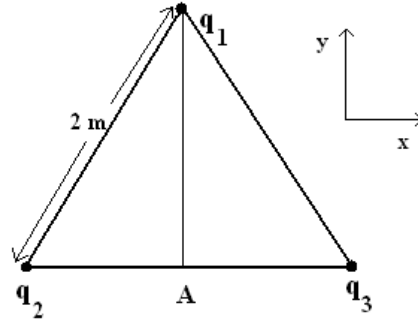


## Exam No. 1 Solutions

I. (20 pts) Three positive charges  $q_1 = +2 \mu\text{C}$ ,  $q_2 = +1 \mu\text{C}$ , and  $q_3 = +1 \mu\text{C}$  are arranged at the corners of an equilateral triangle of side 2 m as shown in the diagram. Calculate:

a) The force exerted on  $q_1$  by the other charges.



**Answer:**

Because charges  $q_2$  and  $q_3$  are equal, and  $q_1$  lies on the line bisecting the two charges, the horizontal components will cancel, leaving only the vertical components, or

$$\begin{aligned} \vec{E}_y(q_2) = \vec{E}_y(q_3) &= \frac{q}{4\pi\epsilon_0 r^2} \cos \theta = \frac{(9 \times 10^9 \text{ Nm}^2\text{C}^{-2})(1 \times 10^{-6} \text{ C})}{(2\text{m})^2} \cos 30^\circ \\ &= (1.95 \times 10^3 \text{ NC}^{-1}) \end{aligned}$$

$$\begin{aligned} F &= q_1 \cdot (\vec{E}_y(q_2) + \vec{E}_y(q_3)) = (2 \times 10^{-6} \text{ C}) \cdot 2 \cdot (1.95 \times 10^3 \text{ NC}^{-1}) \\ &= \mathbf{7.8 \times 10^{-3} \text{ N}} \end{aligned}$$

b) The electrostatic potential energy of the system of charges

**Answer:**

There are a number of ways to describe the potential energy associated with assembling a system of charges,

$$U = \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}} = \frac{1}{2} \sum_{i,j,i \neq j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} = \frac{1}{2} \sum_i q_i V_i$$

where  $V_i$  is the potential at  $q_i$  due to all the other charges, but the answers will all be the same.

For this assembly,

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}}$$

Since  $r_{12} = r_{13} = r_{23} = r$ , this becomes

$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0 r} (q_1 q_2 + q_1 q_3 + q_2 q_3) \\ &= \frac{(9 \times 10^9 \text{ Nm}^2\text{C}^{-2})}{2} \left( 2((2 \times 10^{-6} \text{ C})(1 \times 10^{-6} \text{ C})) + (1 \times 10^{-6} \text{ C})(1 \times 10^{-6} \text{ C}) \right) \\ &= \mathbf{2.25 \times 10^{-2} \text{ J}} \end{aligned}$$

- c) A charge is now placed midway between  $q_2$  and  $q_3$  at point A. If the object is released from rest at this point, what is its velocity when it is a large distance from the three charges? (Take  $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ )

**Answer:**

This portion of the question requires recognizing that, far from the surface, the potential energy of a particle initially placed at position A will all be converted to kinetic energy, and the question is asking to solve for the velocity associated with that kinetic energy.

The potential energy,  $U$ , that the 4<sup>th</sup> charge will have at A will be  $U = q_4 V$ , where  $V$  is the potential at A produced by the three positive charges,  $q_1$ ,  $q_2$  and  $q_3$ .

$$\begin{aligned} U &= q_4 \left( \frac{q_1}{4\pi\epsilon_0 r_1} + \frac{q_2}{4\pi\epsilon_0 r_2} + \frac{q_3}{4\pi\epsilon_0 r_3} \right) \\ &= (3 \times 10^{-6} \text{ C})(9 \times 10^9 \text{ Nm}^2\text{C}^{-2}) \left( \frac{(2 \times 10^{-6} \text{ C})}{2\text{m} \cos 30^\circ} + \frac{(1 \times 10^{-6} \text{ C})}{1\text{m}} + \frac{(1 \times 10^{-6} \text{ C})}{1\text{m}} \right) \\ &= \mathbf{8.5 \times 10^{-2} \text{ Nm}} \end{aligned}$$

The loss of this potential energy will result in a gain in kinetic energy, so that far from the surface, the kinetic energy will simply equal the initial potential energy  $U$ .

