## Physics 102 Spring 2006: Final Exam; May 3, 2006 <br> Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiplechoice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 3 HOURS
- The test consists of three free-response questions and 25 multiple-choice questions.
- The test is graded on a scale of 180 points; the free-response questions account for 105 points ( 35 points each), and the multiple-choice questions account for 75 points ( 3 points each).
- Answer the three free-response questions in your blue book. Answer the multiple-choice questions by marking a dark $X$ in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, nonprogrammed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labeled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

I. A spherical capacitor consists of an inner solid conducting sphere of radius a surrounded by a spherical conducting shell of inner radius $b$ and outer radius $c$. The capacitor is charged with $-Q$ on the inner sphere and $+Q$ on the outer spherical shell. The coordinate $r$ measures the distance from the center of the solid sphere.
(a) Determine the electric field $\vec{E}(r)$ everywhere in space due to this charge configuration, indicating both direction and magnitude. Sketch $E(r)$ vs. $r$ for all $r$.
(b) In part (c) you will calculate the potential difference $\Delta V$ between the inner and outer conductors. Before you calculate the value, indicate what the sign (positive or negative) of $\Delta V=V_{b}-V_{a}$ must be and give a clear explanation as to why.
(c) Determine the potential difference $\Delta V=V_{b}-V_{a}$ between the inner and outer conductors.
(d) From the result in (c), determine the capacitance of this device and the energy stored in it.
(e) Taking the electrostatic potential to be zero at infinity, determine $V(r)$ for all $r$ and sketch $V(r)$ vs. $r$.
(f) Determine the energy density in the electric field in the region between $a$ and $b$. Integrate the energy density over the volume between $a$ and $b$ and compare to your answer in (d).

II. Two very long wires, each carrying a current $I$ out of the page, are located on the $x$-axis at $x= \pm a$.
(a) Determine the magnetic field $\vec{B}$ at points $P_{1}$ and $P_{2}$ on the $y$ axis, a distance $b$ above and below the origin.


Now consider the situation of an infinite sheet of current which lies in the $x-z$ plane, with the current flowing toward the positive $z$ direction, as shown below. The current density across the sheet is $\lambda$ Amp $/ \mathrm{m}$.
(b) Using symmetry arguments and the result from (a), determine the direction of the resulting magnetic field $\vec{B}$ both above and below the sheet of current. Indicate clearly on a sketch the direction of $\vec{B}$ both above and below the current sheet.
(c) Given the direction of the magnetic field as determined in (b), use Ampere's law to show that the magnetic field both above and below the current sheet is uniform in space. Determine the magnitude of the magnetic field in terms of the current density $\lambda$ and other constants.
(d) A particle with positive charge $+Q$ and mass $m$ enters the region below the current sheet, moving upward along the negative $y$ axis, with initial velocity in the positive $y$ direction, $\vec{v}=v_{0} \hat{j}$. The particle executes uniform circular motion in the $y-z$ plane. Determine the radius of the circle, the time required to complete one revolution (the period $T$ ), and the sense of rotation (clockwise or counterclockwise) as viewed from the positive $x$ axis.
(e) How would the radius of circular motion and the period change if the initial velocity of the particle were doubled in magnitude?
(f) Suppose instead that the particle enters the region of field at $45^{\circ}$ to the $x$-axis, $\vec{v}=\left[v_{0} \hat{i}+v_{0} \hat{j}\right] / \sqrt{2}$. Describe the subsequent motion of the particle qualitatively and determine the radius of circular motion in this case.

III. A dipole magnet creates a large region of uniform magnetic field $\vec{B}$, pointing to the right as shown below. Outside of the magnet the magnetic field is zero. A loop of wire (mass $m$, resistance $R$, width $b$ and height $a$ ) is held vertically and dropped into the region of uniform magnetic field. Express your answers in terms of $R, m, a, b$, the magnitude of the magnetic field $B$, and the acceleration of gravity $g$.
First consider the period of time during which the bottom of the loop is within the magnetic field but the top is not.
(a) At the instant when the loop is falling with speed $v_{o}$, determine the magnitude of the current induced in the loop and the direction of the current (clockwise or counter-clockwise) when viewed from right to left. (b) At the instant when the loop is falling with speed $v_{o}$, determine the magnetic force $F_{B}$ on the loop, being sure to specify the direction.
(c) Assuming that the region of magnetic field is large enough, and that the loop is large enough, the loop will reach a terminal velocity $v_{t}$. Determine $v_{t}$.

The loop continues to fall at the terminal velocity until the top enters the region of magnetic field. Consider the period of time during which the loop is completely within the magnetic field.
(d) During this period of time, determine the magnitude of the current induced in the loop and the direction of the current.
(e) Determine the acceleration of the loop during this period of time.
(f) As the loop continues to fall, the bottom will leave the region of magnetic field while the top is still in the field. Explain qualitatively what happens in this situation.


