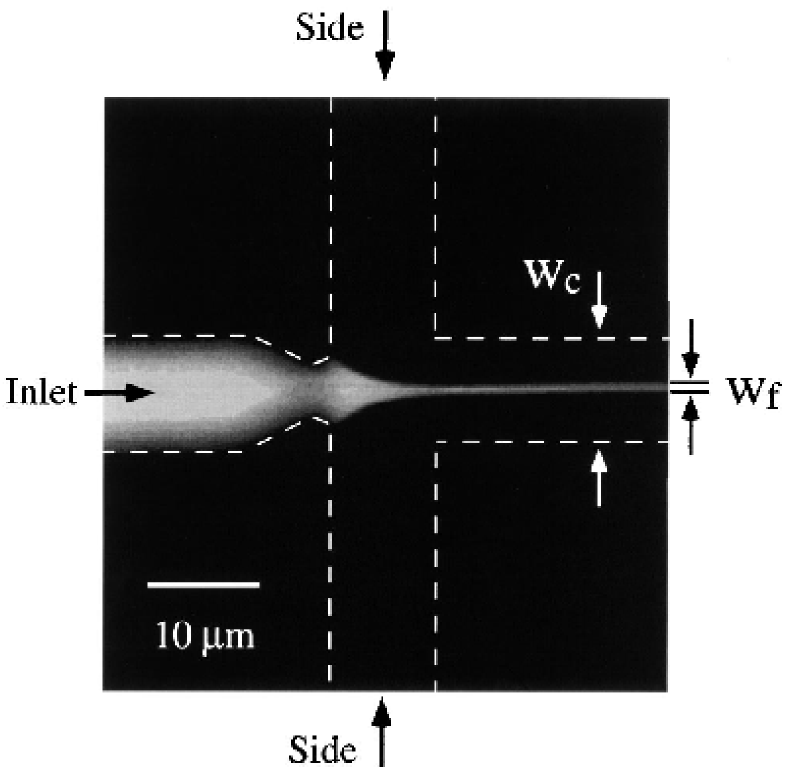
Very low Reynolds numbers = extremely laminar flow.

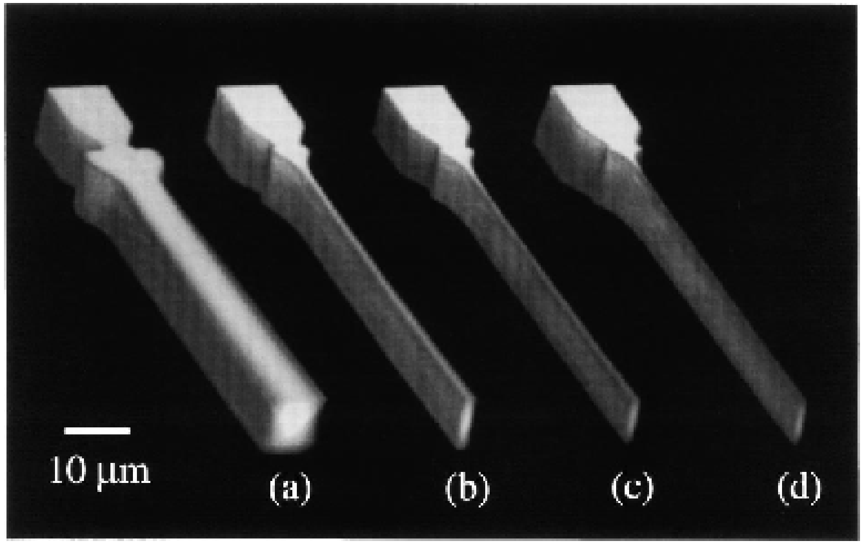


Kenis *et al.*, Science **285**, 83 (1999)

Microfluidic devices

Hydrodynamics allows manipulation and confinement of nanoscale quantities of fluids.



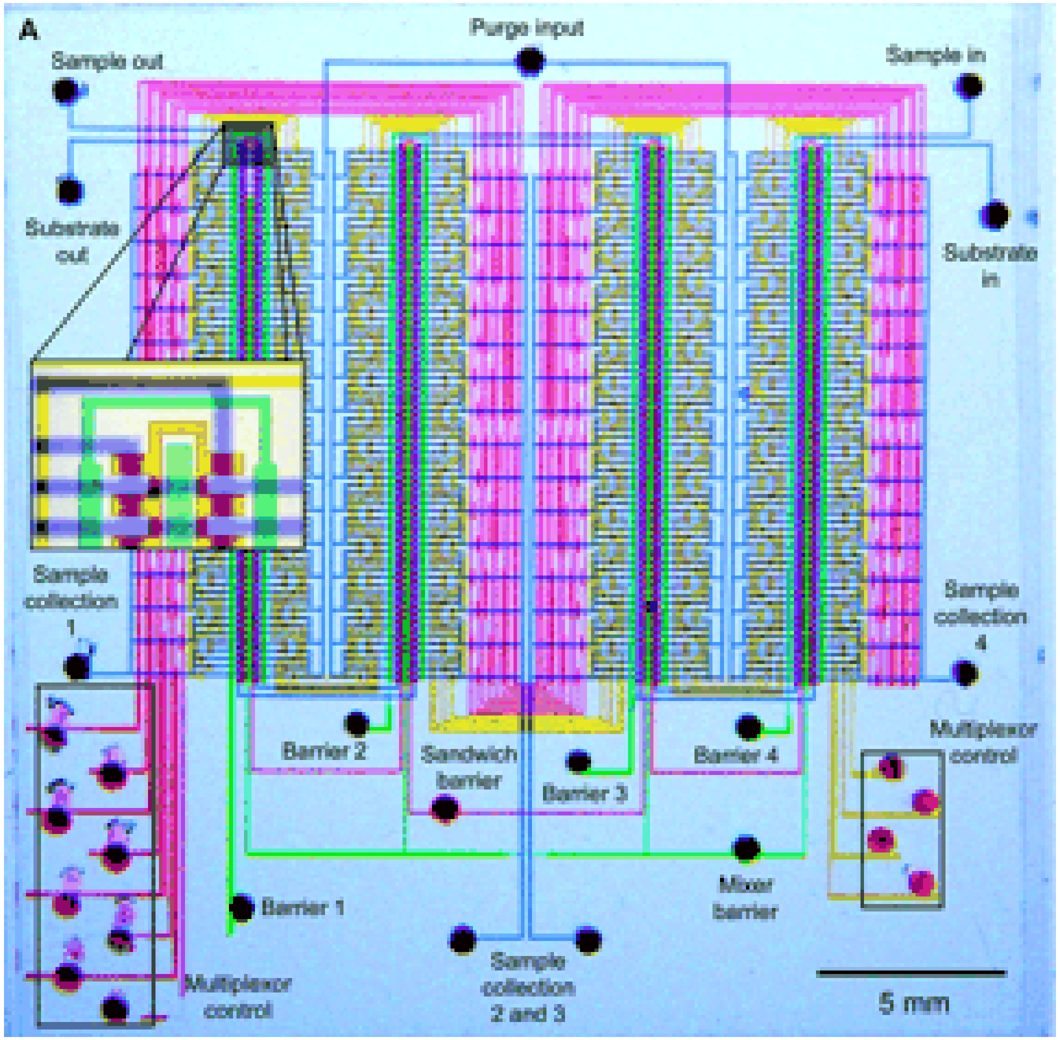


Knight *et al.*, PRL **80**, 3863 (1998).

Microfluidic devices

Can make valves, pumps, etc. using elastomers and microfabrication.

