Last Name: ______________________ First Name: ______________________

Physics 102 Spring 2005: Test 2—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and 15 multiple-choice questions.
- The test is graded on a scale of 100 points; the free-response questions are worth a total of 70 points, and the multiple-choice questions account for 30 points.
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

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Show your work for the free-response problems, including neat and clearly labeled figures, in your blue book.

Answers without explanation (even correct answers) will not be given credit.

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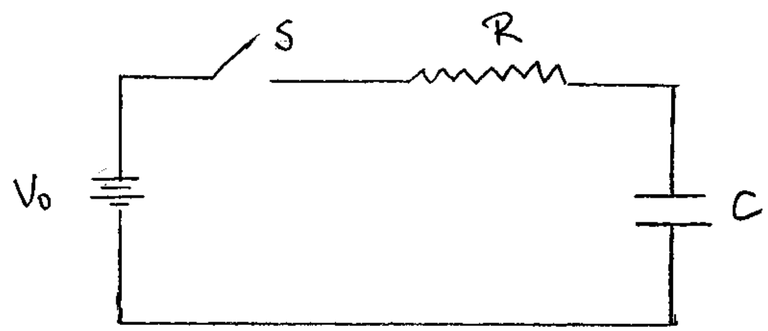
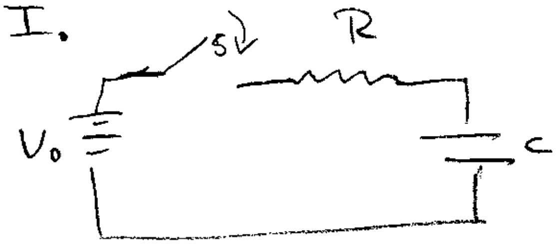
I. (35 pts) The sketch below shows a simple RC circuit. A battery V_0 is connected in series to a resistor R and capacitor C through a switch S. The capacitor is initially uncharged, and at $t = 0$ the switch S is closed. Express your answers in terms of V_0, R, and C.

5 (a) Determine $I(t)$, the current through R as a function of time. Sketch $I(t)$ vs t.
5 (b) Determine $Q(t)$, the charge on C as a function of time. Sketch $Q(t)$ vs. t.
(c) Determine $P_V(t)$, the power delivered by the battery as a function of time. Integrate $P_V(t)$ from $t = 0$ to $t = \infty$ to determine the total energy delivered by the battery as the capacitor charges.
(d) Determine $P_R(t)$, the power dissipated in the resistor as a function of time. Integrate $P_R(t)$ from $t = 0$ to $t = \infty$ to determine the total energy dissipated in the resistor as the capacitor charges. If the value of R is doubled, how does the total energy dissipated in the resistor change?
4 (e) Determine the total energy stored in the capacitor at $t = \infty$.
5 (f) Compare your answers for energies in (c), (d), and (e). Are the results as expected?



Phys 102 - Exam 2

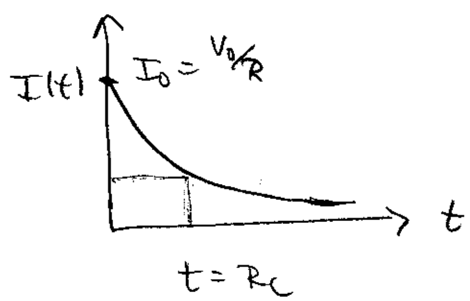
I.



(a) $I_0 = \frac{V_0}{R}$ $\Rightarrow$ C acts like short at $t=0$.

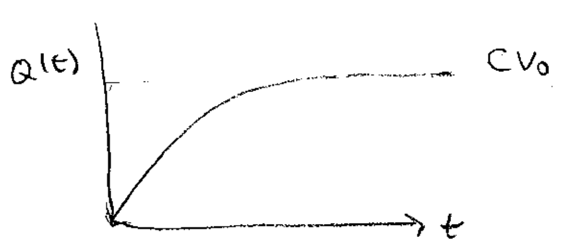
Current decays exponentially w/ $\tau = RC$

$$\boxed{I(t) = \frac{V_0}{R} e^{-t/RC}}$$



(b) Q(t) also behaves exponentially, but $Q_f = CV_0$, that is as $t \to \infty$ the potential across the capacitor $= V_0$.