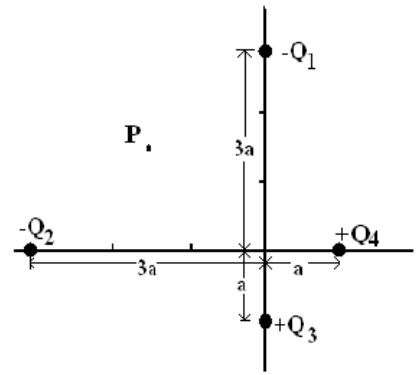
The velocity is derived from  $-\Delta U = \Delta KE = \frac{1}{2} m v^2$ , or  $v = \sqrt{2\Delta KE/m}$

Plugging in the numbers, one gets

$$v = \sqrt{2 \cdot 8.5 \times 10^{-2} \text{ Nm} / 10^{-3} \text{ kg}} = \mathbf{13 \text{ m/s}}$$

II. (25 pts) Four charged particles, carrying the same magnitude  $Q$  of charge but of different signs, are arranged on the  $x$  and  $y$  axes as shown in the figure.

- a) Determine the direction of the electric field vector at the point  $P$  midway between the negative charges. (You should be able to answer this part without any numerical calculation.)



**Answer:**

First, the fields associated with the 2 negative charges (located at  $(0, 3a)$  and  $(-3a, 0)$ ) will cancel by inspection. Second, because  $P$  is on the line bisecting the 2 positive charges (located at  $(a, 0)$  and  $(0, -a)$ ), the portions of the field perpendicular to that bisecting line will cancel, leaving a field pointing along that line, or at  $45^\circ$  above the  $-x$ -axis.

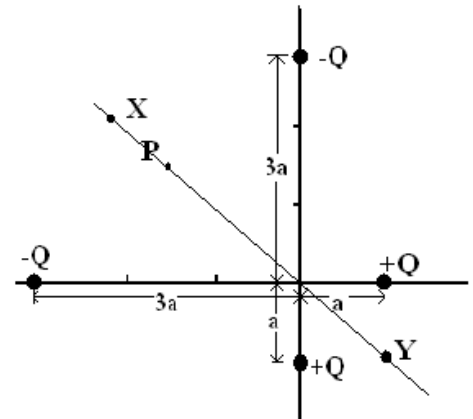
**Alternative approach** –The direction is determined by symmetry, and points away, at  $45^\circ$  above the  $-x$ -axis.

- b) Indicate the approximate location of the point(s) at which the electric field is zero.

**Answer:**

The first point (marked  $X$  on the figure below) will be on the line going through  $P$  and the origin and further away from the origin than  $P$ . At that point, the net fields created by the negative charges (which will be pointed toward the origin) will be offset by the net fields associated with the positive charges (which will be away from the origin).

There will be a similar point along the same line (marked  $Y$  on the figure below) outside the point midway between the 2 positive charges, where again, the net field along that line associated with the positive charges (which will be pointed away from the origin) will be offset by the net fields associated with the negative charges (which will point toward the origin.)



- c) Devote an entire page in your bluebook to a sketch of electric field lines in the  $xy$  plane.

**Answer:**

See attached.

III. (25 pts) A solid metal sphere of radius  $R_1$  carries a charge  $-Q_1$ , where  $Q_1 > 0$ . Surrounding this sphere is a metal shell of inner radius  $R_2 = 2R_1$  and outer radius  $R_3 = 3R_1$  that carries a total charge of  $Q_2 = +3Q_1$ .

a) Determine the electric field at all values of  $r$ .

**Answer:**

For  $r < R_1$ . Because this area is inside a metal conductor, the electric field will be zero.