Physics 102 Spring 2006: Final Exam -Multiple-Choice Questions
For questions 1 and 2 , refer to the graph below, depicting the potential on the $x$-axis as a function of $x$


1. Which of the charge configurations depicted below would give rise to the potential shown above?

2. What is the $x$ component of the electric field for points on the $x$-axis given by the potential depicted above?



D


E
3. In each of the pictures below, the arrows represent the electric field lines and the distance between $A$ and $B$ is the same. Assuming the electric field is the same at point $A$ in all cases (1-3), list in order from least to greatest the potential difference $\Delta V_{A B} \equiv V(B)-V(A)$.


1


2


3
(A) $\operatorname{case}(1)<\operatorname{case}(2)<\operatorname{case}(3)$.
(B) case $(2)<\operatorname{case}(3)<$ case $(1)$.
(C) $\operatorname{case}(3)=\operatorname{case}(1)<\operatorname{case}(2)$.
(D) $\operatorname{case}(2)<\operatorname{case}(3)=$ case $(1)$.
(E) case $(3)<\operatorname{case}(1)<$ case $(2)$.
4. The current through an infinitely long solenoid is increased linearly as a function of time in the direction indicated in the figure below. The figure below represents a cross section of the solenoid indicating the direction of the magnetic field. The electric field inside the solenoid is
(A) in the form of counter-clockwise rotating circles centered on the axis of the solenoid.
(B) in the form of clockwise rotating circles centered on the axis of the solenoid.
(C) parallel to the axis of the solenoid.
(D) radially directed outward from the center on the axis of the cylinder.
(E) zero.

5. The switch in the circuit below has been opened a very long time. What is the current through resistor $R_{2}$ immediately after the switch is closed?
(A) $V / R_{1}$.
(B) $V\left[\frac{R_{1}+R_{2}}{R_{1} R_{2}}\right]$.
(C) $V /\left(R_{1}+R_{2}\right)$.
(D) $V / R_{2}$.
(E) 0 .

6. Consider four very long wires carrying equal steady currents going into or out of the page as indicated in the figure below. Rank the line integral of the magnetic field $\oint \vec{B} \cdot d \vec{l}$ (from greatest to least) taken in the clockwise direction.
(A) $A>C>B>D$
(B) $D>B>C>A$
(C) $A=C>B=D$
(D) $A=B>C=D$
(E) $C=D>A=B$

7. A capacitor is fully charged to $24 V$ and then connected between points A and B in the figure below, with its positive plate located at A . What is the current through the $2 \mathrm{k} \Omega$ resistor immediately after the capacitor is connected?
(A) 0 mA .
(B) 2 mA .
(C) 4 mA .
(D) 8 mA .
(E) 12 mA .


Refer to the circuit below for Questions 8-10. At time $t=0$, the switch is closed.

8. The current, $I_{0}$, through resistor $R_{1}$ at $t=0$ is
(A) $I_{0}=V /\left(R_{1}+R_{2}\right)$.
(B) $I_{0}=V / R_{1}$.
(C) $I_{0}=V / R_{2}$.
(D) 0 .
9. After the switch has been closed for a very long time, the potential difference, $V_{C}$, across the capacitor is
(A) $V_{C}=V$.
(B) $V_{C}=V\left(R_{1} / R_{2}\right)$.
(C) $V_{C}=V\left(R_{2} / R_{1}\right)$.
(D) $V_{C}=V\left(\frac{R_{1}}{R_{1}+R_{2}}\right)$.
(E) $V_{C}=V\left(\frac{R_{2}}{R_{1}+R_{2}}\right)$.
10. Which of the following graphs best represents the behavior of the charge on the capacitor as a function of time?
(A)

(B)

(C)

(D)

11. The figure below shows an open loop of wire in a magnetic field. There are no charged particles near the loop. Charge has piled up at the loop gap with the polarity indicated in the figure. What can be concluded?
(A) The magnetic field strength is static.
(B) The magnetic field strength is increasing.
(C) The magnetic field strength is decreasing.
(D) There can be no charge piled up at the loop gap since this is not a closed loop.
(E) More information is needed to answer the question.

12. A loop of wire carries a constant current $I$ in the clockwise direction when viewed from the right as shown below. A very long straight wire passes near the loop, also carrying constant current $I$ indicated in the figure. The loop will experience a torque in what direction?

(A) The loop experiences a net torque about axis A .
(B) The loop experiences a net torque about axis B.
(C) The loop experiences a net torque about axis C.
(D) The loop experiences no net torque.
13. Charges $+4 Q$ and $-Q$ are situated as shown below. The net electric field is zero nearest which point?

(A) A .
(B) B .
(C) C.
(D) D .
(E) E.
14. A neutral particle (labeled 1) in the figure below is moving with constant speed in a region of constant magnetic field (the direction is indicated in the figure). At a given instant in time, particle 1 "decays" into two different particles (labeled 2 and 3). The two particles follow paths indicated in the figure such that initially they have the same radius of curvature. Which of the following statement(s) is (are) true?

