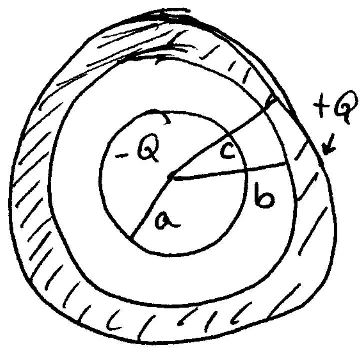
I. (a)



(a) Since we have two conductors, the electric field inside the conductors is zero. All charge resides on surface of conductors.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$r < a \quad E = 0 \qquad$ No charge enclosed

$b < r < c \quad E = 0$

$a < r < b \quad \oint \vec{E} \cdot d\vec{A} = -E \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = -\frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$

$r > c \quad E = 0$ since $Q_{enc} = 0$

$E = 0$ for all space except for $a < r < b$ where $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$

(b) $\Delta V = V_b - V_a$ Since $\vec{E}$ points from high potential to low potential $\Rightarrow V_b > V_a$ so $\boxed{\Delta V = V_b - V_a > 0 \quad \text{positive}}$

(c) $\Delta V = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \left(-\frac{Q}{4\pi\varepsilon_0 r^2}\right) dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\Big|_a^b\right) = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) = \underline{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)} \; \#$$

(d) from (c)

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) \qquad \text{Note } \Delta V > 0 .$$

$$C \equiv \frac{Q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{1}{a}-\frac{1}{b}\right)^{-1}$$

$$\frac{1}{a}-\frac{1}{b} = \frac{b-a}{ab} \Rightarrow \boxed{C = \frac{4\pi\varepsilon_0\, ab}{b-a}}$$

$$\boxed{U = \frac{1}{2}QV = \frac{1}{2}Q^2\left(\frac{b-a}{4\pi\varepsilon_0 ab}\right)} \qquad \boxed{U = \frac{Q^2(b-a)}{8\pi\varepsilon_0\, ab}}$$

\#\#

(e) $\Delta V_{\infty,r} = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} \Rightarrow \Delta V_{\infty,r} = V(r) - V(\infty)$

$$\Delta V_{\infty,r}(r>c) = -\int_{\infty}^{r} 0\, dl = 0 \Rightarrow V(r) = 0$$

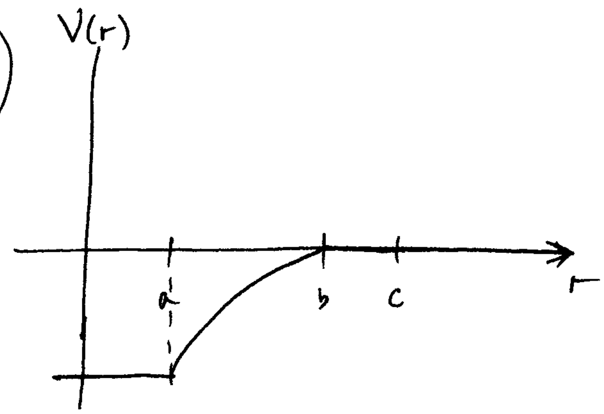
$$\Delta V_{\infty,r}(b<r<c) = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{r} 0\, dl = 0 \Rightarrow V(r) = 0$$

$$\Delta V_{\infty,r}(a<r<b) = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{b} 0\, dr + -\int_{b}^{r}\left(\frac{-Q}{4\pi\varepsilon_0 r^2}\right)dr$$

$$\Delta V_{\infty,r}(a<r<b) = +\int_{b}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$$

$$\Delta V_{\infty,r}(r<a) = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

(Note: @ $r=b$ $\Delta V = 0$)



(f) $u_E = \frac{1}{2}\varepsilon_0 |E|^2 = \frac{1}{2}\varepsilon_0 \left| \frac{-Q}{4\pi\varepsilon_0 r^2} \right|^2 \qquad a < r < b$

$$u_E = \frac{1}{2}\varepsilon_0 \frac{Q^2}{16\pi^2\varepsilon_0^2 r^4} = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$

$U = \int_a^b u_E \, dV \qquad dV \equiv$ infinitesimal volume of sphere

$$dV = 4\pi r^2 \, dr$$

$$U = \frac{Q^2}{32\pi^2\varepsilon_0} \cdot 4\pi \int_a^b \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0}\left(-\frac{1}{r}\Big|_a^b \right)$$

$$\boxed{U = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)}$$

Same as in (d).