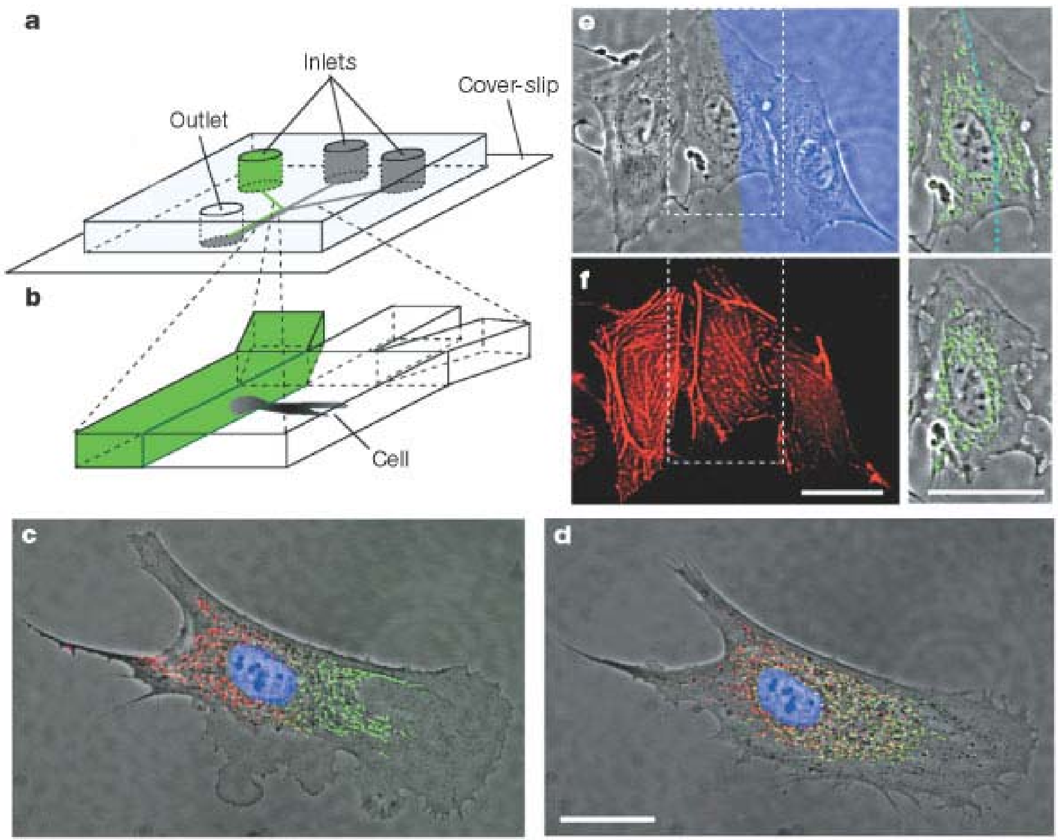
Result: “Lab-on-a-chip” capabilities.



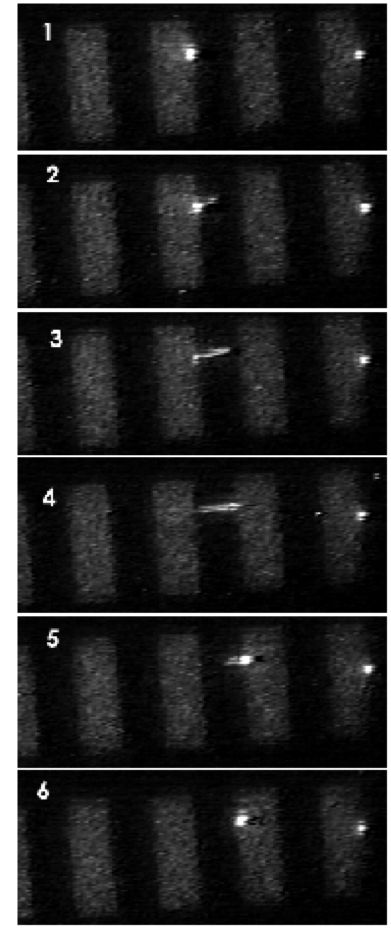
Thorsen *et al.*, Science **298**, 580 (2002).

Microfluidic devices

Takayama *et al.*, Nature **411**, 1016 (2001)



Craighead group, Cornell



Makes possible studies of individual cells, sorting of cells, manipulation of individual macromolecules, etc.

Nanoscale fluid manipulation

Pure science issues, too:

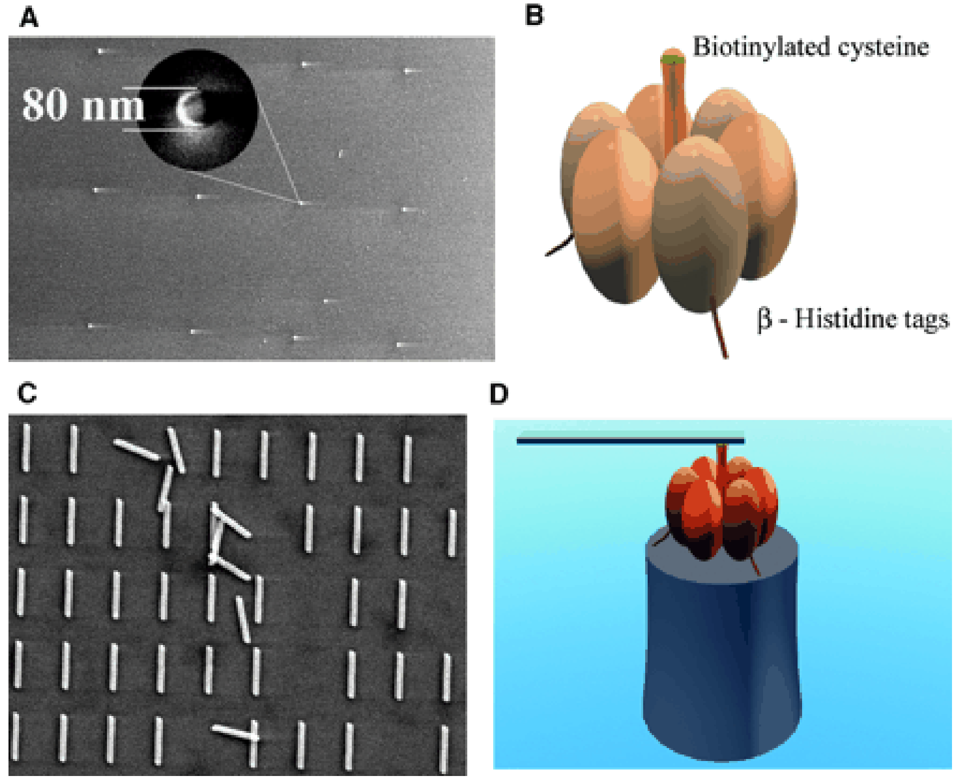
- No-slip condition at walls?
- Breakdown of continuous medium properties in confined geometries?

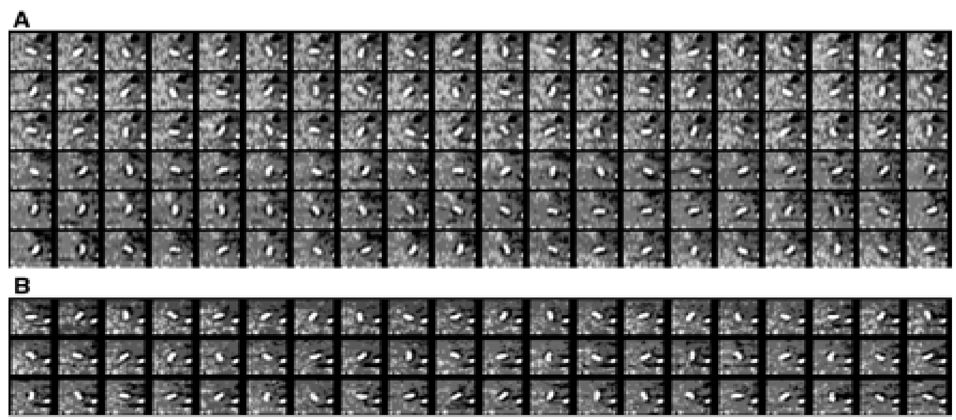
Soong *et al.*, Science **290**, 1555 (2000)

Nanobiotechnology

Combine top-down nanofabrication (Ni wires, SiO_2 posts) with biochemical tools (histadine tags, F_1-ATPase biomolecular motor).