For  $R_1 < r < R_2$  The charge enclosed will be  $-Q_1$ , and the electric field can be determined either from using a concentric spherical Gaussian surface with radius  $r$  in this region, or simply noting that the field will be the same as if all the charge were located at the origin.

$$\vec{E} = -\frac{Q_1}{4\pi\epsilon_0 r^2} \hat{r}$$

For  $R_2 < r < R_3$ . The electric field will again be zero because this area is inside a metal conductor.

For  $r > R_3$ . Because the charge distribution is spherically symmetric, a concentric spherical Gaussian surface should be used. The total charge enclosed will be

$$Q_{encl} = -Q_1 + Q_2 = -Q_1 + 3Q_1 = 2Q_1.$$

$$\oint \vec{E} \cdot d\vec{A} = E4\pi r^2 = \frac{Q_{encl}}{\epsilon_0} = \frac{2Q_1}{\epsilon_0}$$

$$\vec{E} = \frac{2Q_1}{4\pi\epsilon_0 r^2} \hat{r}$$

b) How is the charge  $Q_2$  divided between the surfaces at  $R_2$  and  $R_3$ ?

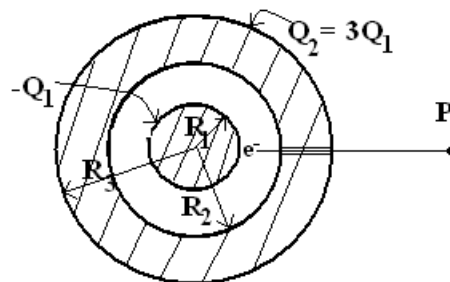
**Answer:**

$+Q_1$  must appear on the inner surface  $R_2$ , to meet the requirement that the electric field inside the outer shell is zero. The remaining part of  $Q_2$ , equal to  $2Q_1$ , will appear on the outer surface, at  $R_3$ .

c) Determine the potential of the inner sphere ( $r < R_1$ ).

**Answer:**

The potential can be derived from the electric field  $-V = -\int_{\infty}^{R_1} \vec{E} \cdot d\vec{l}$ ; where the path to be integrated over will need to be divided into each of the 3 areas  $R_1 < r < R_2$ ,  $R_2 < r < R_3$ , and  $r > R_3$ .



$$\begin{aligned}
V &= - \int_{\infty}^{R_3} \frac{2Q_1}{4\pi\epsilon_0 r^2} dr - \int_{R_3}^{R_2} 0 dr - \int_{R_2}^{R_1} \frac{-Q_1}{4\pi\epsilon_0 r^2} dr \\
&= - \frac{Q_1}{4\pi\epsilon_0} \left( 2 \int_{\infty}^{R_3} \frac{1}{r^2} dr + \int_{R_2}^{R_1} \frac{1}{r^2} dr \right) \\
&= \frac{Q_1}{4\pi\epsilon_0} \left( 2 \left( \frac{1}{R_3} \right) + \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \right) = \frac{Q_1}{4\pi\epsilon_0} \frac{1}{6R_1}
\end{aligned}$$

An alternative approach is to sum the potentials at  $R_1$  due to the separate charge distributions on the shells located at  $R_1$ ,  $R_2$ , and  $R_3$ .

$$V = \frac{1}{4\pi\epsilon_0} \frac{-Q_1}{R_1} + \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_2} + \frac{1}{4\pi\epsilon_0} \frac{2Q_1}{R_3} = \frac{Q_1}{4\pi\epsilon_0} \frac{1}{6R_1}$$

- d)** An electron is emitted from the surface of  $R_1$  with negligible velocity. It is accelerated to the right and escapes through a small hole in the metal shell. It comes to rest at point P. How far is P from the center of the metal sphere?

