Collisions in Two Dimensions

"In the field of observation, chance only favors those minds which have been prepared"

L. Pasteur

OBJECTIVES

To quantitatively examine some collisions between macroscopic objects.

THEORY

Much of our knowledge of atoms and nuclei and all of our knowledge of elementary particles is derived from observations of carefully arranged collisions. The basic sequence of a collision is shown in Fig. 1, along with a convenient set of reference axes. The goal of a collision experiment is to find out how the particles interact during the collision by measuring the masses and velocities of the particles before and after the collision. In the case of microscopic objects, we often do not have direct access to what the particles do during the collision because the lengths and times involved are too small for observation, so there is no more direct way to obtain this information. In this experiment you will study several simple collisions between macroscopic objects to gain some understanding of how these experiments work, and the effects of various conservation laws.

In many cases conservation laws restrict, but do not totally determine, the results of a collision. For example, if the force between m_1 and m_2 is the only force which can change their motion, then both linear and angular momentum will be conserved in the system consisting of m_1

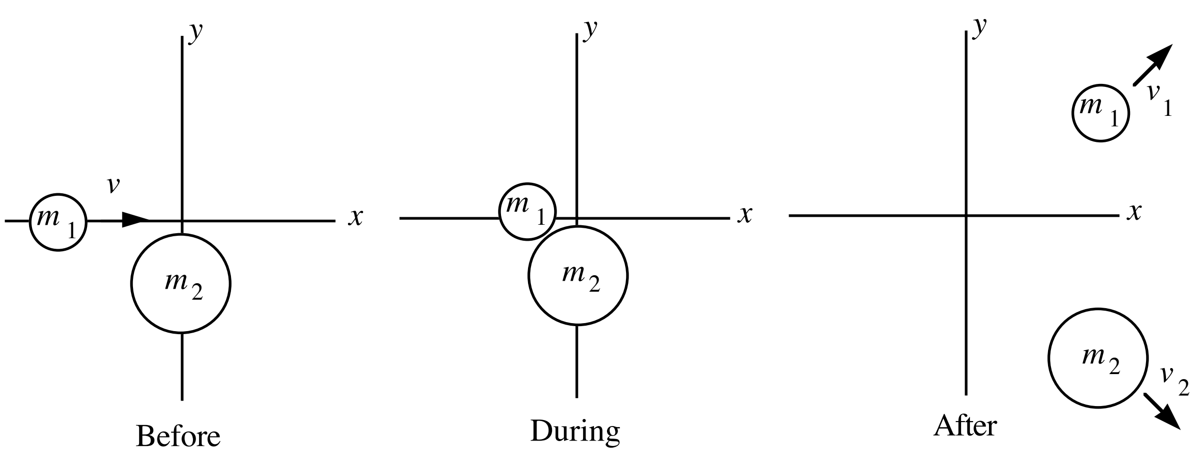

Fig. 1. A collision sequence showing initial and final velocities. The particles may also rotate before and after the collision.

and m_2. In the situation shown in Fig. 1, where only m_1 is moving before the collision, this tells us immediately that at least one of the masses must be moving after the collision to carry off the initial momentum.

More generally, we can express the conservation of linear momentum by the vector relationship

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \tag{1}$$

which asserts that all components of linear momentum must be conserved regardless of what happens during the actual collision. We will work on an air table which constrains the motion to the *x-y* plane and reduces the frictional forces in that plane nearly to zero, so we would expect Eq. 1 to hold for our examples.

Angular momentum is a little more complicated because there are two pieces for a rigid body. The part due to translational motion is

$$\vec{L}_t = \vec{r} \times m\vec{v} \tag{2}$$

where $\vec{r}$ is a vector from the chosen, fixed, origin to the center of mass and $\vec{v}$ is the velocity of the center of mass. There is also angular momentum due to the rotation or 'spin' of the object about the center of mass, given by

$$\vec{L}_s = I\vec{\omega} \tag{3}$$

where I is the moment of inertia about the center of mass and $\vec{\omega}$ is the angular velocity about the center of mass. The total angular momentum is then $\vec{L} = \vec{L}_s + \vec{L}_r$ and we can write the conservation of angular momentum in the form

$$\vec{L}_{1i} + \vec{L}_{2i} = \vec{L}_{1f} + \vec{L}_{2f} \tag{4}$$

If the motion is limited to two dimensions, the translational angular momentum is necessarily perpendicular to the plane of motion. On an air table, the sliders can only rotate about the vertical direction, so the spin angular momentum is also perpendicular to the plane of motion, and Eq. 4 reduces to a single scalar equation. Since only internal forces act in the plane of motion, there are no external torques to change the angular momentum of the system, and we expect Eq. 4 to hold for the collisions we will study.

