PHYS 416 – Basic Numerical Methods for ODE's

From Garcia, Chapter 2

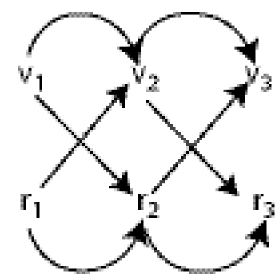
Define $f_n = f(t_n)$; $t_n = (n-1)\tau$

Note: the arrows in the diagrams indicate flow of information.

Euler Method

$$\vec{v}_{n+1} = \vec{v}_n + \tau\vec{a}_n + O(\tau^2)$$

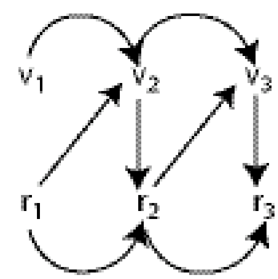
$$\vec{r}_{n+1} = \vec{r}_n + \tau\vec{v}_n + O(\tau^2)$$



Euler-Cromer Method

$$\vec{v}_{n+1} = \vec{v}_n + \tau\vec{a}_n + O(\tau^2)$$

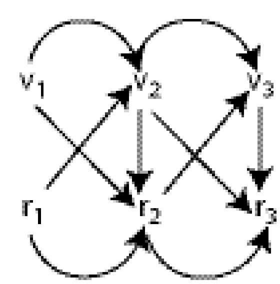
$$\vec{r}_{n+1} = \vec{r}_n + \tau\vec{v}_{n+1} + O(\tau^2)$$



Midpoint Method

$$\vec{v}_{n+1} = \vec{v}_n + \tau\vec{a}_n + O(\tau^2)$$

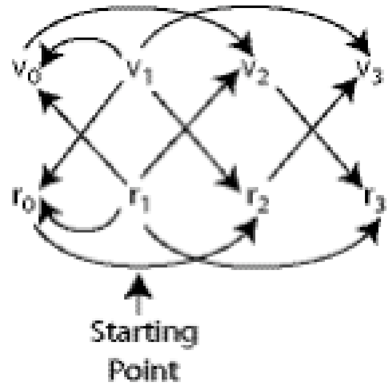
$$\vec{r}_{n+1} = \vec{r}_n + \tau\frac{(\vec{v}_n + \vec{v}_{n+1})}{2} + O(\tau^2)$$
$$= \vec{r}_n + \tau\vec{v}_n + \frac{1}{2}\vec{a}\tau^2 + O(\tau^2)$$



Leap-Frog Method

$$\vec{v}_{n+1} = \vec{v}_{n-1} + 2\tau\vec{a}_n(\vec{r}_n) + O(\tau^3)$$

$$\vec{r}_{n+1} = \vec{r}_{n-1} + 2\tau\vec{v}_n + O(\tau^3)$$



or alternatively

$$\vec{v}_{n+\frac{1}{2}} = \vec{v}_{n-\frac{1}{2}} + \tau\vec{a}_n(\vec{r}_n) + O(\tau^3)$$
$$\vec{r}_{n+1} = \vec{r}_n + \tau\vec{v}_{n+\frac{1}{2}} + O(\tau^3)$$

to start the method, use a backward Euler scheme

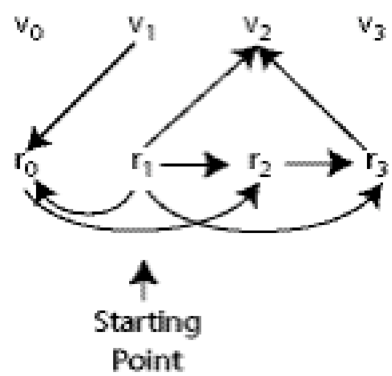
$$\vec{r}_0 = \vec{r}_1 - \tau\vec{v}_1$$
$$\vec{v}_0 = \vec{v}_1 - \tau v\vec{a}_1=$$

or better still use the same starting scheme for the Verlet scheme described below for the position.

Verlet Method

$$\vec{v}_n = \frac{\vec{r}_{n+1} - \vec{r}_{n-1}}{2\tau} + O(\tau^2)$$

$$\vec{r}_{n+1} = 2\vec{r}_n - \vec{r}_{n-1} + \tau^2\vec{a}_n + O(\tau^4)$$



to start the method, use

$$\vec{r}_0 = \vec{r}_1 - \tau\vec{v}_1 + \frac{\tau^2}{2}\vec{a}_1$$