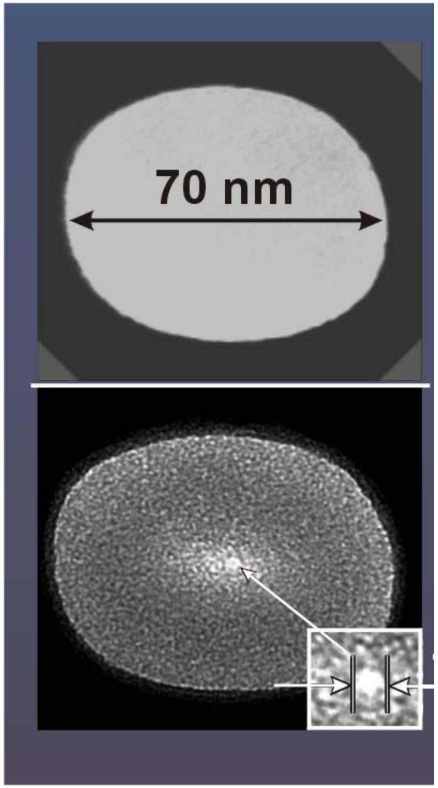
Result: ATP-powered "propellers".

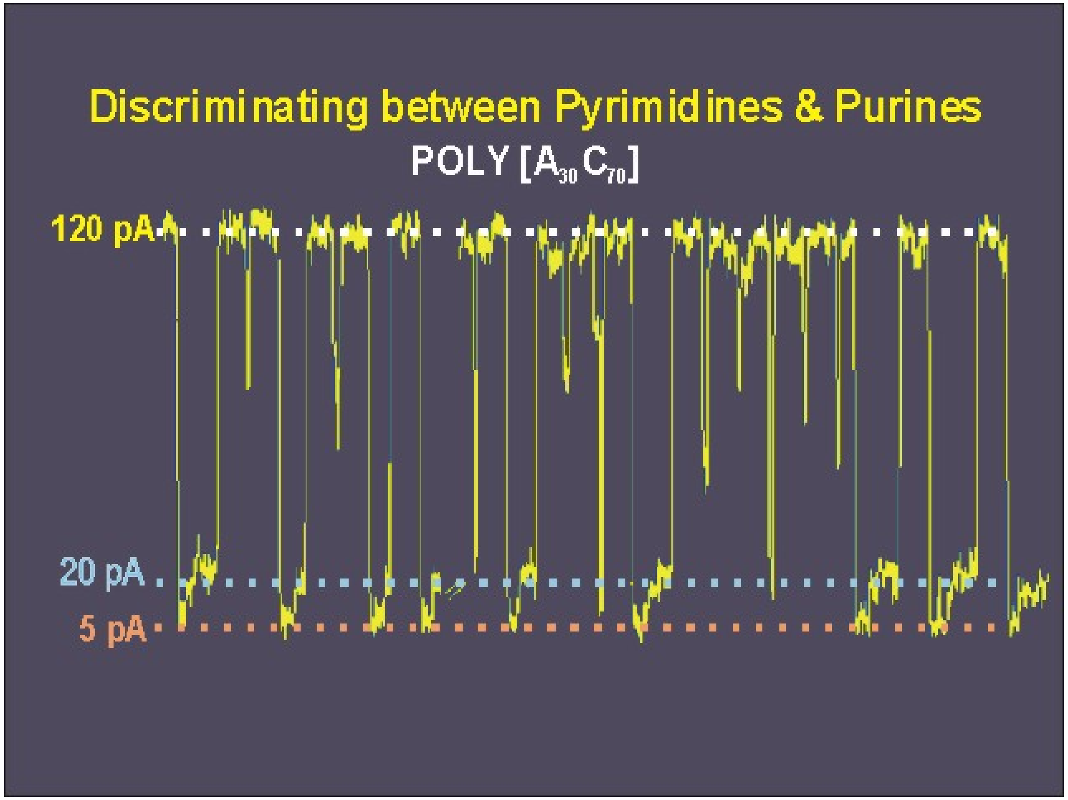




Nanobiotechnology

http://golgi.harvard.edu/branton/index.htm

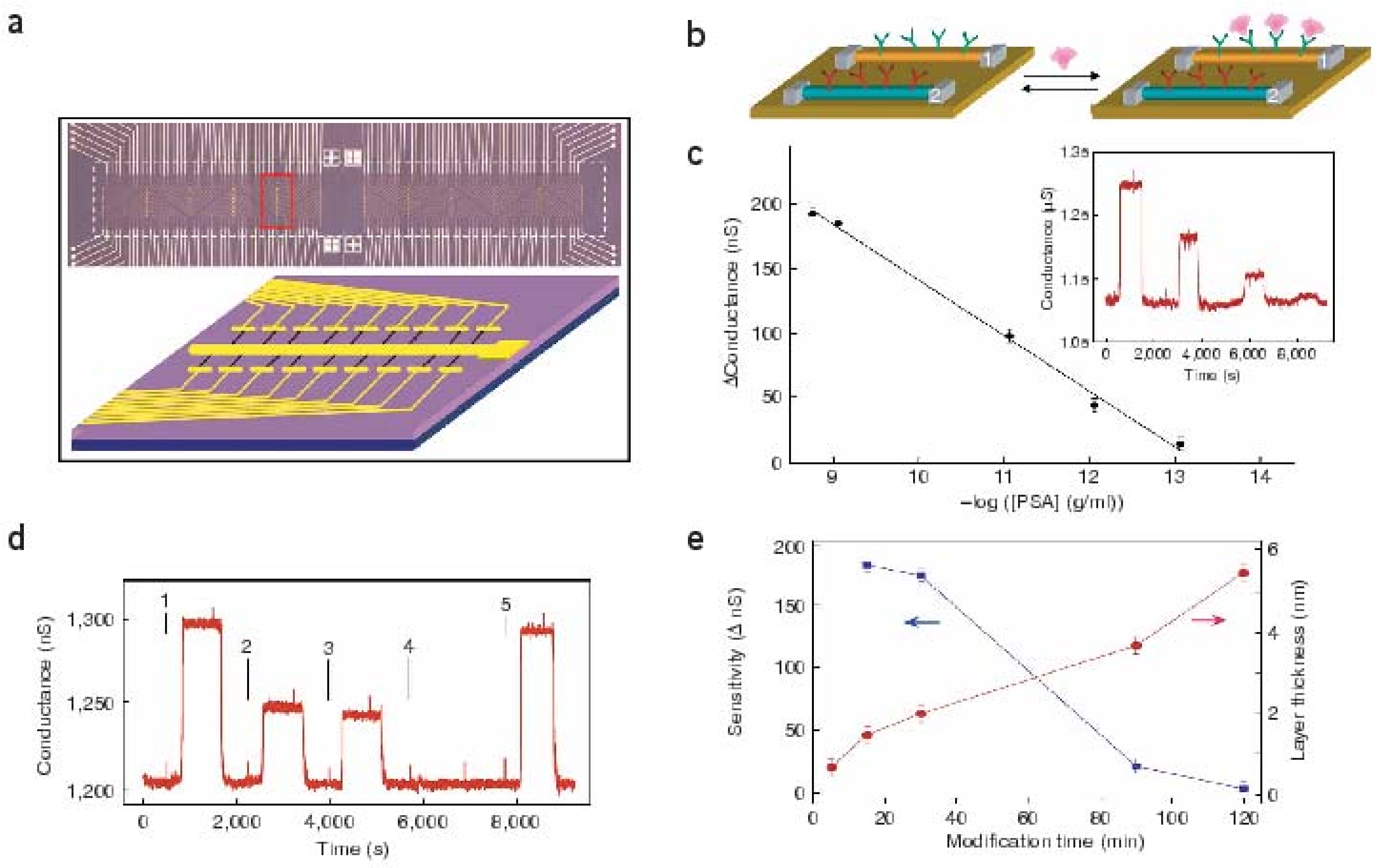




Using artificial nanopores and sensitive electrical measurements to do single-strand DNA and RNA sequencing.

Nanobiotechnology

Zheng *et al.*, Nature Biotech. **23**, 1294 (2006)



Using nanowires and surface chemistry to detect cancer markers.

Summary:

We’re going to look at several other areas of technology, and see where the ability to manipulate matter on the nm scale is increasing our capabilities and understanding.

These include:

- Photonics
- Micro-and nanomechanics
- Microfluidics
- Nanobiotechnology

Will conclude with some overview and perspective on nanotechnology.