Although unlikely for macroscopic objects, it may also happen that the kinetic energy remains constant during the collision, so that

$$E_{1i} + E_{2i} = E_{1f} + E_{2f} \tag{5}$$

As with the angular momentum, the kinetic energy of a rigid object can be thought of as the sum of a piece due to translation of the center of mass and a piece due to rotation about the center of mass,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{6}$$

If Eq. 5 is obeyed, the collision is said to be elastic, otherwise it is inelastic. (Some people restrict the term elastic to situations where the translational energy alone is constant.) Since we do not know much about what happens when two objects hit each other, we will have to discover experimentally whether our collisions are elastic or inelastic.

For a collision in two dimensions with known starting conditions there are four unknown linear velocity components and two angular speeds after the collision. Eqs. 1, 4 and 5 supply at most four restrictions on these six quantities, and in fact only three if the collision is not known to be elastic. The final velocities must, therefore, be determined by the details of the forces acting during the collision. By measuring the final velocities for various initial conditions one can learn about the interaction force between the particles, at least in principle. During the course of the experiment you will see qualitatively how this works, and also see how the conservation laws are applied to a real collision.

EXPERIMENTAL PROCEDURE

You should first investigate several types of collisions qualitatively, to get an idea of the phenomena you will want to measure. After that, you should quantitatively study one nearly-elastic collision between equal-mass sliders, and two inelastic collisions with different initial conditions. In each case, we will be interested in seeing if the momentum conservation laws hold and in finding the kinetic energy losses.

1. Physical Arrangement

The collisions will take place on an air table, a surface with numerous small holes through which air is blown to allow plastic disks to slide with very little friction. Disks are available with two different masses, and they can be fitted with Velcro® bumpers to change the character of their collisions. Qualitative data can be obtained by visual observation of collisions. A video system is provided for quantitative analysis.

Before starting the experiment, there are some precautions you must be aware of. Never slide anything on the surface of the air table when the air flow is off. This will damage the surface, and may plug the small holes. You should also handle the sliders carefully to avoid nicks and deep scratches which would increase the sliding friction. Best results will be obtained if the collisions are relatively gentle. Hard collisions cause the edges of the sliders to hit the surface of the table, introducing external forces which complicate the experiment.

The table should be approximately level. To check, turn on the air supply to the table and release a slider. It will probably wander a bit because the surface is not exactly flat and because of air currents in the room, but there will be no consistent direction at all points on the table. If there seems to be a problem, contact the instructor as the leveling is awkward to adjust.

The video camera is already mounted above the air table, looking vertically downward. When you are ready to use it, you will need to plug in the power supply to turn it on. You can then check the orientation and field of view from the computer screen. Consult the instructor if adjustments seem to be needed. Please unplug the camera when you finish using it, but leave it otherwise undisturbed for the next group of students.

2. Qualitative observations

The first part of your work will be qualitative. You have two types of sliding disks available, differing in mass by about a factor of two. Set up a variety of collisions, ranging from head-on to near-miss, with each possible projectile-target pair: heavy on heavy, heavy on light and light on heavy. Observe the collisions carefully, and sketch the outcome of a head-on and a glancing collision for each of the pairs. Be sure to note the relative speeds of projectile and target after the collisions.

When a heavy projectile strikes a lighter, stationary target will the projectile ever bounce backward? Is the same true for a light projectile striking a heavy target? Describe the conditions required for the projectile to recoil backward from the collision. At this point you should begin to see how measurements on scattered projectile particles could tell us something about the relative mass of projectile and target.

It is also interesting to experiment with collisions in which the target and projectile stick together after the collision. This is known as a completely inelastic collision, not because all the initial kinetic energy is lost, but because as much is lost as is consistent with conservation of momentum. To create a completely inelastic collision, slip one Velcro® collar over each of the heavy sliders, being careful that the collars do not drag on the surface. Sketch the outcome of a head-on and a glancing collision. Is the final translational velocity roughly half the initial velocity of the projectile when the target is initially stationary? Why should this be true? In a near-miss collision you will be able to see quite clearly where some of the "lost" translational kinetic energy goes, a point you should discuss.