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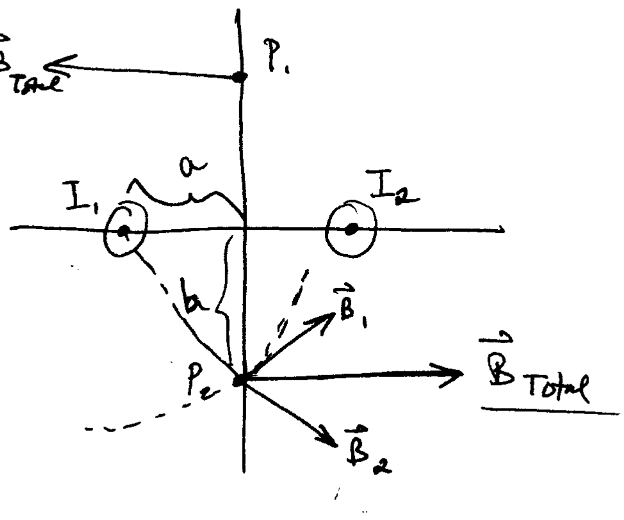
II.



Use Ampere's law for one straight wire carrying current I

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \cancel{\frac{d\oint \vec{E}\cdot d\hat{A}}{dt}}^{\,0}$$

$$\oint \vec{B}\cdot d\hat{l} = B\cdot 2\pi r = \mu_0 I_{enc} \Rightarrow B = \frac{\mu_0 I_{enc}}{2\pi r}$$

$r = \sqrt{a^2+b^2}$ # @ point P_2 below the origin.

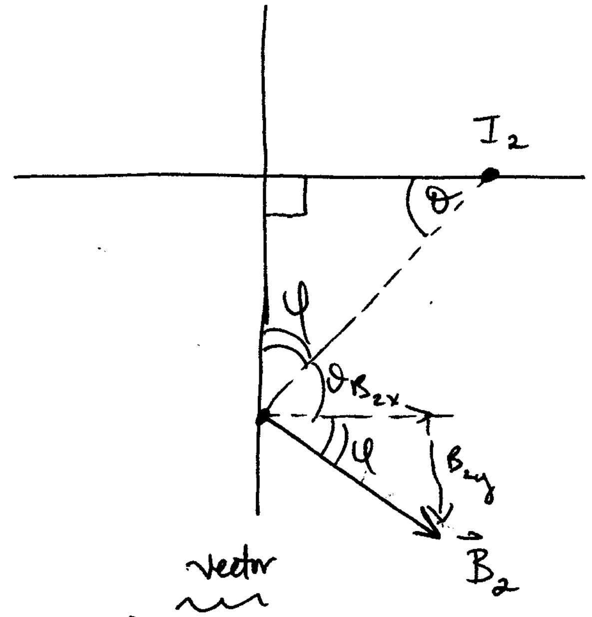
$$\Rightarrow |\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0 I}{2\pi\sqrt{a^2+b^2}}$$

If we find the horizontal component of $\vec{B}_1$ or $\vec{B}_2$, then we have our answer to (a).