Next time:

Review of physical optics

Light at interfaces

Physical optics

Maxwell's equations and waves

Fourier analysis and dispersion

Boundary conditions and interfaces

Total internal reflection

Evanescent waves

Microscopics of media: dispersion

Diffraction

The near-field

Maxwell's equations (no sources)

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mu\mathbf{H} \qquad \nabla \cdot \mu\mathbf{H} = 0$$

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\varepsilon\mathbf{E} \qquad \nabla \cdot \varepsilon\mathbf{E} = 0$$

Combining these quickly gives wave equations for the fields:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times (-\frac{\partial}{\partial t}\mu\mathbf{H}) \qquad \nabla \times (\nabla \times \mathbf{H}) = \nabla \times (\frac{\partial}{\partial t}\varepsilon\mathbf{E})$$

$$= -\mu\frac{\partial}{\partial t}(\nabla \times \mathbf{H}) \qquad = \varepsilon\frac{\partial}{\partial t}(\nabla \times \mathbf{E})$$

$$= -\mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad = -\mu\varepsilon\frac{\partial^2 \mathbf{H}}{\partial t^2}$$

Recall: $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

For uniform media, $\nabla^2 \mathbf{E} = \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2}$ $\qquad \nabla^2 \mathbf{H} = \mu\varepsilon\frac{\partial^2 \mathbf{H}}{\partial t^2}$

Wave equations

$$\nabla^2\mathbf{E} = \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad \nabla^2\mathbf{H} = \mu\varepsilon\frac{\partial^2\mathbf{H}}{\partial t^2}$$

These are wave equations. For uniform media, the solutions consistent with these and Maxwell's equations are plane waves.

Let's start in the linear world, where we believe in superposition. Then we can break down any general solution into Fourier components:

$$\mathbf{E} = \mathbf{E}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{H} = \mathbf{H}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

We can then translate differential operations into algebra: $\frac{\partial}{\partial t} \rightarrow -i\omega$

$\nabla \rightarrow i\mathbf{k}$

Plugging into our wave equations, we see

$$(-k^2 + \mu\varepsilon\omega^2)\mathbf{E}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = 0$$

$$(-k^2 + \mu\varepsilon\omega^2)\mathbf{H}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = 0$$

Phase velocity and field strengths

$$(-k^2 + \mu\varepsilon\omega^2)\mathbf{E}_{0\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = 0 \qquad (-k^2 + \mu\varepsilon\omega^2)\mathbf{H}_{0\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = 0$$

This is the *dispersion* relation, and it tells us the phase velocity of the waves:

$$v_p \equiv \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{In vacuum,} \quad v_p \to \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

Define index of refraction, $n \equiv \sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}} \Rightarrow v_p = \dfrac{c}{n}$

Can use Faraday's law to relate $\boldsymbol{E}_{0\mathbf{k}}$ and $\boldsymbol{H}_{0\mathbf{k}}$:

$$i\mathbf{k} \times \mathbf{E}_{0\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = i\mu\omega\mathbf{H}_{0\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\to \mathbf{H}_{0\mathbf{k}} = \frac{1}{\mu\omega}\mathbf{k} \times \mathbf{E}_{0\mathbf{k}} = \sqrt{\frac{\varepsilon}{\mu}}\frac{\mathbf{k}}{k} \times \mathbf{E}_{0\mathbf{k}} = \sqrt{\frac{\varepsilon}{\mu}}\hat{\mathbf{e}}_{\mathbf{k}} \times \mathbf{E}_{0\mathbf{k}}$$

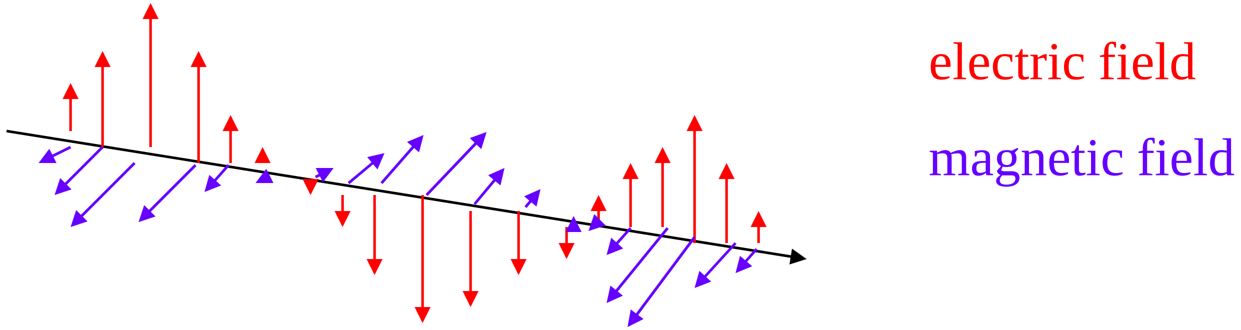
This quantity is called the characteristic admittance of a medium.

Field directions (uniform media)

Gauss' law: $\nabla \cdot \varepsilon \mathbf{E} = 0 \rightarrow \mathbf{k} \cdot \mathbf{E}_{0\mathbf{k}} = 0$

Electric field is *transverse* to direction of propagation.

Similarly, since $\mathbf{H}_{0\mathbf{k}} \sim \mathbf{k} \times \mathbf{E}_{0\mathbf{k}}$ magnetic field is also transverse.



This wave is *linearly polarized*: electric (magnetic) field is always oriented along *y*-axis (*x*-axis). Wave propagates in -*z* direction.

Complex vectors

From our notation, you've already guessed that $\boldsymbol{E}_{0k}$ and $\boldsymbol{H}_{0\boldsymbol{k}}$ are complex vectors, where each cartesian component is complex, and the physical field is the real part of the complex vector.

For example,

$$\mathbf{E}_{0\mathbf{k}} e^{-i\omega t} = (E_x \hat{\mathbf{x}} + E_y e^{i(\pi/2)} \hat{\mathbf{y}}) e^{-i\omega t}$$

So, at $\omega t = 0$, the (real part of the) field points along x.
At $\omega t = \pi/2$, the field points along $-y$.

One could rewrite the real part of this as:

$$\mathbf{E}_{0\mathbf{k}}(t) = E_x \cos(\omega t)\hat{\mathbf{x}} + E_y \sin(\omega t)\hat{\mathbf{y}}$$

With different magnitudes E_x and E_y, this is elliptically polarized light.

Superposition

In *linear* media, we can write a generic field configuration as a superposition of plane waves. Starting from a given electric field at $t = 0$, and using the properties of Fourier series,

$$\mathbf{E}_{0\mathbf{k}} + \mathbf{E}^{*}_{0-\mathbf{k}} = \frac{2}{(2\pi)^{3/2}} \int \mathbf{E}(\mathbf{r},0) e^{-i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{r}$$

Knowing direction of propagation, can then pick out Fourier terms we care about, and figure out the future configuration of the wave:

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{(2\pi)^{3/2}} \int \mathbf{E}_{0\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega(\mathbf{k})t)} d^3\mathbf{k}$$

Group velocity and dispersion

Consider a generic wave built up from components this way:

$$\psi(\mathbf{r},0)=\frac{1}{(2\pi)^{3/2}}\int A(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}$$

We want to time-evolve this:

$$\psi(\mathbf{r},t)=\frac{1}{(2\pi)^{3/2}}\int A(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega(\mathbf{k})t)}d^3\mathbf{k}$$

Obviously, if ω is proportional to ***k***, then the whole shape of ψ has just been translated by a distance ωt/*k*.

If ω depends on ***k***, and the wavepacket is “localized” around some $\boldsymbol{k}_0$, we can Taylor expand:

$$\begin{aligned}\omega(\mathbf{k})&\approx\omega(\mathbf{k}_0)+(\mathbf{k}-\mathbf{k}_0)\cdot\nabla_{\mathbf{k}}\omega\big|_{\mathbf{k}_0}\\&=\omega_0+(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{v}_g\end{aligned}$$

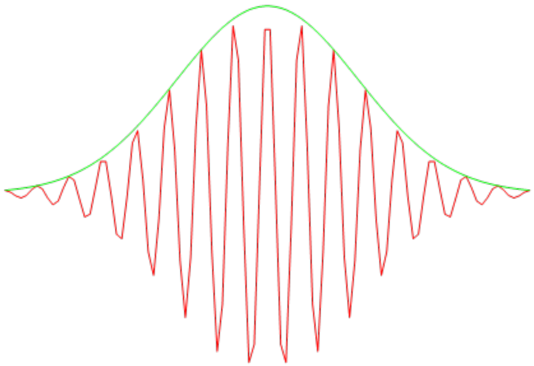
Group velocity and dispersion

$$\psi(\mathbf{r},t) \approx \frac{1}{(2\pi)^{3/2}} \int A(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{r}-(\omega_0+\mathbf{v}_g\cdot(\mathbf{k}-\mathbf{k}_0)t))} d^3\mathbf{k}$$

$$= e^{i(\mathbf{v}_g\cdot\mathbf{k}_0-\omega_0)t} \frac{1}{(2\pi)^{3/2}} \int A(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{v}_g t)} d^3\mathbf{k}$$

$$= e^{i(\mathbf{v}_g-\mathbf{v}_p)\cdot\mathbf{k}_0 t} \psi(\mathbf{r}-\mathbf{v}_g t, 0)$$

So, the *envelope* of the wave moves forward at velocity $\boldsymbol{v}_g$.
The individual components move with their phase velocity.