This question asks at what point outside the sphere is the potential the same as the value just calculated in part (c). Outside the sphere,

$$V(r > R_3) = - \int_{\infty}^r \frac{2Q_1}{4\pi\epsilon_0 r'^2} dr' = \frac{2Q_1}{4\pi\epsilon_0 r}$$

Thus,

$$\frac{Q_1}{4\pi\epsilon_0} \frac{1}{6R_1} = \frac{2Q_1}{4\pi\epsilon_0 r} \text{ when } r = 12R_1$$

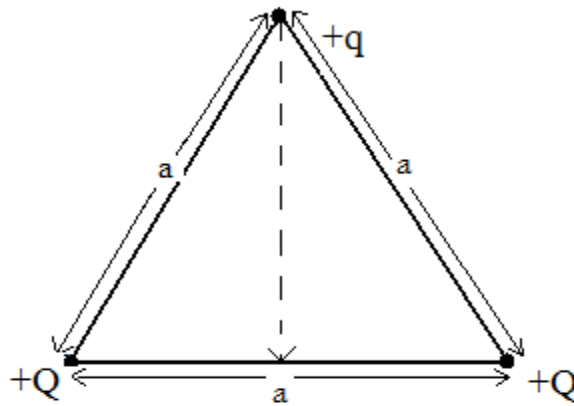
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### Physics 102 Spring 2008: Exam #1 – Multiple-Choice Questions

1. Three charges are initially positioned at the vertices of an equilateral triangle with sides of length  $a$  as shown. The work required to move the charge  $+q$  to the midpoint of the line joining the charges  $+Q$  is:

- (A) 0
- (B)  $\frac{1}{4\pi\epsilon_0} \frac{Qq}{a}$
- (C)  $\frac{1}{4\pi\epsilon_0} \frac{Qq}{a^2}$
- (D)  $\frac{1}{4\pi\epsilon_0} \frac{2Qq}{a}$  ✓
- (E)  $\frac{1}{4\pi\epsilon_0} \sqrt{2} \frac{Qq}{a}$

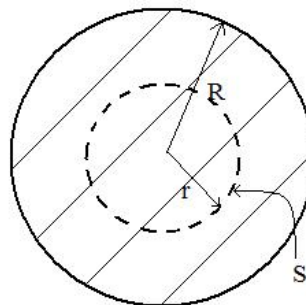


2. Two well separated metal spheres of radii  $R_1$  and  $R_2$  carry equal electric charges  $Q$ . The spheres are at potentials  $V_1$  and  $V_2$ , respectively, with  $V_1=3V_2$ . The spheres are then connected by a conducting wire. When equilibrium is reached, what is the charge on the sphere of radius  $R_2$ ?

- (A)  $3Q/2$  ✓
- (B)  $Q/3$
- (C)  $2Q$
- (D)  $Q/2$
- (E)  $Q$

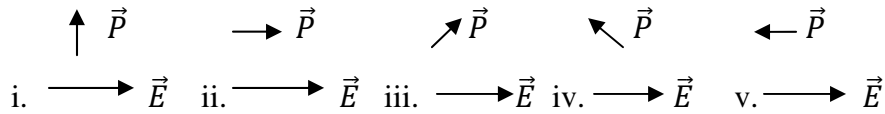
3. Consider a solid conducting sphere of radius  $R$  on which a net charge  $+Q$  is deposited. The total flux of the electric field through a spherical surface  $S$  of radius  $r < R$  is proportional to:

- (A)  $Q(r/R)$



- (B)  $Q(r/R)^2$
- (C)  $Q(R/r)$
- (D)  $Q(R/r)^2$
- (E) **zero** ✓.

4. The diagrams show five possible orientations of an electric dipole  $\vec{P}$  in a uniform electric field  $\vec{E}$ -

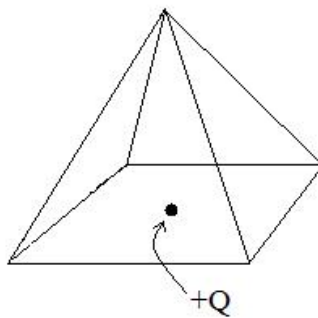


where the electric field  $\vec{E}$  is represented by the longer, horizontal vector and the dipole  $\vec{P}$  is represented by the shorter vector with differing orientation. For which of these orientations does the torque on the dipole have its greatest magnitude?

- (A) **i.** ✓
- (B) ii.
- (C) iii.
- (D) iv.
- (E) v.