(a) $\vec{B}(P_2) = 2|B_x|\,\hat{\imath}$ @ $y = -b$

and $\vec{B}(P_1) = -2|B_x|\,\hat{\imath}$ @ $y = +b$ Since $I_1 = I_2$ and $r_1 = r_2$ #

(a)

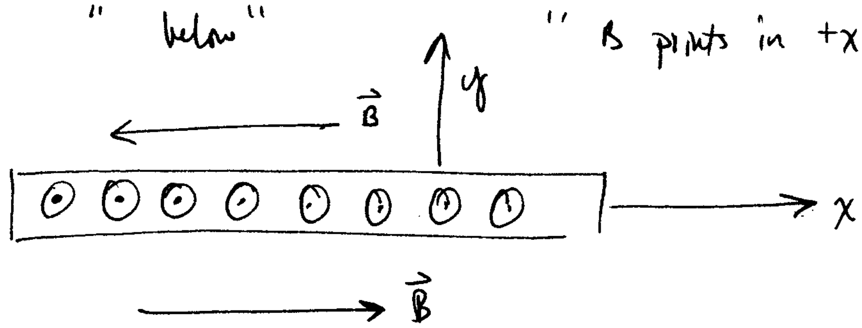


$$B_{2x} = \underbrace{|\vec{B}_2| \cos\varphi}_{\text{vector}} \qquad \text{not} \qquad \underbrace{\cos\theta}_{\text{geometrical}}$$

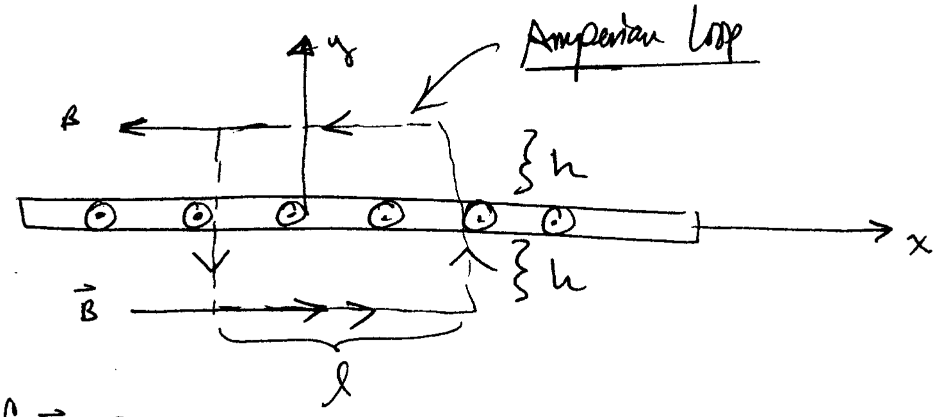
But $\cos\varphi = \dfrac{b}{r} = \dfrac{b}{\sqrt{a^2+b^2}}$

$$\boxed{\begin{aligned} \Rightarrow \quad & \vec{B}(P_1) = -\frac{\cancel{2}\mu_0 I}{\cancel{2}\pi\sqrt{a^2+b^2}}\left(\frac{b}{\sqrt{a^2+b^2}}\right)\hat{i} = \frac{-\mu_0 I b}{\pi(a^2+b^2)}\hat{i} \quad \text{for } \underline{y=-b} \\ + \quad & \vec{B}(P_2) = +\frac{\mu_0 I b}{\pi(a^2+b^2)}\hat{i} \quad \text{for } \underline{y=+b} \end{aligned}}$$

(b) from (a) we know above the current sheet B points in $-x$ dir$^{\underline{n}}$.
" " below " " B points in $+x$ dir$^{\underline{n}}$.



