Rolling Motion

"Our business is with the causes of sensible effects"

I. Newton (1642-1727)

OBJECTIVES

To derive and test a model of rolling with slip.

THEORY

A round object placed on a tilted flat surface as shown in Fig. 1 can both roll and slip, depending on the frictional force available and the angle of tilt. The actual behavior should be derivable from Newton's laws and a model of the frictional force.

Referring to the quantities labeled in Fig. 1, the equations of motion are

$$Ma = Mg\sin\phi - f \qquad (1)$$

$$I\alpha = -Rf \qquad (2)$$

where M is the mass and I is the moment of inertia about the center of mass. The minus sign in Eq. 2 arises because the torque is in the $-\hat{k}$ direction for the coordinates shown. The frictional force can be approximated with the usual coefficient of friction

$$f = \mu N = \mu Mg\cos\phi \qquad \text{slipping} \qquad (3)$$

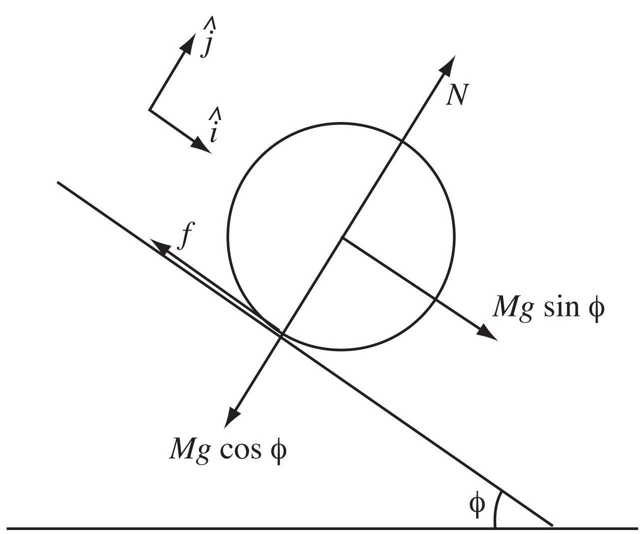

Fig. 1 A round object of mass M, radius R and moment of inertia I which can roll and slip on the inclined plane.

$$f \leq \mu Mg\cos\phi \qquad \text{not slipping} \qquad (4)$$

neglecting any difference between static and sliding coefficients. If the conditions are such that the object rolls without slipping, there is also a geometric relationship between the displacement of the center of mass and the angle turned or, equivalently, between the accelerations:

$$x = -R\theta \quad \Rightarrow \quad a = -\alpha R \qquad (5)$$

where the minus sign again occurs because of our choice of coordinates.

The qualitative features of the motion can now be deduced from the equations. For small tilt angles the linear acceleration down the incline will be small. The frictional force can then be large enough to produce the needed angular acceleration to cause rolling without slip and satisfy Eq. 5. As the tilt increases, the angular acceleration required to avoid slip and the frictional force needed to produce that angular acceleration also increase. At a critical angle the maximum possible frictional force, given by Eq. 4, will be reached and the object will start to slip. The object will still move down the incline with calculable linear and angular accelerations, but they will no longer be related by Eq. 5.

Quantitative analysis of each regime is straightforward. For tilts below the critical angle, Eqs. 1, 2 and 5 can be solved for a, α and f in terms of the mass, gravitational acceleration g, and the various geometric factors. You will find, unsurprisingly, that the motion occurs with constant linear and angular accelerations, whose magnitudes increase with tilt. Above the critical angle, f is maximum at the value given by Eq. 3. Substituting the maximal f into Eqs. 1 and 2 yields different accelerations, which now depend on μ as well as the other parameters. Interestingly, the accelerations are still constant in this case. Finally, the critical angle can be found by equating the expression for f in the no-slip case to the maximum possible non-slip f given by Eq. 4.

