Last Name: First Name:

Physics 102 Spring 2006: Test 2—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and 15 multiple-choice questions.
- The test is graded on a scale of 100 points; the free-response questions are worth a total of 70 points, and the multiple-choice questions account for 30 points (two points each).
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

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Show your work for the free-response problems, including neat and clearly labeled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

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I. (35 pts) The sketch below shows a simple RC circuit. The capacitor is initially uncharged, and at $t = 0$ the switch S is closed. Express your answers in terms of V_0, R_1, R_2, and C.

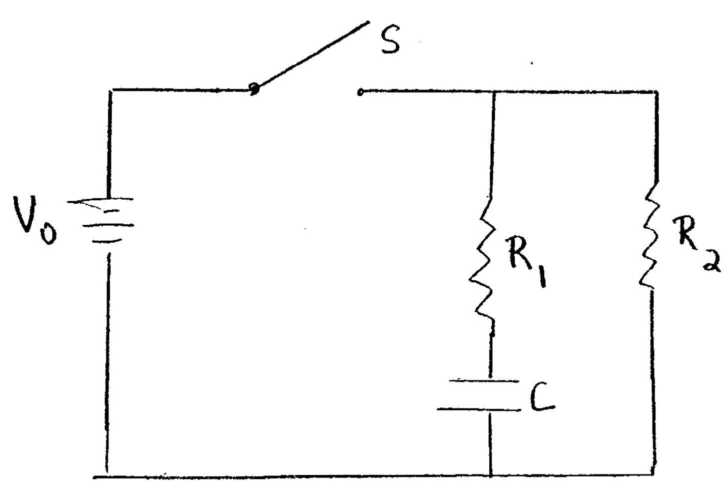
5 (a) Determine the current through each resistor immediately after the switch is closed.

10 (b) Determine the charge on the capacitor $Q(t)$ as a function of time after the switch is closed. What are the initial charge $Q(t = 0)$, the final charge $Q(t \to \infty)$, and the time constant? Sketch $Q(t)$ vs. t.

5 (c) Determine the current through the capacitor $I(t)$ as a function of time after the switch is closed. What are the initial current $I(t = 0)$, the final current $I(t \to \infty)$, and the time constant? Sketch $I(t)$ vs. t.

5 (d) Determine the current through each resistor a long time after the switch is closed.

10 (e) After the switch has been closed for a long time, it is opened again. Determine the current through the capacitor $I(t)$ as a function of time after the switch is opened. What are the initial current $I(t = 0)$, the final current $I(t \to \infty)$, and the time constant in this case? Sketch $I(t)$ vs. t.



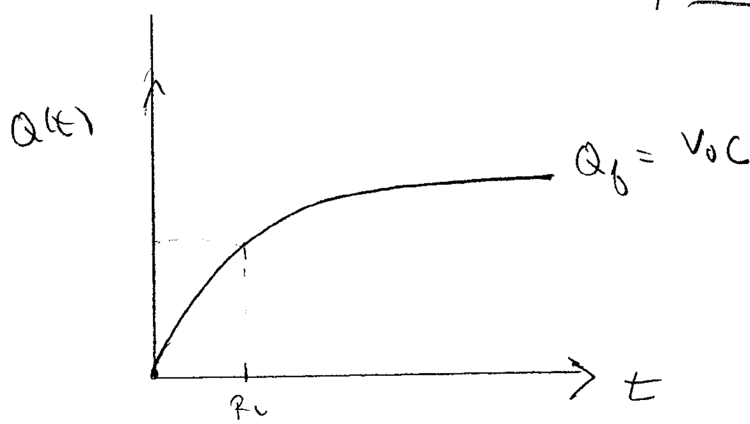
$$\ln(Q - V_0 C) = -t/R_1 C + A$$

$$Q(t) = V_0 C + B e^{-t/R_1 C}$$

$$Q(t=0) = 0 \Rightarrow B = -1$$

$$\boxed{Q(t) = V_0 C\left(1 - e^{-t/R_1 C}\right)}$$

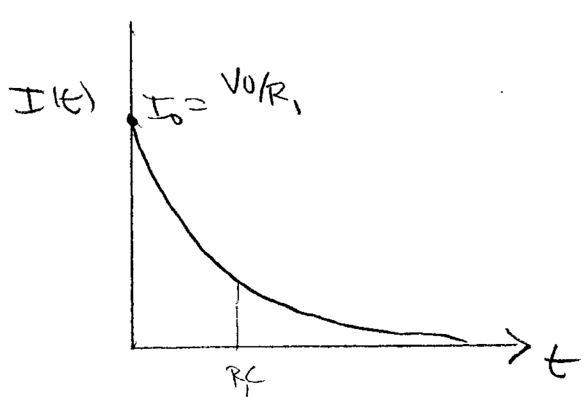
$$\boxed{\begin{array}{l} Q(t=0) = 0 \\ Q(t\to\infty) = V_0 C \end{array}}$$