(c)



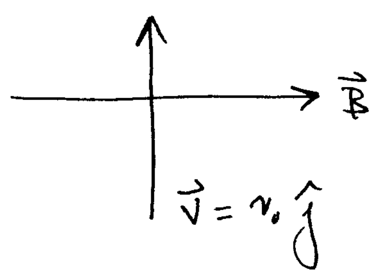
$$\oint \vec{B}\cdot d\vec{l} = Bl + 0 + Bl + 0 = \underline{2Bl = \mu_0 I_{enc}}$$

$$I_{enc} = \lambda \cdot l \Rightarrow 2Bl = \mu_0 \lambda l$$

$$\Rightarrow \boxed{B = \frac{\mu_0 \lambda}{2} \quad \text{independent of } h}$$

(d)

Below the sheet

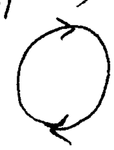


$$\vec{F} = q\vec{v}\times\vec{B} \Rightarrow F_z = -qvB = -\frac{mv_0^2}{r} \Rightarrow \boxed{r = \frac{mv_0}{qB} = \frac{2mv_0}{q\mu_0\lambda}}$$

$T = d/v_0$ No change in v_0 $\Rightarrow T = \frac{2\pi r}{v_0} = 2\pi\left(\frac{2m}{q\mu_0\lambda}\right)$

$$\boxed{T = \frac{4\pi m}{q\lambda\mu_0}}$$

When viewed from $+x$ axis



<u>clockwise rotn.</u>

(e) If initial velocity we doubled in magnitude then r would be twice as large!

& $T = T$ No change in period.

$r = \frac{m v_0}{qB}$ ← depends on v_0.

$T =$ constant for given B, q, m

(f) If $\vec{v} = \frac{v_0 \hat{i} + v_0 \hat{j}}{\sqrt{2}}$ the particle would have a component of $\vec{v} \parallel$ to $\vec{B}$ $\Rightarrow$ No force in that direction.

Particle will move in $+x$ direction with speed $v_0/\sqrt{2}$

Particle will also undergo circular motion in y-z plane as before in (d) (Helical trajectory) with a different r!

$$\vec{F} = q\vec{v} \times \vec{B} = \left[q \frac{v_0}{\sqrt{2}} \hat{i} \times |B| \hat{i}\right] + \left[q \frac{v_0}{\sqrt{2}} \hat{j} \times |\vec{B}| \hat{i}\right] = 0 - \frac{q v_0 |B|}{\sqrt{2}} \hat{k}$$

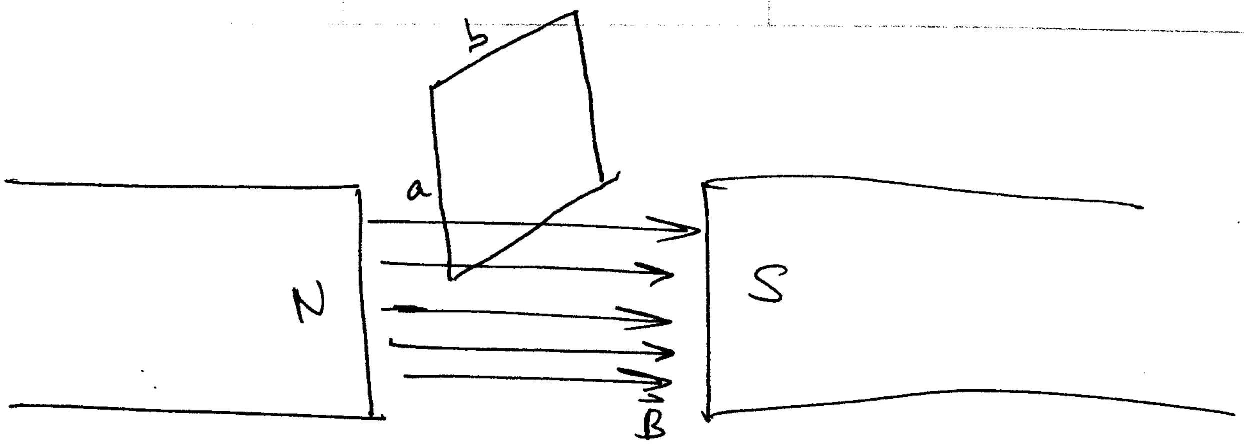
$$\Rightarrow \frac{m v_y^2}{r_1} = \frac{q v_0 B}{\sqrt{2}}$$

$$\boxed{v^2 = \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{v_0}{\sqrt{2}}\right) + \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{v_0}{\sqrt{2}}\right) = v_0^2}$$