Other velocities: *front*, *signal*, *energy*….

Poynting's theorem and vector

Start from Ampere's law and dot with ***E***:

$$\mathbf{E}\cdot(\nabla\times\mathbf{H}) = \mathbf{E}\cdot\mathbf{J} + \mathbf{E}\cdot\frac{\partial}{\partial t}\varepsilon\mathbf{E}$$

Note: $\nabla\cdot(\mathbf{E}\times\mathbf{H}) = \mathbf{H}\cdot(\nabla\times\mathbf{E}) - \mathbf{E}\cdot(\nabla\times\mathbf{H})$

Combining and rearranging,

$$\mathbf{H}\cdot(\nabla\times\mathbf{E}) - \nabla\cdot(\mathbf{E}\times\mathbf{H}) = \mathbf{E}\cdot\mathbf{J} + \frac{1}{2}\frac{\partial}{\partial t}\mathbf{E}\cdot\varepsilon\mathbf{E}$$

Plugging in Faraday's law,

$$\mathbf{H}\cdot\left(-\frac{\partial}{\partial t}\mu\mathbf{H}\right) - \nabla\cdot(\mathbf{E}\times\mathbf{H}) = \mathbf{E}\cdot\mathbf{J} + \frac{1}{2}\frac{\partial}{\partial t}\mathbf{E}\cdot\varepsilon\mathbf{E}$$

Rearranging,

$$\nabla\cdot\underbrace{(\mathbf{E}\times\mathbf{H})} = -\mathbf{E}\cdot\mathbf{J} - \frac{1}{2}\frac{\partial}{\partial t}\underbrace{\mathbf{E}\cdot\varepsilon\mathbf{E}} - \frac{1}{2}\frac{\partial}{\partial t}\underbrace{\mathbf{H}\cdot\mu\mathbf{H}}$$

$$\mathbf{S} \equiv \mathbf{E}\times\mathbf{H}$$

Poynting vector

$$\mathsf{E}_E \equiv \frac{1}{2}\mathbf{E}\cdot\varepsilon\mathbf{E}$$

electric energy density

$$\mathsf{E}_H \equiv \frac{1}{2}\mathbf{H}\cdot\mu\mathbf{H}$$

magnetic energy density

Energy densities

Evaluating this for the plane wave case, we have to remember that it's the *real* parts of the complex fields that matter.

In *free space*,

$$\mathsf{E}_E \equiv \frac{1}{2}\mathbf{E}\cdot\varepsilon_0\mathbf{E} = \frac{\varepsilon_0}{2}\left(\frac{1}{2}\left(\mathbf{E}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \mathbf{E}_{0\mathbf{k}}^{*}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right)\right)^2$$

Ugly! If all we care about, though, is the *time-average*,

$$\left\langle \mathsf{E}_E \right\rangle = \frac{\varepsilon_0}{4}\mathbf{E}_{0\mathbf{k}}\cdot\mathbf{E}_{0\mathbf{k}}^{*}$$

Same idea for magnetic energy density:

$$\left\langle \mathsf{E}_H \right\rangle = \frac{\mu_0}{4}\mathbf{H}_{0\mathbf{k}}\cdot\mathbf{H}_{0\mathbf{k}}^{*}$$

Time averaged Poynting vector

Can follow same idea:

$$\mathbf{S} \equiv \mathbf{E}\times\mathbf{H} = \left(\frac{1}{2}\left(\mathbf{E}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \mathbf{E}_{0\mathbf{k}}^{*}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right)\times\frac{1}{2}\left(\mathbf{H}_{0\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \mathbf{H}_{0\mathbf{k}}^{*}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right)\right)$$

Again, looking just at time average,

$$\langle\mathbf{S}\rangle = \frac{1}{2}\left(\left(\mathbf{E}_{0\mathbf{k}}\times\mathbf{H}_{0\mathbf{k}}^{*}\right)+\left(\mathbf{E}_{0\mathbf{k}}^{*}\times\mathbf{H}_{0\mathbf{k}}\right)\right)$$

In free space,

$$\mathbf{H}_{0\mathbf{k}} = \sqrt{\frac{\varepsilon}{\mu}}\hat{\mathbf{e}}_{\mathbf{k}}\times\mathbf{E}_{0\mathbf{k}}$$

So,

$$\langle\mathbf{S}\rangle = \hat{\mathbf{e}}_{\mathbf{k}}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\mathbf{E}_{0\mathbf{k}}\right|^2$$

Energy densities and superposition

What about superposing waves with different wavevectors?

General rule: for nonlinear quantities (like energy density), need to write down full expression for *real* fields, then square to find energy densities.

For linear media, we get a bit lucky, and the final answer ends up looking like:

$$\langle \mathsf{E}_E \rangle = \frac{\varepsilon}{4} \int \mathbf{E}_{0\mathbf{k}} \cdot \mathbf{E}^*_{0\mathbf{k}} d^3\mathbf{k}$$

That is, we can find the energy content for each frequency (or wavevector), and add them. Again, this is something of a lucky break.

Interactions with media

The previous expressions can be complicated when realistic media are involved.

We'll ignore magnetic media, and assume $\mu \to \mu_0$.

Potential complications:

- Dielectric "constant" can depend on frequency (dispersion).
- Dielectric "constant" can depend on *direction* (tensor!).
- Dielectric "constant" can be *complex* (conductors).
- Dielectric "constant" can be spatially varying (interfaces).

Complex dielectric function

- Remember, a complex ε(ω) just means that polarization doesn't have to be in phase with the electric field.

- The changing polarization can take energy from the electric field, causing damping.

In general, $$\mathbf{D}(t)/\varepsilon_0 = \mathbf{E}(t) + \int_0^\infty f(\tau)\mathbf{E}(t-\tau)\,d\tau$$

Fourier transforming,

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega),$$

$$\varepsilon(\omega)/\varepsilon_0 = 1 + \int_0^\infty f(\tau)e^{i\omega\tau}\,d\tau$$

Writing $\varepsilon(\omega) \equiv \varepsilon'(\omega) + i\varepsilon''(\omega)$ we see

$$\varepsilon(-\omega) = \varepsilon^*(\omega) \Rightarrow$$

$$\varepsilon'(-\omega) = \varepsilon'(\omega), \quad \varepsilon''(-\omega) = -\varepsilon''(\omega)$$

Kramers-Kronig relations

It is possible to use the analytic properties of $\varepsilon(\omega)$ (which come from the causal definition of the dielectric function) to relate the real and imaginary parts to each other.