3. Quantitative analysis

Use LoggerPro to obtain and analyze movies of the collisions you intend to analyze. The distance between red dots on the air table is used for calibration, and the masses of the sliders are marked. The remaining procedures depend on the type of collision.

a) Nearly-elastic. This should be a collision between sliders without Velcro collars. Their masses need not be equal, and either can be the projectile. Do not use a head-on collision as it is less interesting than a 2D collision.

You will want to follow the motions of the centers of mass of the sliders for times before and after the collision. Point series 1 should follow the center of one slider through the collision sequence. Then add a point series and follow the center of the other slider through the collision.

To check the kinetic energy and linear momentum you need the velocity components for both the target and projectile before and after the collision. Since the velocities should be constant outside the collision region, we can find them by fitting straight lines to a position-time plot. Set up a graph of X and Y vs Time, and do separate linear fits to the data before and after the collision. The slopes are the x and y components of velocity for that slider. Repeating the process with a graph of X2 and Y2 vs Time 2 will yield the velocities for the other slider.

Use your measured velocity components and the known masses to see if linear momentum is constant within errors, as expected. Calculate the change in translational kinetic energy and determine whether or not the collision is elastic. You may neglect the rotational motions for this part of the exercise.

b) Completely inelastic, head-on, equal-mass. Install Velcro collars on two equal-mass sliders, so that the sliders will stick on contact. This maximizes the loss of kinetic energy, hence the term "completely inelastic". The collision should be as nearly 'head-on' as possible, so that the joined sliders leave the collision with very little rotation. Neither projectile nor target should be rotating before impact. The analysis is slightly simplified if the target is stationary, but that is not essential.

Before the collision you need to track the centers of mass of the two sliders. Afterward, you need the position of the center of mass of the pair, which will be at the point of contact between sliders. It is convenient to use the first point series to mark the projectile center before the collision, and the center of mass of the pair after the collision. If the target is moving, you can use a second point series to track it before the collision.

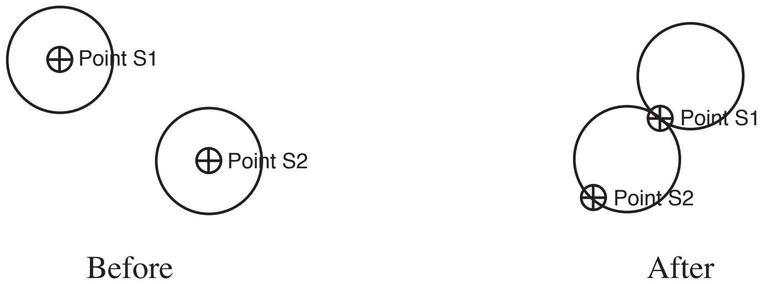

Fig. 2 Point-series labels for inelastic collision.

Find the velocity components of the centers of mass as you did for the nearly-elastic collision. Use the components to find the translational kinetic energy before and after, and to determine whether or not the linear momentum is constant. The rotational part of the kinetic energy should be small, so you may ignore it.

c) Completely inelastic, grazing incidence, equal-mass. Using the Velcro collars again, record a collision where the joined sliders spin rapidly about their center of mass. Neither projectile nor target should be rotating before the collision, and, again, the analysis is simpler if the target is stationary.

Use the first point series to mark the center of the projectile before collision and the center of mass of the pair after the collision. If you chose to use a moving target, use a second point series to track the center of the target. This data can be analyzed to find the momentum and the translational part of the kinetic energy before and after the collision, as you did before.

It is also of interest to determine the rotational part of the kinetic energy after the collision, since this may be significant. To do this, use the second point series to mark the edge of one of the sliders opposite the point of contact, as suggested in Fig. 2. Translate and rotate the coordinate system so that the origin is at the contact point in the first image after the collision with the x-axis along the line of motion of the contact point. The y-coordinate of the edge point should then follow a sinusoidal trajectory which you can fit with the Sine function. The angular speed, in rad/s, of the joined sliders is just the coefficient of t in this fit. Note: The coordinate system applies to all of the data. Do not mix vector components found in two different coordinate systems.

The moment of inertia of joined sliders with equal mass is

$$I = 2 \times mr^2 + 2 \times \tfrac{1}{2} mr^2 = 3mr^2 \qquad (6)$$

where m and r are the mass and radius of the individual slider. Use this with your angular speed to determine the rotational kinetic energy of the pair. You can then calculate the total kinetic energy before and after the collision for both the glancing and head-on examples. Is some of the total initial kinetic energy lost? How does the loss compare with the head-on collision?

You may also attempt to check for conservation of angular momentum, but it is difficult to accurately measure the translational contribution using our equipment, so it is not required.