Not

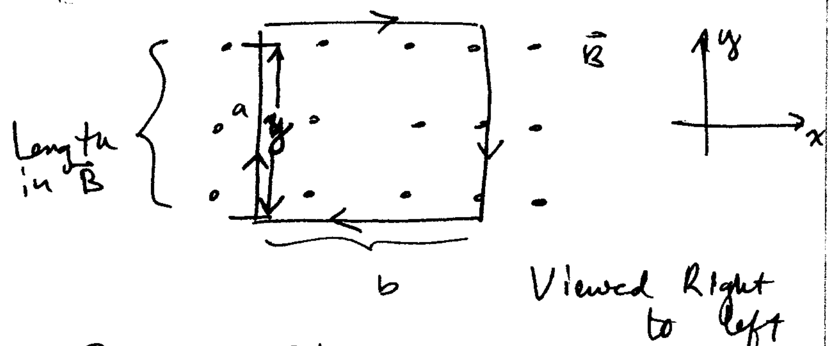
$$\Rightarrow \frac{\sqrt{2}\, m v_y^2}{q v_0 B} = r_1 = \frac{\sqrt{2}}{2} \frac{m v_0}{qB} = \frac{\sqrt{2}}{2} r_0 \;\#$$

$$\Rightarrow \boxed{r_1 = r_0/\sqrt{2}} \;\#$$



(a) bottom of loop in $\vec{B}$ but not top of loop

$$\Phi_B = B \cdot A_{\substack{loop \\ in\ \vec{B}}}$$



$$\Phi_B = B \cdot b \cdot y$$

$$\mathcal{E} = -\frac{d}{dt}\Phi_B = -Bb\frac{dy}{dt} = -Bbv_y$$

@ instant $v_y = v_0$ $\quad \boxed{\mathcal{E} = -Bbv_0}$

$$\Rightarrow \boxed{I_{induced} = \mathcal{E}/R = \frac{Bbv_0}{R}}$$ clockwise when viewed from Right to left.

(b) At the same instant $\vec{F} = q\vec{v}\times\vec{B} \Rightarrow I\vec{L}\times\vec{B}$

The two sides (labeled y) have equal but opposite forces

$$\Rightarrow \quad \vec{F}_B = I_{induced}\, b\,(-\hat{\imath}) \times B(\hat{k}) = \left(\frac{Bbv_0}{R}\right) b \cdot B\,\hat{\jmath}$$

$$\boxed{\vec{F}_B = \frac{B^2 b^2 v_0}{R}\hat{\jmath} \quad \text{(upward force)}}$$

(c) v_t is reached when $a \to 0 \Rightarrow \vec{F}_{net} = 0$.

$$\Rightarrow mg = F_B = \frac{B^2 b^2 v_t}{R}$$

$$\Rightarrow \boxed{v_t = \frac{mgR}{B^2 b^2}}$$

#

(d) Once loop is entirely inside $\vec{B}$-field $\frac{d\Phi_B}{dt} = 0$ $\left(\Phi_B \neq 0\right)$

Since $\mathcal{E} = -\frac{d\Phi_B}{dt} = 0 \Rightarrow \boxed{I_{induced} = 0}$

(e) If there is no I in loop then $\vec{F}_B = 0 \Rightarrow$ only $\vec{F}_g$

$$\boxed{a_y = -|g|\hat{j}}$$

(f) Φ_B will decrease through loop so $\frac{d\Phi_B}{dt} \neq 0 \Rightarrow I_{induced} \neq 0$.

Current induced in loop (direction opposite to (a)) c-c-w

$\vec{F}_B$ now acts up with this induced current $\rightarrow$ Object can once ~~again~~ again reach terminal velocity while top of loop is still interacting with $\vec{B}$-field.