I. The magnitude of momentum of particle 2 is less than the magnitude of momentum of particle 3.
II. The magnitude of momentum of particle 2 is the same as the magnitude of momentum of particle 3.
III. The magnitude of momentum of particle 2 is greater than the magnitude of momentum of particle 3.
IV. Particle 2 is negatively charged and particle 3 is positively charged.
V. Particle 2 is positively charged and particle 3 is negatively charged.
VI. The speed of particle 2 is increasing.
VII. The speed of particle 2 is decreasing.
(A) Only I, IV, and VI are correct.
(B) Only II, IV, and VII are correct.
(C) Only II, V, VII are correct.
(D) Only I, V, and VI are correct.
(E) Only I, V, and VII are correct.
15. The figure below shows two arcs of equal radius centered about point P . Positive charge $+Q$ is uniformly distributed on a the upper arc and negative charge $-Q$ is uniformly distributed on the lower arc, as indicated in the figure below. Which arrow bests indicates the direction of the electric field at point P , the center of the arcs?

16. A solenoid has an inductance $L_{0}$. A second solenoid is identical to the first except that its geometrical dimensions have been magnified by a factor of two and the total number of loops remains the same. What is the ratio of the inductance of the magnified $\left(L_{1}\right)$ solenoid to the original solenoid (i.e., what is $L_{1} / L_{0}$ )?
(A) 4 .
(B) 2 .
(C) 1 .
(D) $1 / 2$.
(E) $1 / 4$.
17. Doubling which of the following quantities quadruples the energy stored in an ideal solenoidal inductor:
I. current.
II. length.
III. radius.
IV. loops per length.
(A) Only I and II.
(B) Only I and III.
(C) Only I, II and III.
(D) Only I, III and IV.
(E) All four.

For Questions 18-20 refer the figure below. Originally there were eight negative charges held equally around a circle of radius $R$. One of the charges is removed and the charge configuration is illustrated in the figure below.

18. What is the magnitude of the electric field at the center of the circle?
(A) 0 .
(B) $(1 / 8) k q / R^{2}$.
(C) $(7 / 8) k q / R^{2}$.
(D) $k q / R^{2}$.
(E) $7 k q / R^{2}$.
19. What is the direction of the electric field at the center of the circle?
(A)
(B) $\pi$
(C)
$\square$
$(\mathrm{D}) \longrightarrow$
(E) 0
20. What is the electric potential at the center of the circle? (Assuming the electric potential approaches zero at a infinite distance away from the center of the circle).
(A) 0 .
(B) $-(1 / 8) k q / R$.
(C) $-(7 / 8) k q / R$.
(D) $-k q / R$.
(E) $-7 k q / R$.
21. Two transformers are arranged as shown below and the power cord is "plugged" into a 120 V AC voltage source. The number of turns $N_{2}$ is twice as large as the number of turns $N_{1}$. If 1 Amp of current is measured in the intermediate wire (the wire containing $N_{2}$ turns), what is the current through the light-bulb?
(A) $1 / 4$.
(B) $1 / 2$.
(C) 1 .
(D) 2 .
(E) 4 .

22. The leads of an AC-generator with angular frequency $\omega$ and peak voltage $V_{p}$ are connected across a capacitor with capacitance $C$ and reactance $X_{C}=1 / \omega C$. What is the average power $<P>$ dissipated by the capacitor?
(A) $\langle P\rangle=0$.
(B) $<P>=V^{2} /\left(\sqrt{2} X_{C}\right)$.
(C) $\langle P\rangle=V^{2} /\left(2 X_{C}\right)$.
(D) $<P>=V^{2} /\left(X_{C}\right)$.
(E) $<P>=2 V^{2} /\left(X_{C}\right)$.
23. Two resistors and an inductor are connected across an AC voltage source as shown below. As the frequency of the AC voltage source increases, the RMS current in the resistor labeled $R_{2}$
(A) increases.
(B) remains the same.
(C) decreases.

24. Three very long wires carrying constant current of equal magnitude and directions as indicated in the figure below are held in place. Which arrow best represents the direction of the force experienced by the wire located at the point $(x=0, y=b)$ ?
(A)
(B)

(E) 0
25. Four positively charged particles with equal charge, $+Q$ are situated near a very long wire carrying a negative uniform linear charge density, $-\lambda$. A sphere of radius $R$ is centered about point $P$ indicated in the figure below. The electric flux, $\Phi_{E}=\oint \vec{E} \cdot d \vec{A}$ through the sphere is:
(A) $\Phi_{E}=0$.
(B) $\Phi_{E}=\frac{8 Q \pi R^{2}-4 \lambda \pi R^{2}}{\varepsilon_{0}}$.
(C) $\Phi_{E}=\frac{2 Q-2 \lambda R}{\varepsilon_{0}}$.
(D) $\Phi_{E}=\frac{-2 \lambda R}{\varepsilon_{0}}$.
(E) $\Phi_{E}=\frac{2 Q}{\varepsilon_{0}}$.


Physics 102 Spring 2006: Final Exam -Multiple-Choice Answers

|  | $A$ | $B$ | $C$ | $D$ | $E$ |
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