(C) $$I = \frac{dQ}{dt} = -V_0 C\left(\frac{-1}{R_1 C}\right) e^{-t/R_1 C}$$

$$\boxed{I(t) = \frac{V_0}{R_1} e^{-t/R_1 C}}$$

$$\boxed{\begin{array}{l} I(t=0) = \dfrac{V_0}{R_1} \\ I(t\to\infty) = 0 \end{array}}$$

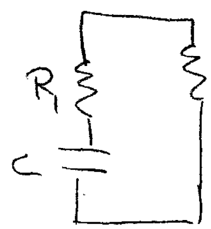


(d) A long time after the switch is closed, C acts like an open circuit:

$$\boxed{I_1(t \to \infty) = 0} \qquad \boxed{I_2(t \to \infty) = \frac{V_0}{R_2}}$$

The current in R_2 is unchanged.

(e) When S is opened again, the capacitor discharges through both R_1 & R_2 in series



$R_{eff} = R_1 + R_2 \qquad \tau = (R_1 + R_2) C$

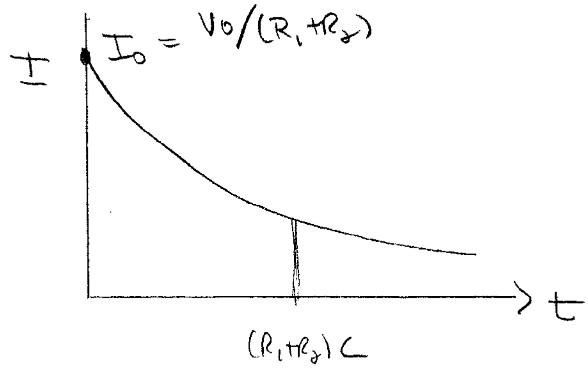
$$I(t) = I_0 \, e^{-t/(R_1+R_2)C}$$

Before S is opened, C is charged to V_0.

$$\boxed{I(0) = \frac{V_0}{R_1 + R_2}}$$ is the initial current

$$\boxed{I(t) = \frac{V_0}{R_1 + R_2} \, e^{-t/(R_1+R_2)C}}$$

$$\boxed{I(t \to \infty) = 0}$$



The time constant is longer for discharging.

35 II. (35 pts) Three very long parallel are arranged as shown below. Two wires, which carry current I out of the page, pass through the points $y = 0$, $x = \pm a$. The third wires carries a current I into the page and is located at the point $x = 2a$, $y = -a$.

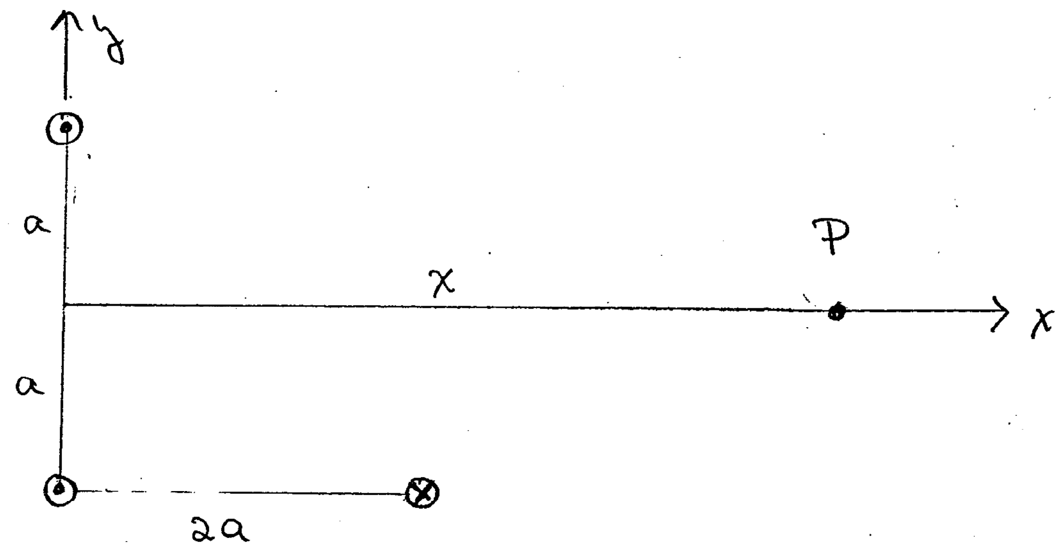
8 (a) Determine the magnetic field at the location of the top left wire ($x = 0, y = a$) due to the other two wires.