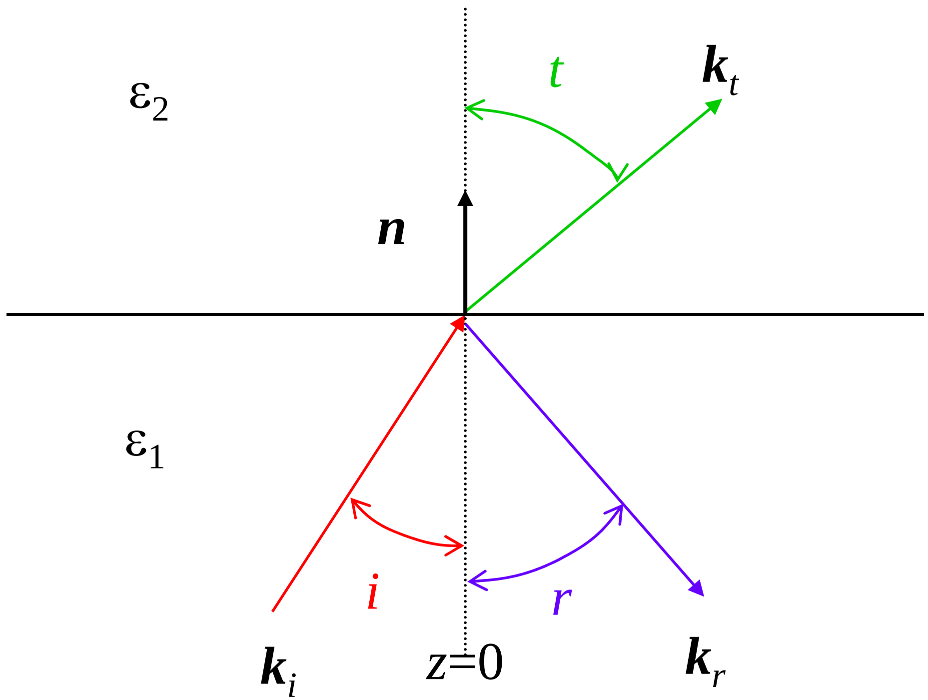
The results are: (see Jackson, p. 310ff)

$$\Re\left[\varepsilon(\omega)/\varepsilon_0\right] = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\Im\left[\varepsilon(\omega)/\varepsilon_0\right]}{\omega' - \omega} d\omega'$$

$$\Im\left[\varepsilon(\omega)/\varepsilon_0\right] = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\Re\left[\varepsilon(\omega)/\varepsilon_0\right] - 1}{\omega' - \omega} d\omega'$$

We will see shortly that the imaginary part of $\varepsilon(\omega)$ is related to *conductivity* and absorption. Thus, one can measure an absorption spectrum and infer dielectric properties, and vice-versa.

Reflection and refraction at interfaces



Want to treat the general problem of discontinuity of dielectric properties.

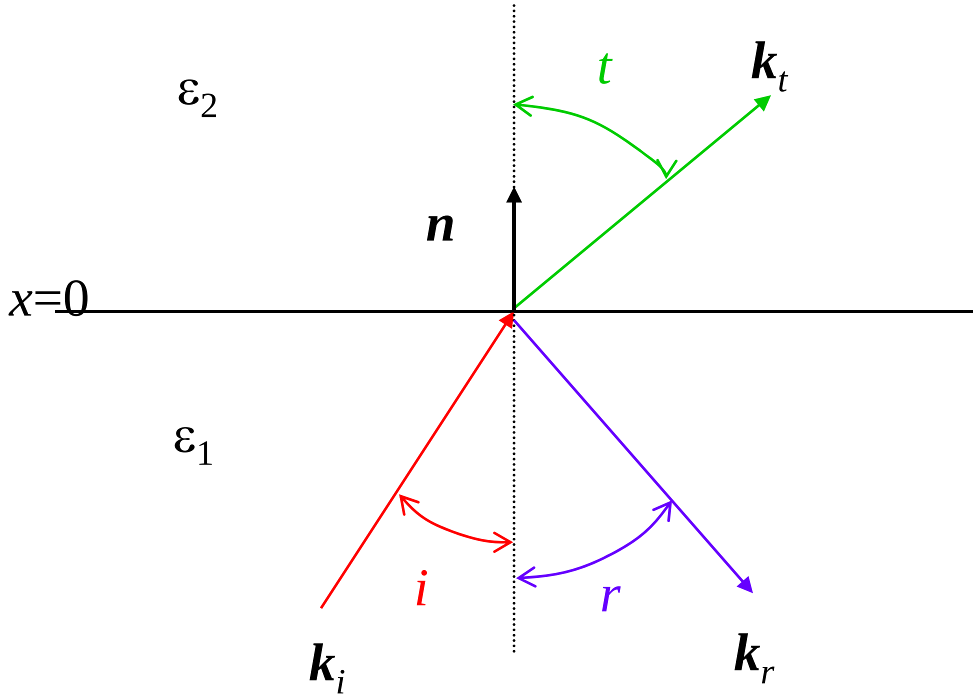
Will find appropriate boundary conditions and treat two specific cases.

Reflection and refraction at interfaces

Incident:

$$\mathbf{E}_i = \mathbf{E}_{0i} e^{i(\mathbf{k}_i \cdot \mathbf{r} - \omega t)}$$

$$\mathbf{H}_i = \sqrt{\frac{\varepsilon_1}{\mu}} \hat{\mathbf{e}}_{\mathbf{k}_i} \times \mathbf{E}_i$$



Reflected:

$$\mathbf{E}_r = \mathbf{E}_{0r} e^{i(\mathbf{k}_r \cdot \mathbf{r} - \omega t)}$$

$$\mathbf{H}_r = \sqrt{\frac{\varepsilon_1}{\mu}} \hat{\mathbf{e}}_{\mathbf{k}_r} \times \mathbf{E}_r$$

Transmitted:

$$\mathbf{E}_t = \mathbf{E}_{0t} e^{i(\mathbf{k}_t \cdot \mathbf{r} - \omega t)}$$

$$\mathbf{H}_t = \sqrt{\frac{\varepsilon_2}{\mu}} \hat{\mathbf{e}}_{\mathbf{k}_t} \times \mathbf{E}_t$$

Wavevector magnitudes:

$$|\mathbf{k}_i| = |\mathbf{k}_r| = k = \omega\sqrt{\mu\varepsilon_1}$$

$$|\mathbf{k}_t| = k_t = \omega\sqrt{\mu\varepsilon_2}$$

Boundary conditions

- The tangential component of ***E*** must be continuous.
- Longitudinal component of ***B*** (and ***H*** here) must be continuous.
- Tangential component of ***H*** must be continuous for equal μ.
- Longitudinal component of ***D*** must be continuous.

$$\left(\mathbf{E}_{0i} + \mathbf{E}_{0r} - \mathbf{E}_{0t}\right) \times \mathbf{n} = 0$$

$$\left(\mathbf{k}_i \times \mathbf{E}_{0i} + \mathbf{k}_r \times \mathbf{E}_{0r} - \mathbf{k}_t \times \mathbf{E}_{0t}\right) \cdot \mathbf{n} = 0$$

$$\left(\mathbf{k}_i \times \mathbf{E}_{0i} + \mathbf{k}_r \times \mathbf{E}_{0r} - \mathbf{k}_t \times \mathbf{E}_{0t}\right) \times \mathbf{n} = 0$$

$$\left(\varepsilon_1\left(\mathbf{E}_{0i} + \mathbf{E}_{0r}\right) - \varepsilon_2 \mathbf{E}_{0t}\right) \cdot \mathbf{n} = 0$$

TE wave at a single interface

Apply cont. of tang. ***E*** at $x = 0$;
must be true for all y and z, including origin.

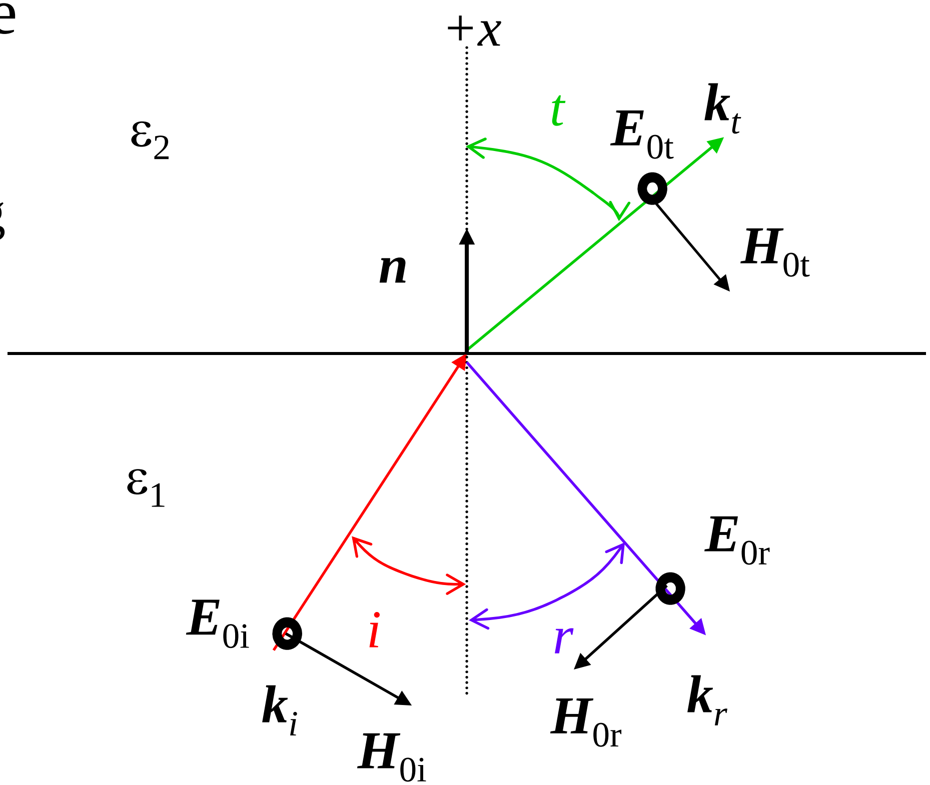
This tells us that $k_{iy} = k_{ry} = k_{ty}$.
Since $k_i = k_r$, this immediately gives that angle of incidence = angle of reflection.

