Electromagnetic Induction

"Concepts without factual content are empty; sense data without concepts are blind.... The understanding cannot see. The senses cannot think. By their union only can knowledge be produced."

<div align="right">I. Kant (1724-1804)</div>

OBJECTIVES

To study Faraday's law quantitatively. To observe the action of a transformer.

THEORY

About 1820 the Danish physicist Oersted noticed that a wire carrying an electric current produced a magnetic field. It occurred to Michael Faraday to ask the converse question: Is a current produced by a magnetic field? The answer he found is yes, but only if the field is changing. Faraday's results can be summarized in a simple form for a loop of wire by the equation

$$V(t) = -\frac{d\Phi_B(t)}{dt} \tag{1}$$

Here *V(t)* is the induced voltage measured at a gap in the loop and $\Phi_B(t)$ is the magnetic flux through the loop. For a multi-turn loop in a uniform magnetic field $\vec{B}$, the flux is given by

$$\Phi_B(t) = BNA\cos\theta \tag{2}$$

where *A* is the area of the loop, *N* is the number of turns and θ is the angle between $\vec{B}$ and a perpendicular to the plane of the loop.

The magnetic flux through a loop might change for a variety of reasons, but in this experiment we will only be concerned with the effect of a sinusoidally varying current in a nearby circuit. This will produce a sinusoidally varying magnetic field, which causes a flux of magnitude

$$\Phi_B(t) = (B\sin\omega t)NA\cos\theta \tag{3}$$

The voltage induced is given by
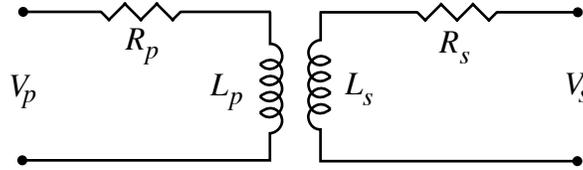
Fig. 1 Circuit diagram of a simplified transformer, including resistance of the wire in the coils.

$$V(t) = (B\omega \cos \omega t) NA \cos \theta \qquad (4)$$

This expression tells us that the amplitude of the induced voltage is proportional to the amplitude of the magnetic field variation B, the frequency ω, and the cosine of the angle θ between the field and the loop. Since B is proportional to the amplitude of the sinusoidally varying current which causes the field, we can investigate all these dependencies experimentally.

Transformers are technically important in both power and signal circuits because they can be used to change voltage and impedance levels as needed. A transformer basically consists of two coils, the primary and secondary, placed in close proximity as indicated in Fig. 1.

In operation the primary is driven by some sort of signal source, and a voltage is induced in the secondary. The individual coils are characterized by their inductance and resistance. Since the field of the primary is not usually uniform over the secondary it is difficult to calculate the induced voltage, so we simply define the mutual inductance M by the relation

$$V_s(t) = -M \frac{dI_p(t)}{dt} \qquad (5)$$

where $V_s(t)$ is the induced voltage and $I_p(t)$ is the current in the primary. Mutual inductance has the same unit, the Henry, as inductance. The relation between the primary and secondary voltages, $V_p(t)$ and $V_s(t)$, is obviously complicated for the transformer as shown. The situation simplifies greatly if we can construct the device so that all the magnetic flux that links the primary also links the secondary and so that the resistances R_p and R_s are small relative to the inductive reactances at the operating frequency. For such an ideal transformer one can show that

$$\frac{V_s(t)}{V_p(t)} = \frac{N_s}{N_p} \qquad (6)$$

where N_s/N_p is the ratio of turns on the coils.
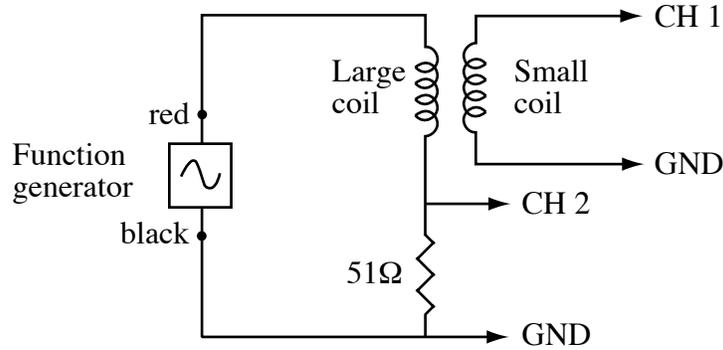
Fig. 2. Circuit for measuring induced voltage. The large coil has 552 turns and an average radius of 0.12 m. The smaller pickup coil has 200 turns and an average radius of 0.033 m.