5 (b) Determine the force per unit length on the top left wire.

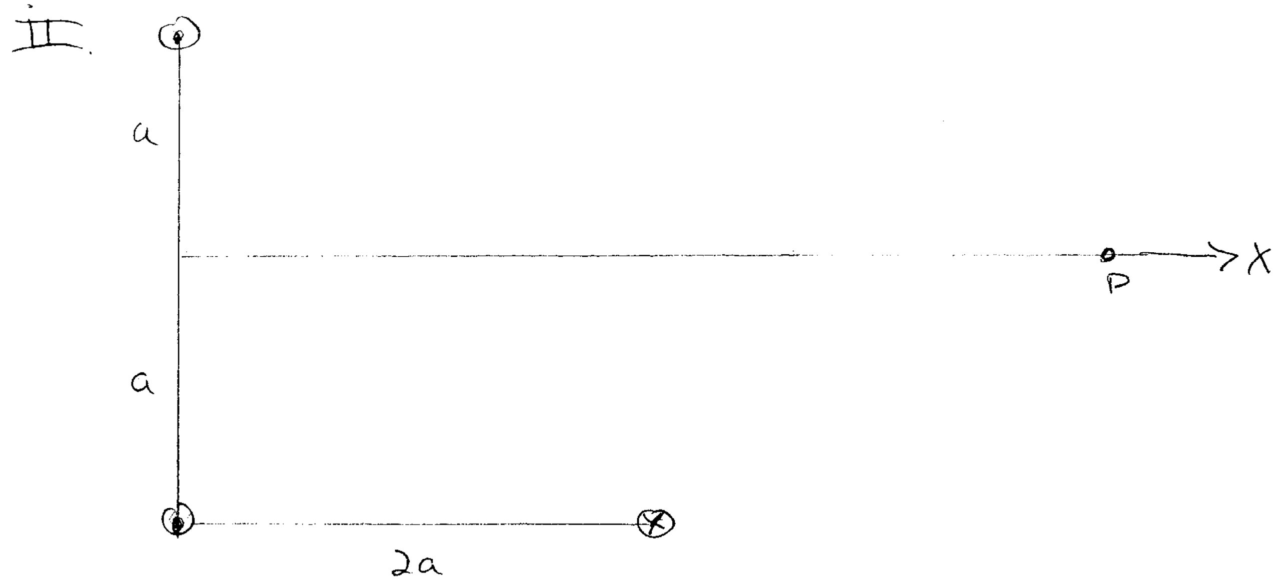
9 (c) Determine the magnetic field $\vec{B}_{12}(x)$ due to the two wires located at $x = 0$, $y = \pm a$ at an arbitrary point P on the x-axis, a distance x from the origin.

10 (d) Determine the magnetic field $\vec{B}_3(x)$ at the point P due to the single wire at $x = 2a, y = -a$.

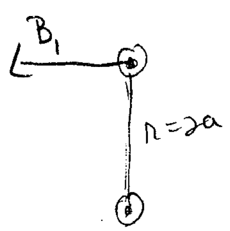
3 (e) From (c) and (d), determine the total magnetic field at the point P.



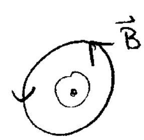
II.



(a) $\vec{B}$ at top left due to the other two wires



From Ampere's law we know that the field due to a long wire forms concentric loops

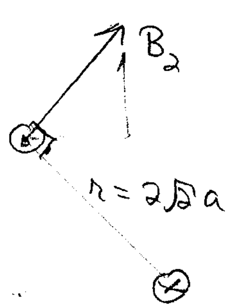


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

This gives $\vec{B}_1$ (due to lower left wire) $= \frac{\mu_0 I}{4\pi a}(-\hat{\imath})$

$\vec{B}_2$ due to the lower right wire will be at 45°:



$$\vec{B}_2 = \frac{\mu_0 I}{4\pi(\sqrt{2}a)}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)$$

$$\boxed{\vec{B}_{TOT} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi a}\left(-\frac{1}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)}$$