Similarly, one finds

$$k_i \sin\theta_i = k_t \sin\theta_t$$

which gives Snell's law: $$n_1 \sin\theta_i = n_2 \sin\theta_t$$

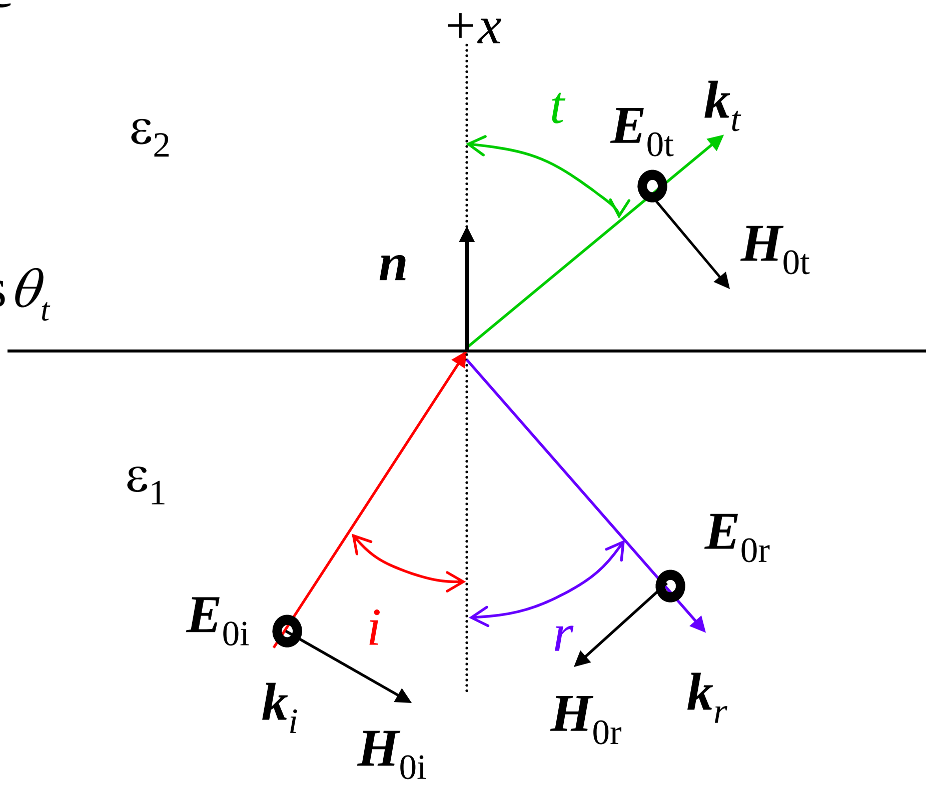
Knowing this tells us $$E_{0i} + E_{0r} = E_{0t}$$



TE wave at a single interface

Using the tang. ***H*** condition gives a second equation,

$$E_{0i} n_1 \cos\theta_i - E_{0r} n_1 \cos\theta_r = E_{0t} n_2 \cos\theta_t$$



We now have two equations, two unknowns, and can find the reflected and transmitted amplitudes in terms of the incident amplitude:

$$E_{0r} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_{0i}$$

$$E_{0t} = \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} E_{0i}$$

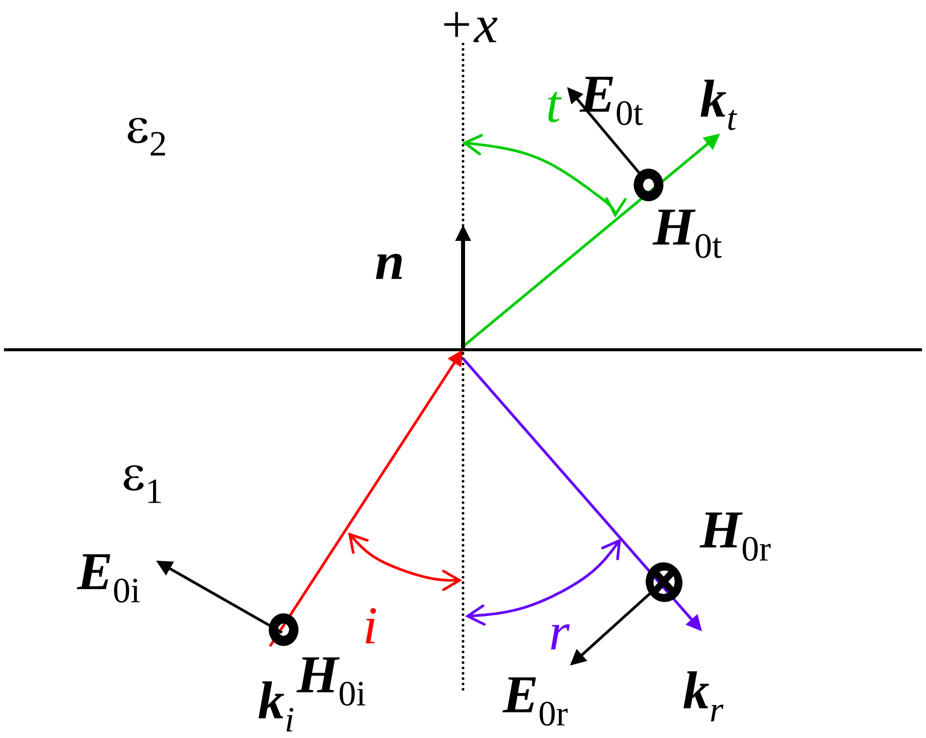
This is an example of a Fresnel formula.

TM wave at a single interface

Can do same thing for case where polarization is offset by 90 degrees.

Can then do any version of polarization by superposition….

Skipping to the result,



$$E_{0r} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_{0i}$$

$$E_{0t} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} E_{0i}$$

Note that an arbitrarily polarized incident wave can lead to a reflected wave (for example) with quite different polarization. Remember this….

Percent polarization

Suppose we have two waves of same amplitude that differ in frequency by some small amount, Ω. The electric field at some fixed position from the superposition of the two looks like:

$$\begin{aligned}\mathbf{E} &= E\hat{\mathbf{y}}e^{-i\omega t} + E\hat{\mathbf{z}}e^{-i(\omega+\Omega)t} \\ &= E(\hat{\mathbf{y}} + \hat{\mathbf{z}}e^{-i\Omega t})e^{-i\omega t}\end{aligned}$$

The polarization at this position looks like it's varying in time at a frequency Ω.

A very slow detector will see an average of all polarizations, and its output would be indistinguishable from one illuminated by unpolarized light.

Now suppose amplitudes of two frequency components differ slightly….

Percent polarization and Stokes parameters

$$\mathbf{E} = E\frac{1+p}{\sqrt{1+p^2}}\hat{\mathbf{y}}e^{-i\omega t} + E\frac{1-p}{\sqrt{1+p^2}}\hat{\mathbf{z}}e^{-i(\omega+\Omega)t}$$

$$= E\frac{1-p}{\sqrt{1+p^2}}\hat{\mathbf{y}}e^{-i\omega t} + E\frac{2p}{\sqrt{1+p^2}}\hat{\mathbf{y}}e^{-i\omega t} + E\frac{1-p}{\sqrt{1+p^2}}\hat{\mathbf{z}}e^{-i(\omega+\Omega)t}$$

If you think about a slow y-polarized detector and the *intensity* it sees, you find that it's equivalent to an unpolarized wave containing the fraction $(1-p)^2/(1+p^2)$ of the energy, while the polarized wave contains $2p/(1+p^2)$ of the energy.