$$\boxed{Q(t) = CV_0\left(1 - e^{-t/RC}\right)}$$



We can also find $Q(t)$ by integrating $I(t)$

$$Q(t) = \int_0^t I(t')\,dt' = \frac{V_0}{R}\int_0^t e^{-t'/RC}\,dt'$$

$$= \frac{V_0}{\cancel{R}}(-\cancel{R}C)\, e^{-t'/RC}\Big|_0^t$$

$$Q(t) = -\frac{V_0}{R}\left(e^{-t/RC} - 1\right)$$

$$Q(t) = \frac{V_0}{R}\left(1 - e^{-t/RC}\right) \quad \text{as above!}$$

(c) $P_V(t) = V_0\, I(t)$ (V_0 constant)

$$\boxed{P_V(t) = \frac{V_0^2}{R}\, e^{-t/RC}}$$

$$\int_0^\infty P_V(t)\,dt = \frac{V_0^2}{R}\int_0^\infty e^{-t/RC}\,dt = \text{total energy delivered by battery}$$

$$U_V = \frac{V_0^2}{R}(-RC)\, e^{-t/RC}\Big|_0^\infty$$

$$U_V = -V_0^2 C\,(0-1)$$

$$\boxed{U_V = V_0^2 C}$$

(d) $P_R(t) = I^2(t) R = \frac{V_0^2}{R^2}\cdot R\, e^{-2t/RC}$

$$\boxed{P_R(t) = \frac{V_0^2}{R}\, e^{-2t/RC}}$$

$$U_R = \int_0^\infty P_R(t)\,dt = \frac{V_0^2}{R}\left(-\frac{RC}{2}\right) \underbrace{e^{-2t/RC}\Big|_0^\infty}_{-1}$$

$$\boxed{U_R = \frac{1}{2}V_0^2 C} = \text{total energy dissipated in } R$$

This result is independent of R!
If $R \to 2R$ the total energy dissipated in R is unchanged. It would take longer to fully charge the capacitor, though.

(e) $\boxed{U_C = \frac{1}{2} C V_0^2}$ at $t \to \infty$ since $V_C \to V_0$

(f) $U_V = U_R + U_C$

Energy delivered by the battery = energy dissipated in R + energy stored in C

So energy conservation works!

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Grading Criteria

I. 35 pts total

(a) 5 pts

3 - expression for current
2 - sketch

(b) 5 pts

3 - expression for $Q(t)$
2 - sketch

(c) 8 pts

3 - expression for $P_V(t)$
5 - integration to get energy

(d) 8 pts

2 - expression for $P_R(t)$
5 - integration to get energy
1 - no R dependence.

(e) 4 pts

No need to derive expression for U_C

(f) 5 pts

Give full credit if they understand that energy should be conserved, even if their expressions for U_V, U_R & U_C don't add up.

II.(35 pts) The figure below shows the cross section of a long conductor of a type called a coaxial cable. The radius of the inner solid cylinder is a, and the outer cylindrical shell has inner radius b and outer radius c, as shown in the figure below. The conductors carry equal but opposite currents I, with the current in the inner conductor flowing out of the page. The currents are uniformly distributed over the cross-sectional area in each case. The coordinate r measures the distance from the axis of the cylinders. Express your answers in terms of I, a, b, c and possibly other constants.

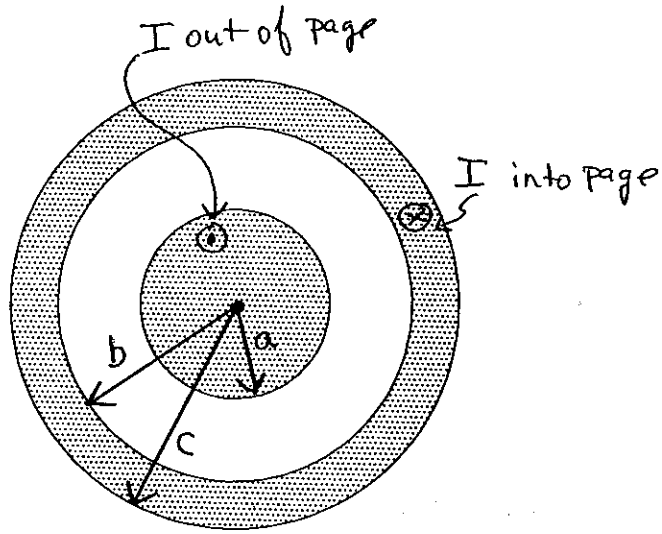
7 (a) Determine the magnitude of the current density $|\vec{J}_{inner}|$ in the region $r < a$. Determine the magnitude of the current density $|\vec{J}_{outer}|$ in the region $b < r < c$.

8 (b) Determine the magnetic field $\vec{B}(r)$ in the range $r < a$, being sure to indicate the direction of $\vec{B}$.