EXPERIMENTAL PROCEDURE

The experimental goal is check the expressions you have derived. You can use the video system to measure $x(t)$ and $\theta(t)$ to determine if the accelerations are constant and of the expected magnitude. You can also find out if the transition to slipping occurs at the expected critical angle.

1. Physical arrangement

The round object is a hoop with reference marks at the center and on the rim. It can roll down a smooth ramp whose tilt can be increased by raising a support bar under one end. Position the ramp so that the motion is perpendicular to the camera axis. The actual tilt can be determined from the height difference of the two calibration marks or of the ends of the ramp.

The camera should already be set up on a support across the room from the ramp. Check that the power is on (plugged in) and the round switch on the back is set to MOVIE. You can check the orientation and field of view when you open the preview screen of the capture program.

The camera has been set for a fast shutter speed (1/500th s) in order to produce a sharp image of rapid motion. Auxiliary lamps, mounted on a stand, provide the additional light needed for a good exposure. The lights should be turned on when you are taking data, and turned off when you are finished taking pictures.

2. Data acquisition

Start LoggerPro from Rolling.cmbl, since it has the needed functions in the fitting menu. Check the preview screen and adjust the camera zoom or position if necessary to get all of the ramp into the picture. Practice releasing the cylinder from near the top of the ramp until you can get a good recording of it rolling at least most of the way down without falling off.

Analyze the data by marking the position of the center of the hoop for as much of the motion as possible. Then add a second point series, and mark the edge of the hoop at the end of the radial bar. Calibrate the distance scales using the two black marks on the edge of the ramp. To facilitate later analysis, move the origin of coordinates to the starting position of the hoop center, and rotate so the positive x axis is along the motion of the center.

To compare with theoretical expectations, you need to determine if the linear and angular accelerations of the hoop are constant, and, if so, how the accelerations depend on ramp angle.

The linear acceleration parallel to the ramp is found by plotting x vs t for the center of the hoop. In LoggerPro, this is X vs Time from the first point series. If the linear acceleration is constant, the data will fit a quadratic curve, and the acceleration is twice the coefficient of t^2 in the fit.

Finding the angular acceleration is trickier since we do not directly measure the rotation angle. However, the y-position of a point on the rim is related to the rotation angle θ by $y = R\sin(\theta)$. If θ is increasing with constant angular acceleration, then y will be given by

$$y(t) = R\sin\left(\tfrac{1}{2}\alpha t^2 + \omega_0 t + \theta_0\right) \qquad (6)$$

If the x-axis is along the line of motion of the center, a plot of Y2 vs Time2 should have the form of Eq. 6. Use the function Accel Roll in the fitting menu to see if this is correct. If the function describes the data well, you can read the angular acceleration from the coefficient of t^2. Note that the fitting parameter R will probably be smaller than the true radius because you have not marked the exact outer edge of the cylinder.

You will want to repeat these measurements so that you have 3-4 angles in the rolling regime, and 3-4 in the slipping regime. The maximum tilt will probably be limited by the erratic motion that occurs when the normal force becomes very small and the hoop can be deflected by minor irregularities in the ramp surface. Since you don't know the critical angle *a priori*, you should analyze your data as you proceed to be sure of adequate coverage.

3. Analysis

There are several ways that you can compare your data with the theoretical model:

a) Do the fits demonstrate that the linear and angular accelerations are constant for each angle?

b) Below the critical angle, the accelerations are independent of the unknown coefficient of friction and a plot of a vs $\sin\phi$ should be a straight line. Is that observed? With the expected slope? Is the magnitude of $a/\alpha R = 1$, as required by Eq. 5?

c) Above the critical angle, you can show that a plot of α vs $\cos\phi$ should be straight, with a slope that depends on μ. Is that observed, and is the value of μ plausible?

d) Above the critical angle, $a/\alpha R$ should be linear in $\tan\phi$, whereas it is constant below the critical angle. You can, therefore, clearly display the transition to slipping by plotting $a/\alpha R$ vs $\tan\phi$. Use this plot to estimate the critical angle.