Colloquially, the wave has a fractional polarization of $4p^2/(1+p^2)$ in the y-direction.

Useful quantities: Stokes parameters (assuming x-dir. propagation)

$$I = \sqrt{\frac{\varepsilon}{\mu}}\left\langle \left|E_y\right|^2 + \left|E_z\right|^2 \right\rangle$$

$$Q = \sqrt{\frac{\varepsilon}{\mu}}\left\langle \left|E_y\right|^2 - \left|E_z\right|^2 \right\rangle$$

$$U = \sqrt{\frac{\varepsilon}{\mu}}\left\langle 2\,\mathrm{Re}(E_y^* E_z) \right\rangle$$

$$V = \sqrt{\frac{\varepsilon}{\mu}}\left\langle 2\,\mathrm{Im}(E_y^* E_z) \right\rangle$$

Fraction of polarized light = $\sqrt{\dfrac{Q^2+U^2+V^2}{I^2}}$

Total internal reflection

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Returning to waves at interfaces, it's clear from Snell's law that something weird happens when $n_1 \sin\theta_i > n_2$.

The result is total internal reflection; cos θ_t becomes purely imaginary. The reflected amplitude is equal to the incident amplitude (conservation of energy).

The reflected wave picks up a phase shift, though, that depends on the direction of polarization.

For TE waves,

$$\frac{E_{0r}}{E_{0i}} = \exp\left\{ i2\tan^{-1}\left[\sqrt{\frac{n_1^2 \sin^2\theta_i - n_2^2}{n_1^2(1-\sin^2\theta_i)}} \right] \right\} = \exp(i\phi_{TE})$$

For TM waves,

$$\frac{E_{0r}}{E_{0i}} = \exp\left\{ i2\tan^{-1}\left[\left(\frac{n_1}{n_2}\right)^2 \sqrt{\frac{n_1^2 \sin^2\theta_i - n_2^2}{n_1^2(1-\sin^2\theta_i)}} \right] \right\} = \exp(i\phi_{TM})$$

Evanescent waves

The EM fields that extend out into the second material are an example of an evanescent wave.

$$\langle \mathbf{S} \rangle = \frac{1}{2}\left(\left(\mathbf{E}_{0\mathbf{k}} \times \mathbf{H}_{0\mathbf{k}}^*\right) + \left(\mathbf{E}_{0\mathbf{k}}^* \times \mathbf{H}_{0\mathbf{k}}\right)\right)$$

$$\mathbf{E}_t = E_t \hat{\mathbf{z}} e^{i(k_{ty} y - \omega t) - \kappa x}$$

The x-component of the wavevector in medium 2 is $k_{tx} = i\frac{\omega}{c}\sqrt{n_1^2 \sin^2\theta_i - n_2^2} \equiv i\kappa$

$$\mathbf{H}_t = \sqrt{\frac{\varepsilon_2}{\mu}}\hat{\mathbf{e}}_{\mathbf{k}_t} \times E_t \hat{\mathbf{z}} e^{i(k_{ty} y - \omega t) - \kappa x}$$

$$= \sqrt{\frac{\varepsilon_2}{\mu}}(k_{ty}\hat{\mathbf{x}} - i\kappa\hat{\mathbf{y}}) E_t e^{i(k_{ty} y - \omega t) - \kappa x}$$

We picked the physically reasonable positive root, so that the fields *decay* exponentially moving into medium 2.

Note: the wave in medium 2 actually propagates in the y-direction, so there's no dissipation here. Look at avg. Poynting vector (TE case):

$$\Rightarrow \langle \mathbf{S}_t \rangle \propto \left|E_t\right|^2 \exp(-2\kappa x)\hat{\mathbf{y}}$$

No average energy flux into medium 2!

Plug in numbers for visible light (500 nm wavelength), n_1=1.5, n_2 = 1, θ_i = 60 degrees gives ~ 600 nm.

Waveguides

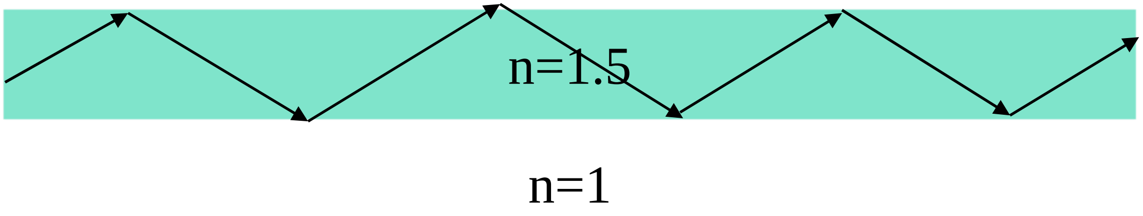
Total internal reflection is the physics that permits one to make optical waveguides (*e.g.* optical fibers).

There are two basic approaches to analyzing optical waveguide structures:

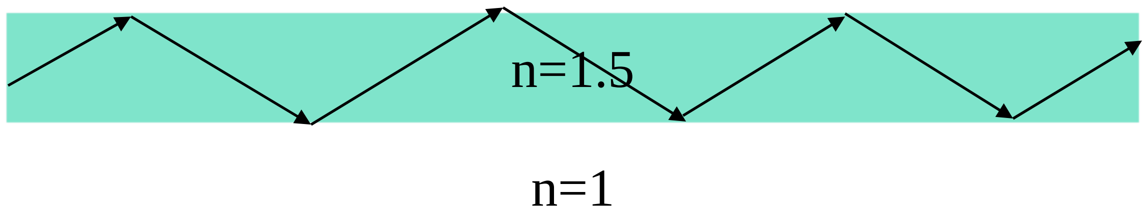
• Ray optics (throws out intensity and phase information, but can be mathematically simpler and revealing sometimes)

• Wave-field method (actually solve Maxwell's equations, keeping track of all boundary conditions)

A couple of general points:



Waveguides



- (Ray optics picture) Because of lateral bounces, longitudinal propagation (of energy, signals, etc.) is typically slower than just c/n.

- (Wave field picture) Solution to boundary value problem leads to well-defined modes that are *naturally orthogonal*. That is, in a completely linear medium, a waveguide will only pass radiation of particular wavelengths; furthermore, populating one mode does not affect propagation in the other modes.

- (Wave field picture) Fields from these modes extend outside the total internal reflection interface. Placing two waveguides in close proximity can cause *mixing* - completely analogous to our work on quantum wells from last semester.

- Real optical fibers can be very complicated: birefringent; dispersive; lossy; etc.

Loss

What do we mean by *loss*? We'll see a specific case in a minute. General idea: EM wave does *work* on something, and that energy is lost from the EM wave.

Manifests itself as a complex index of refraction. Remember,

$$n(\omega) \equiv \sqrt{\frac{\varepsilon(\omega)\mu}{\varepsilon_0\mu_0}}$$

Suppose there's a damped charged impurity bound by a harmonic potential that interacts with the EM wave:

$$m\left[\ddot{\mathbf{r}} + \gamma\dot{\mathbf{r}} + \omega_0^2\mathbf{r}\right] = -e\mathbf{E}(\mathbf{r},t)$$

Can solve for the polarization, assuming *N* of these per unit volume:

$$\mathbf{P} = \frac{Ne^2}{m}\left(\omega_0^2 - \omega^2 - i\omega\gamma\right)^{-1}\mathbf{E}$$

Loss

Dielectric constant is then:

$$\varepsilon \equiv \varepsilon_0 + \frac{\mathbf{P}}{\mathbf{E}} = \varepsilon_0 + \frac{Ne^2}{m}\left(\omega_0^2 - \omega^2 - i\omega\gamma\right)^{-1}$$

Can take square root of this, and look at real and imaginary parts of $k = n(\omega)\,\omega/c$ to find the absorption coefficient.

Real and imaginary parts of dielectric constant look like: