Ballistic nanotransistors

Modeled on analysis by Mark Lundstrom (ECE, Purdue). Unless otherwise indicated, all images are his.

Interesting hybrid of classical transistor behavior, quantum confinement effects, and Landauer-Buttiker type ideas.

Device parameters:
Length: $L > \sim 10$ nm to avoid contributions of direct source-drain tunneling.

First look at effects of confinement and see what we can neglect.

Subbands and quantum confinement

This problem is essentially something we’ve done several times before.

One can solve the Schrödinger equation for neutral, noninteracting particles confined in two dimensions but extended in the third.

This is effectively a 1d quantum system, with 1d subbands separated in energy by spacings that look like those between low-lying levels of a 2d particle in a box.

As usual, we recall that one can consider the envelope function of Bloch waves, and arrive at an identical equation for the single-particle levels for non-interacting electrons.
Subbands and quantum confinement

Individual 1d subbands are significantly higher in energy than the floor of the conduction band.

Much weaker dependence (~0) on transverse position, too.

Subbands and quantum confinement

Must worry about energy vs. position of individual subbands, not just conduction band as in usual FETs.
Complete solution

In generality, one must self-consistently solve the electrostatic and quantum mechanical problems.

Generally this is requires numerical solution, but it is possible to come up with analytical expressions in certain limits.

First, we’ll look qualitatively at what happens as gate and drain bias are varied.

Then we’ll consider three regimes:

\[ T \gg 0 \] -- nondegenerate carriers

\[ T \sim 0 \] -- degenerate carriers

\[ T > 0 \] -- general case.

Nonequilibrium velocity (momentum) distributions

Remember our Landauer formula discussions? We worked in 1d and considered the chemical potential at different places along a device:

Can do same thing here, but plot velocity (momentum) distributions as a function of position:
Nonequilibrium velocity (momentum) distributions

Now fix gate bias.
Examine distributions as function of drain bias at top of barrier.
At low $V_D$, carriers pass over the barrier in both directions.
As $V_D$ is increased, higher fraction of carriers getting over barrier are from the source.
At sufficiently large $V_D$, no carriers from drain reach top of barrier.
Further increases in $V_D$, don’t change the distribution of carriers at the top of the barrier! **Velocity saturation** near source, without pinchoff!
Velocity saturation without scattering

Further increases in $V_D$ don’t change the distribution of carriers at the top of the barrier! **Velocity saturation** near source, without pinchoff!

- Carrier distribution at top of barrier varies with $V_D$, but total density in “good” FET still determined by gate voltage.
- Can use these pictures to derive quantitative model of ballistic FET.

\[
T > 0 \text{ case, nondegenerate carriers} \\
\]

For electrostatically “nice” FET (which we can check if we want by solving Poisson eqn. everywhere), charge density in inversion layer controlled by $V_G - V_T$, even at top of barrier:

\[
C_s(V_G - V_T) = n_{2d}^\text{tot} = n_{2d}^+(x, E_F) + n_{2d}^-(x, E_F - eV_D)
\]

right-moving carriers from source

left-moving carriers from drain

As $V_D$ increases, $n_{2d}$ decreases.

To maintain the equality, $E_F$ for the source effectively increases - more carriers come in from the source.

$x =$ position along channel; $x=0$ defined as top of barrier.
\( T > 0 \) case, nondegenerate carriers

Net current is, similarly, given by an expression familiar from our Landauer picture:

\[
I_D = I^+ (E_F) - I^- (E_F - eV_D)
\]

Velocity distribution of right moving carriers is hemi-Maxwellian:

\[
v_T = \sum_{p_x \geq 0, p_y} v_x \cdot f_M(E) = \sqrt{\frac{2k_B T}{\pi n_s}}
\]

Maxwell-Boltzmann distribution.

Same argument works for left-moving carriers, so their average speed is essentially identical to that of the right-movers.

\( T > 0 \) case, nondegenerate carriers

Resulting current density:

\[
I_D = \frac{n_{2d}^e}{W} \left( v_T - n_{2d}^e \right) v_T
\]

\[
I_D = \frac{n_{2d}^e}{W} \frac{\left( 1 - n_{2d}^e(0)/n_{2d}^e(0) \right)}{\left( 1 + n_{2d}^e(0)/n_{2d}^e(0) \right)} v_T
\]

Ahh, but we can figure out the ratio \( n_{2d}^e/n_{2d}^e \):

\[
n_{2d}^e = \left( \frac{N_{2d}}{2} \right) \exp \left( \frac{E_F - E}{k_B T} \right)
\]

\[
n_{2d}^e = \left( \frac{N_{2d}}{2} \right) \exp \left( \frac{E_F - eV_D - E}{k_B T} \right)
\]

where \( N_{2d} = \left( \frac{m_0}{\pi \hbar^2} \right) k_B T \)

Effective density of states
$T > 0$ case, nondegenerate carriers

Result for current density:

$$I_D / W = en_{2d}^\text{tot} v_T \left( \frac{1-n_{2d}^+(0)/n_{2d}^-(0)}{1+n_{2d}^+(0)/n_{2d}^-(0)} \right) \rightarrow I_D / W = en_{2d}^\text{tot} v_T \left( \frac{1-e^{-eV_D/k_BT}}{1+e^{-eV_D/k_BT}} \right)$$

Plugging in our expression for carrier density in a “nice” FET gives:

$$I_D = WC_s (V_G - V_T) v_T \left( \frac{1-e^{-eV_D/k_BT}}{1+e^{-eV_D/k_BT}} \right)$$

$T > 0$ case, nondegenerate carriers - saturation

There is saturation at high $V_D$, because all current is determined by charge density at top of barrier, where effective velocity saturates out to the hemi-Maxwell mean velocity.

Unlike the standard MOSFET, $V_{Dsat}$ is independent of $V_G$:

$$I_D = WC_s (V_G - V_T) v_T \left( \frac{1-e^{-eV_D/k_BT}}{1+e^{-eV_D/k_BT}} \right) \rightarrow V_{Dsat} = \frac{2k_BT}{e}$$

For $V_D \gg V_{Dsat}$:

$$I_{Dsat} = WC_s (V_G - V_T) v_T$$
$T > 0$ case, nondegenerate carriers – linear regime

We can expand

$$I_D = WC_x (V_G - V_T) v_T \left( \frac{1 - e^{-eV_D/k_BT}}{1 + e^{-eV_D/k_BT}} \right)$$

for small $eV_D/k_BT$ to find linear regime behavior:

$$I_D \approx \left[ WC_x (V_G - V_T) \frac{v_T}{2k_BT/e} \right] V_D$$

So, channel conductance

$$G = WC_x (V_G - V_T) \frac{v_T}{2k_BT/e} = \frac{I_{Dsat}}{2k_BT/e}$$

Regular MOSFET has

$$G = WC_x (V_G - V_T) \frac{\mu}{L}$$

Since regular MOSFET can never be better than ballistic case,

$$\mu \frac{2k_BT}{L} < v_T$$

Upper limit on mobility….

$T > 0$ case, nondegenerate carriers – linear regime

Note that channel conductance is finite even for ballistic case, as in Landauer picture.

Here, it’s a direct consequence of the thermionic emission model used here when examined at small bias.

Left-moving current down from right-moving current by $\exp(-eV_D/k_BT)$

$$I^+ = I^- e^{-eV_D/k_BT}$$

$\Delta E$ $\quad \mathcal{E}_1(x) \quad \Delta E + qV_D$
$T \sim 0$, degenerate case, linear regime

Still have $I_D = I^+(E_F) - I^-(E_F - eV_D)$

$$I^+(E_F) - I^-(E_F - eV_D) = \left( \frac{\partial I^+}{\partial E_F} \right) eV_D$$

Assuming hard transverse walls, transverse modes spaced by $(\pi/W)$ in $k$ space,

$$I^+(E_F) = eW \frac{\hbar k^3_f}{3\pi^2 m_e} = eW \frac{(2m_e E_F)^{3/2}}{3\pi^2 \hbar^2}$$

Result: $I_D = \left( \frac{2e^2}{h} \right) \left( \frac{Wk_F}{\pi} \right) V_D$

This is the Landauer expression, with $M$, the number of channels, given by

$$M = \frac{k_F}{\pi/W}$$

$$I_D = M \left( \frac{2e^2}{h} \right) V_D$$

$T \sim 0$, degenerate case, “saturated” regime

If transistor is “on” all the way, current is just $I^+$:

$$I^+(E_F) = eW \frac{\hbar k^3_f}{3\pi^2 m_e}$$

For 2d gas (one vert. subband, many transverse modes), all the right-moving carriers must be due to gate:

$$n_{2d}^\text{rr} = \frac{k^2_f}{4\pi} = \frac{C_s (V_G - V_T)}{e}$$

Plugging in,

$$I_{\text{Dual}} = WC_s (V_G - V_T) \left( \frac{8\hbar}{3m_e} \right) \sqrt{C_s (V_G - V_T)/q\pi}$$
$T \sim 0$, degenerate case, “saturated” regime

Generically, for ballistic FET, $I_{\text{dsat}}$ varies like $(V_G - V_T)^\alpha$.

For nondegenerate case, $\alpha = 1$.

For degenerate case, $\alpha = 3/2$.

Saturation happens when $V_D$ pulls right contact Fermi level below bottom of conduction band.

This happens here:

$$V_{\text{dsat}} = \frac{C_s}{e^\gamma(V_{2d}/2)}(V_G - V_T)$$

General finite temperature results:

Defining the general Fermi-Dirac integral of order $s$ as:

$$F_s(\eta) \equiv \int_0^\infty \frac{x^s \, dx}{\exp(x - \eta) - 1}$$

and the normalized drain voltage: $U_D \equiv V_D/(k_B T/e)$

and the normalized Fermi energy: $\eta_F \equiv (E_F - \epsilon_F)/k_B T$

we find:

$$I_D = eW C_s (V_G - V_T) \bar{\nu}_T \left[ \frac{1 - F_{1/2}(\eta_F - U_D)}{1 + F_0(\eta_F - U_D)} \right]$$

where:

$$\bar{\nu}_T = \frac{2k_B T}{\pi m_c} \frac{F_{1/2}(\eta_F)}{F_0(\eta_F)}$$
General finite temperature results:

Saturation regime: \[ I_{Dsat} = eWC_s(V_G - V_T)\bar{v}_T \]

Linear regime: \[ I_D = \left[ WC_s(V_G - V_T)\bar{v}_T \right] \frac{\bar{v}_T}{2(k_B T / e)} \left[ \frac{F_{\gamma/2}(\eta_F)}{F_0(\eta_F)} \right] V_D \]

Summary:

- Quantum confinement effects strongly affect transmission in ballistic nanoscale MOSFETs.
- Ignoring source-drain tunneling, velocity saturation happens near source at high \( V_D \).
- For good electrostatic design, result is current determined just by \( V_G \) and source properties.
- Can derive analytic expressions under these conditions for nondegenerate, degenerate, or arbitrary \( T \) conditions.
- Conductance near zero source-drain bias is still finite, even when device is ballistic.
- A melding of classical MOSFET theory and a Landauer way of thinking about such problems….
Next time:

- Coulomb Blockade physics
- Single-electron devices as successors to MOSFETs?