5. A point charge  $+Q$  is positioned at the center of the base of a pyramid as shown. The flux through each of the four identical upper faces of the pyramid is

- (A)  $Q/\epsilon_0$
- (B)  $Q/2\epsilon_0$
- (C)  $Q/4\epsilon_0$
- (D)  $Q/5\epsilon_0$
- (E)  **$Q/8\epsilon_0$**  ✓



6. Three possible configurations of three point charges of equal magnitude on the corners of a right isosceles triangle are depicted below. Rank the electrostatic potential energy of the charge configurations from most positive to most negative.

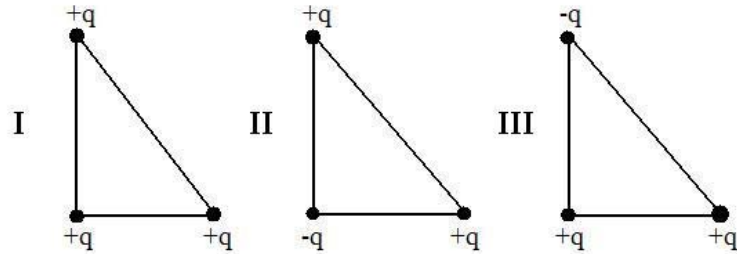
(A) I > II > III

(B) II > I > III

(C) **I > III > II** ✓

(D) III > II > I

(E) II > III > I



7. A positive charge  $+Q$  is placed on a spherical conducting shell with inner radius  $R_1$ , outer radius  $R_2$ . A point charge  $-q$  is placed at the center of the cavity. The total charge on the inner face (radius  $R_1$ ) of the shell is

(A)  $Q$

(B)  **$q$**  ✓

(C)  $Q + q$

(D)  $Q - q$

(E)  $q - Q$

8. The direction of the electric field at a point equidistant from two charged bodies A and B is given by a vector parallel to the line joining A and B. From this we can deduce that

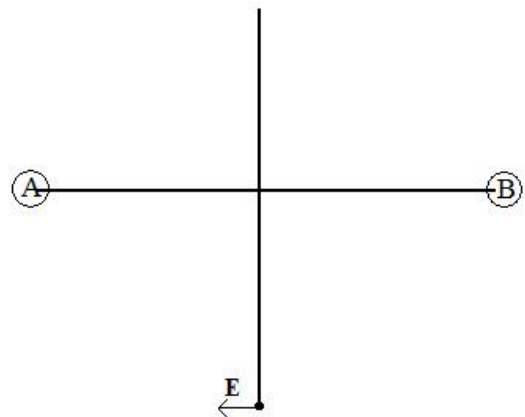
(A) Both A and B are positively charged.

(B) Both A and B are negatively charged.

(C) A is positive and B is negative.

(D) **A is negative and B is positive.** ✓

(E) The information does not lead to any of the above conclusions.

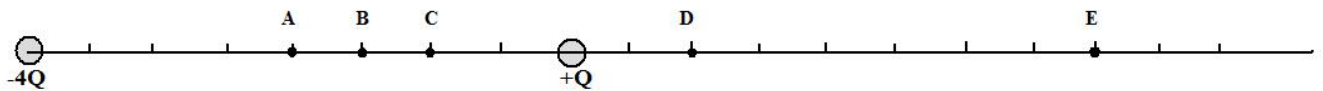




9. Two identical, small, conducting spheres are separated by a distance of 1 m. The spheres originally have equal but opposite charges, and the force between them is  $F_0$ . Half of the charge on one sphere is then moved to the other sphere. The force between the spheres is now

- (A)  $F_0/4$ . ✓
- (B)  $F_0/2$ .
- (C)  $3F_0/4$ .
- (D)  $3F_0$ .
- (E)  $3F_0/2$ .

10. Two small spheres having charges  $+Q$  and  $-4Q$  are situated as shown. The electrostatic force on a small charge will be zero if it is placed at



- (A) A
- (B) B
- (C) C
- (D) D
- (E) **E** ✓