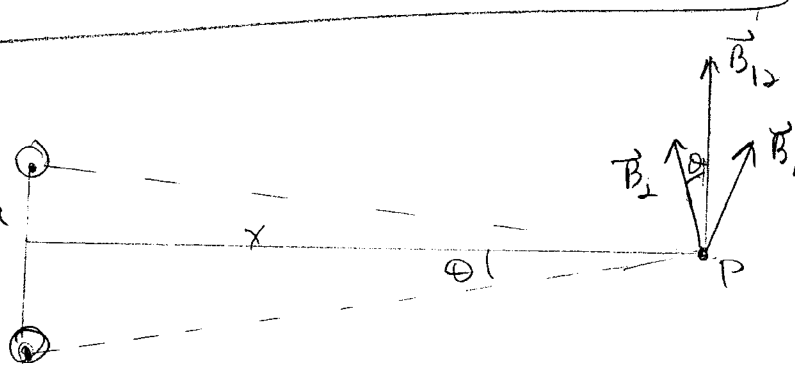
(b) F/L on top left wire

$\vec{F} = I\vec{L}\times\vec{B}$ $\quad\quad \vec{L} = L\hat{k}$ current in $+z$ direction

$$\vec{F} = IL\left(\frac{\mu_0 I}{4\pi a}\right)\Big(\underbrace{-\tfrac{1}{2}\hat{k}\times\hat{i}}_{-\frac{1}{2}\hat{j}} \; \underbrace{+\tfrac{1}{2}\hat{k}\times\hat{j}}_{-\frac{1}{2}\hat{i}}\Big)$$

$$\boxed{\frac{\vec{F}}{L} = \frac{\mu_0 I^2}{4\pi a}\left(-\tfrac{1}{2}\hat{i} - \tfrac{1}{2}\hat{j}\right)}$$

(c)



The horizontal components cancel - vertical components add

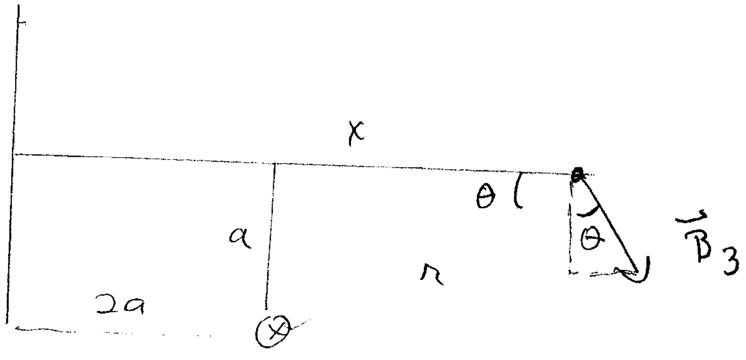
$B_{2y} = B_{1y} = B_1\cos\theta$ $\quad\quad \cos\theta = \frac{x}{\sqrt{x^2+a^2}}$

$$|B_1| = \frac{\mu_0 I}{2\pi(x^2+a^2)^{1/2}}$$

$$\boxed{\vec{B}_{12}(x) = \frac{2\mu_0 I x}{2\pi(x^2+a^2)}\hat{j} = \frac{\mu_0 I x}{\pi(x^2+a^2)}\hat{j}}$$

factor of 2 comes from adding the two contributions

(d)



$$|B_3| = \frac{\mu_0 I}{2\pi r} \qquad \text{where } r = \left[a^2 + (x-2a)^2\right]^{1/2}$$

$\vec{B}_3$ has both x & y components

$$B_{3x} = B_3 \sin\theta \qquad B_{3y} = -B_3 \cos\theta$$

$$\sin\theta = \frac{a}{\left[(x-2a)^2 + a^2\right]^{1/2}}$$

$$\cos\theta = \frac{x-2a}{\left[(x-2a)^2 + a^2\right]^{1/2}}$$

$$\vec{B}_3 = \frac{\mu_0 I}{2\pi\left[(x-2a)^2 + a^2\right]^{1/2}} \left[\frac{a}{\left[(x-2a)^2+a^2\right]^{1/2}}\hat{\imath} + \frac{x-2a}{\left[(x-2a)^2+a^2\right]^{1/2}}(-\hat{\jmath})\right]$$

$$\boxed{\vec{B}_3 = \frac{\mu_0 I}{2\pi\left[(x-2a)^2 + a^2\right]} \left[a\,\hat{\imath} - (x-2a)\,\hat{\jmath}\right]}$$

Note that the y-component changes sign for $x < 2a$, our expression works for both cases.

(e) Superposition tells us we just add the contributions

$$\boxed{\vec{B}_{TOT} = \frac{\mu_0 I}{2\pi}\left[\frac{a}{(x-2a)^2+a^2}\,\hat{\imath} + \left(\frac{x}{x^2+a^2} - \frac{x-2a}{(x-2a)^2+a^2}\right)\hat{\jmath}\right]}$$