EXPERIMENTAL PROCEDURE

Basic tests of Faraday's Law

The circuit of Fig. 2 will allow us to quantitatively determine the induced voltage in a simple system. The function generator produces a sinusoidal current which causes a varying field inside one of a pair of large mounted coils. A smaller coil on a pivoting mount is centered in the larger coil that is being driven. The scope is used to measure the induced voltage $V_s(t)$ and the voltage across the resistor, $V_R(t)$, which is proportional to the current in the large coil and hence the field.

Wire the circuit as shown, and set the function generator for about 400 Hz sine output. Use the TTL output as an external trigger signal and adjust the scope to display the signal from the small coil. By turning the pickup coil on its mount you should be able to see the angular dependence of the induced voltage.

The assumption of a uniform field across the pickup coil is reasonably satisfied because the driving coil is substantially bigger than the pickup coil. The amplitude of the uniform field is approximately that at the center of the driving coil

$$B = \frac{\mu_0 N_p I_p}{2a_p} \tag{7}$$

where a_p is the average radius of the driving coil. The peak current in the driving coil, I_p, is related to the peak voltage across the resistor by $V_R = I_p R$, so Eq. 4 can be written in terms of measurable quantities as

$$V_s(t) = \frac{\pi^2 \mu_0 N_p N_s a_s^2 f V_R}{a_p R} \cos\omega t \cos\theta \tag{8}$$

where $f = \omega/2\pi$ is the driving frequency.

We can test this deduction from Faraday's law by varying the amplitude and frequency of the function generator output while measuring $V_s(t)$. Be sure the pickup coil is in the plane of the large coil and coaxial with it. By adjusting the function generator amplitude, you can vary the current in the large coil. Use the scope to observe the amplitude of the induced voltage and the amplitude of the voltage across the resistor for several currents at a fixed f, noting that they are proportional. You can also check the phase shift between the primary current and the induced voltage. Do you observe the expected ninety degree (one-quarter cycle) phase shift?

A more interesting test is to check the frequency dependence of $V_s(t)$ from about 100 Hz to about 2 kHz. Using the scope, measure the amplitudes of $V_s(t)$ and $V_R(t)$ at several frequencies and compare your measurements with a calculation from Eq. 8. Is the prediction reasonably accurate?

You should also verify the angular dependence predicted by Eq. 8. Set the function generator to a convenient frequency near 1000 Hz and vary the angle of the pick-up coil relative to the driving coil. Is a plot of the amplitude of $V_s(t)$ vs $\cos\theta$ a straight line, as expected?

If you have time, you might also want to investigate what happens when you drive the coil with a low frequency triangle wave, about 30 Hz, rather than a sine wave. Can you explain your observations in terms of Faraday's law?

Transformers

The model transformer consists of two identical coils mounted coaxially on a small box, along with two resistors. Two pieces of magnetic material, a solid iron bar and a bundle of iron wires, are also provided.
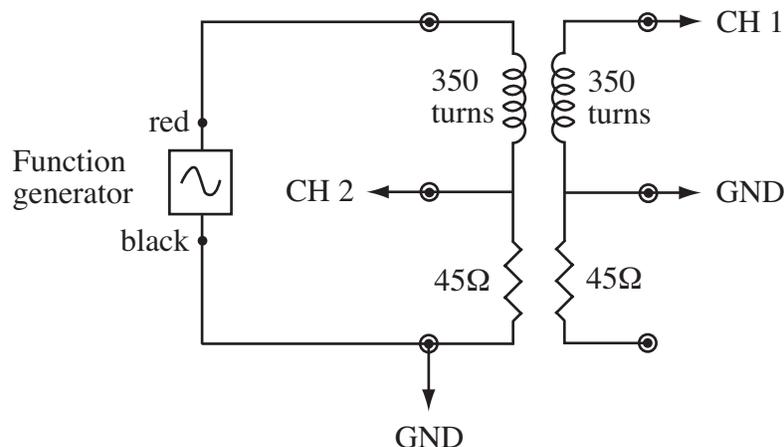
Fig. 3 Wiring diagram for the model transformer, with plug connectors as indicated. The 45 Ω resistors are provided for current measurement. The effective coil resistance is not shown.

Connect the model transformer as shown in Fig. 3, including the 45 Ω resistor in series with the primary to measure the current. Using the definition in Eq. 5, determine M for several frequencies between 100 Hz and 100 kHz, using the scope as your voltmeter to measure amplitudes. Repeat the measurements with the solid iron rod and then the bundle of wires inserted through the pair of coils. What is the effect of the magnetic material on M? Can you explain the difference in the frequency dependence for the solid bar and the wires? (Hint: Look up eddy currents in your text.)

As a last exercise, you can see how close your transformer is to the "ideal", as defined by Eq. 6. Connect the primary coil directly to the function generator and use the scope to measure the primary and secondary voltage amplitudes. Pick a fairly low frequency, about 1000 Hz, and determine the amplitude ratio V_s/V_p with and without the iron cores. Which core makes the transformer most like the ideal model?