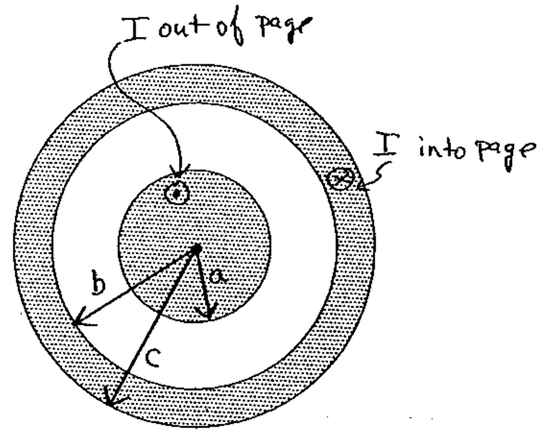
7 (c) Determine $\vec{B}(r)$ in the region $a < r < b$.

8 (d) Determine $\vec{B}(r)$ in the region $b < r < c$.

5 (e) Determine $\vec{B}(r)$ in the region $r > c$.



II.



(a) I uniformly distributed over area

$$\boxed{|\vec{J}_{inner}| = \frac{I}{\pi a^2}}$$

$$\boxed{|\vec{J}_{outer}| = \frac{I}{\pi(c^2-b^2)}}$$

since area of outer cylindrical shell $= \pi(c^2-b^2)$

(b) Because of the symmetry of an infinite wire, we can use Ampere's Law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl.}$$

$\vec{B}$ forms concentric loops, ccw in this case.

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = 2\pi r B = \mu_0 \int_0^r J_{inner}\, \underbrace{dA}_{2\pi r\, dr}$$

$(\vec{B} \parallel d\vec{l})$

$$2\pi r|\vec{B}| = \frac{\mu_0 I}{\pi a^2}\int_0^r 2\pi r' dr'$$

Only the current enclosed in $0 \to r$ contributes.

$$2\pi r|\vec{B}| = \frac{\mu_0 I}{a^2}\cdot\frac{\cancel{2} r^2}{\cancel{2}}$$

$$\boxed{|\vec{B}| = \frac{\mu_0 I r}{2\pi a^2}}$$

Direction is counter-clockwise loops.

for $\underline{r < a}$

(c) $\underline{a < r < b}$

In the region between the conductors, $\underline{\underline{\text{all}}}$ of the inner current is enclosed & $\underline{\text{none}}$ of the outer current is enclosed.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r}}$$

direction is $\underline{\text{ccw loops}}$

(d) $\underline{b < r < c}$

In the region inside the outer conductor, $\underline{\text{all}}$ of the inner current is enclosed & $\underline{\text{part}}$ of the outer current is enclosed. The outer current will $\underline{\text{subtract}}$ from the inner current since they are in opposite directions

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I - \mu_0 \int_b^r J_{outer} \overbrace{dA}^{2\pi r' dr'}$$

$$2\pi r B = \mu_0 I - \mu_0 \left(\frac{I}{\pi (c^2 - b^2)} \right) \int_b^r 2\pi r' dr'$$

$$2\pi r B = \mu_0 I - \frac{\mu_0 I}{\pi (c^2 - b^2)} \left(\frac{\cancel{2}\pi r^2}{\cancel{2}} - \frac{\cancel{2}\pi b^2}{\cancel{2}} \right)$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi (c^2 - b^2)} \left[r - \frac{b^2}{r} \right]$$

$$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r}\left[1 - \frac{r^2-b^2}{c^2-b^2}\right]}$$

Direction is ccw loops ↺

$b<r<c$

$$\left(\text{Also} = \frac{\mu_0 I}{2\pi r}\left[\frac{c^2-r^2}{c^2-b^2}\right]\right)$$

(e) For $r>c$, $I_{encl}=0$ since the total currents are equal and opposite. Then

$$\boxed{\vec{B} = 0 \text{ for } r>c}$$

Phys 102 - Exam 2
Grading Criteria

II. 35 pts

(a) 7 pts

3 - J_{inner}
4 - J_{outer}

(b) 8 pts

3 - Ampere's Law
3 - Correct integral & correct $|\vec{B}|$
2 - Correct direction of $\vec{B}$

(c) 7 pts

2 $I_{enclosed} = I_{inner}$
2 Ampere's Law
2 Correct integral & $|\vec{B}|$
1 Correct direction of $\vec{B}$

(d) 8 pts

2 $I_{enclosed} = I_{inner}$ — part of I_{outer}
1 Amperes Law
4 Correct integral & correct $|\vec{B}|$
1 Correct direction

(e) 5 pts

$I_{encl} = 0$ They either get it or they don't.