Ampere's Law

Who shall teach thee, unless it be thine own eyes?

Euripides (480?-406? BC)

OBJECTIVES

To characterize a Hall probe for magnetic field measurements. To develop an operational understanding of a line integral and to examine the operation of Ampere's Law.

THEORY

At first glance Ampere's Law,

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 I \tag{1}$$

just seems to be a complicated way to relate a magnetic field to a current. We are instructed to divide an imaginary closed curve into equal segments of length $d\vec{s}$. Compute the dot products $\vec{B} \bullet d\vec{s}$ for all the segments, add them up, and we get a number proportional to the total current I through the surface that spans the closed curve. The law becomes odder when you think about it more carefully. Most obviously, the integral does not depend on the path chosen, but only on the current encircled. Further, the current that appears on the right hand side need not be the only source of magnetic fields along the path. Somehow the field produced by other currents does not contribute to the integral. Finally, it is only the net current through the spanning surface that is important. If we choose a path so that the current loops through it in opposite directions, the right hand side will be zero even though there is field along the path and a current is encircled. In this experiment we will see how these results come about.

The instrument we will use to measure magnetic fields is called a Hall probe. The Hall effect is a potential difference which appears at right angles to the current flow when a conductor is immersed in a magnetic field. The potential is a consequence of the Lorentz force, acting on the charge carriers, so it is linearly proportional to the component of the applied field normal to both the current and the line between the sensing electrodes. By finding the orientation that gives the maximum Hall voltage we can determine the magnitude and direction of $\vec{B}$.

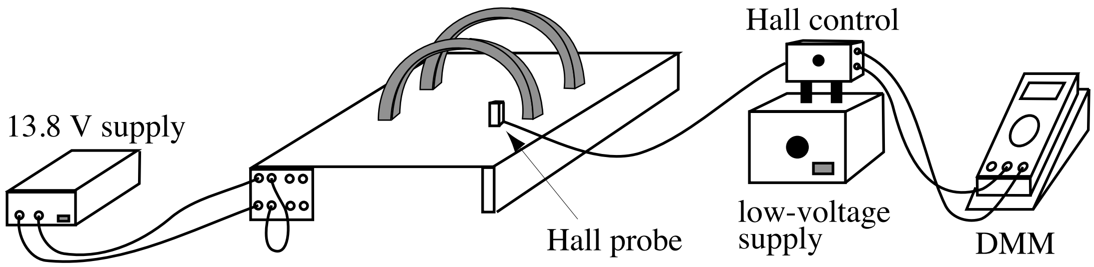

Fig. 1 Overall layout of apparatus for Ampere's Law measurements.

EXPERIMENTAL PROCEDURE

We will study the fields produced by one or two coils, mounted on a wooden plotting surface, as sketched in Fig. 1. The desired path can be marked off on a piece of paper cut to fit around the coils, and a Hall probe used to determine the dot product. By being a bit clever we can calibrate the Hall output relative to the current in the coil, and thereby carry out a quantitative test of Eq. 1.

The coils we will study are mounted on a wooden plotting table. Current is provided by a fixed 13.8 V supply. The Hall probe is powered from the adjustable low-voltage supply, which should be set for 8 V output before the Hall control box is plugged into it. Use the DMM to measure the Hall voltage output from the control box. The Hall element itself is mounted in a plastic box designed to place the Hall probe at the center of the coil diameter when the box is standing with the longest dimension vertical on the plotting table.

Hall probe set-up

The first job is to zero the Hall probe output when no magnetic field is present. Put the plastic probe box inside the magnetic shield and adjust the zero control until the DMM reads as close to zero as you can manage. When you remove the shield you should be able to detect the effect of earth's field on the readings.

Now connect either coil and the 0.5 Ω resistor in series with the 13.8 V power supply, so that the same current will flow through the coil and resistor, and turn on the supply. Put the Hall probe in the center of the energized coil and observe how the output changes as you rotate the probe relative to the field. The reading on the DMM should vary from nearly zero to a maximum of about 0.5 to 1.0V. If this doesn’t occur, check that there is current through the coils by measuring the voltage drop across the resistor and that the Hall probe is getting 8 V input from the small power supply. Consult the instructor if you cannot get a satisfactory signal.

The manufacturer claims that the probe has been designed to measure the component of the magnetic field normal to the surface of the white ceramic sensor chip. Knowing that the field of the coil must be normal to the plane of the coil, check that this claim is reasonable.

Single coil energized

To evaluate the left hand side of Eq. 1, cut a piece of paper to fit around the coils and draw a closed path on the paper. Mark off equal distances of about 1 cm along the chosen path. These intervals will approximate the small intervals $d\vec{s}$ in the line integral.

Recall that the output of the Hall sensor is proportional to $B\cos\theta$, where θ is the angle between $\vec{B}$ and the normal to the probe surface. This means that when the normal to the probe surface is parallel to $d\vec{s}$, the Hall output is linearly proportional to the desired dot product. All we need to do is line up the probe at each mark, and add up the readings to get a number proportional to the value of the line integral. (Don't forget to zero the probe inside the magnetic shield before starting measurements.)

To make a quantitative comparison in situations where the left hand side of Eq. 1 is not zero, we must calibrate the Hall probe output. An easy way to do this is to note that the field at the center of the coil is given by

$$B_0 = \mu_0 N I_c / 2R \tag{2}$$

when the coil has N turns each carrying current I_c and average radius R. For a path which encircles one side of the coil the total current appearing in Ampere's Law is NI_c, which can be found from Eq. 2, substituted into Eq. 1, and rearranged to yield

$$\frac{1}{B_0}\oint \vec{B} \bullet d\vec{s} = 2R \tag{3}$$

This means that we can add up the contributions around the path, divide by the Hall reading at the center of the coil, and compare the result with $2R$ to verify Eq. 1. For our coils, $R = 12.1$ cm.

Carry out this procedure for several paths in the vicinity of the coil. It would be of interest to try paths which encircle one side, both sides and neither side of the coil. Provide diagrams of the paths in your report, and describe how various parts of the path contribute to the final sum. Are the overall integrals quantitatively consistent with Eq. 3?

Normalization and data for two-coil geometry

You can do a similar exercise for the more complicated situation where both coils are energized, but you will need to know how much the current is reduced because the two coils together have more resistance than one alone. Before dismantling the single coil connection, measure the voltage across the 0.5 Ω resistor and call it V_1. Now connect the 0.5 Ω resistor and the two coils in series with the 13.8 V power supply so that the same current will flow through all four components. Measure the new voltage across the 0.5 Ω resistor and call it V_2. Determine if the currents in the coils are flowing in the same or opposite directions by using the Hall probe to find out if the fields tend to add or to cancel on the coil axis at the mid plane between the coils.

For a path which encircles one side of one coil we would get a relation like Eq. 3, except that the factor B_0 must be smaller because the current is less and B_0 is proportional to current. The voltage across the 0.5 Ω resistor is also proportional to current, so the new factor must be

$$B_0' = \frac{V_2}{V_1} B_0 \qquad (4)$$

Using this value you can deduce relations like Eq. 3 for paths which encircle parts of one, both or neither coil. You could also repeat the integration along one of the paths you used before to see if the results are consistent.