Physics 102 Spring 2002: Final Exam, May 6, 2002 Free Response and Instructions

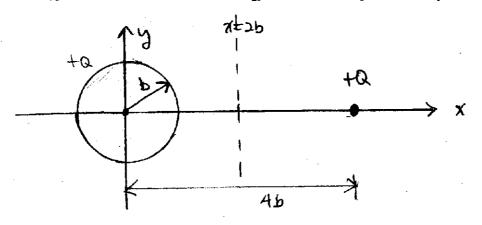
- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 3 HOURS
- The test consists of four free-response questions and 20 multiple-choice questions.
- The test is graded on a scale of 200 points; the free-response questions account for 120 points, and the multiple-choice questions account for 80 points.
- Answer the four free-response questions in your blue book. Answer the multiple-choice questions by
 marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

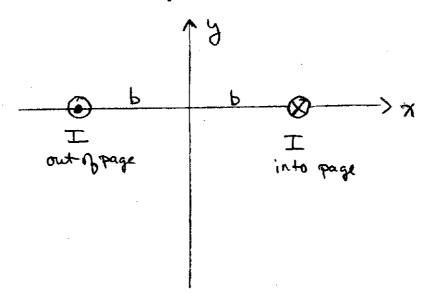
- I. (40 pts) A thin, insulating spherical shell of radius b is centered at the origin. It carries a total charge +Q, uniformly distributed over its surface.
- (a) Using Gauss' law and symmetry arguments, derive, giving complete details of the derivation, the electric field everywhere in space. Sketch the magnitude of the field, E(r), for all r.
- (b) Taking the zero of the electrostatic potential to be at infinity, determine the electrostatic potential V(r) for all r. Sketch V(r) for all r.

Now a second charge, a point particle of charge +Q is placed on the x-axis at x=4b, as shown below. The presence of this charge does not affect the distribution of charge on the spherical shell, which is still located at the origin.

- (c) Determine the total electric field $\vec{E}(y)$ as a function of y on the vertical line x=2b. Don't forget to include both positive and negative values for y.
- (d) Determine the total electric field $\vec{E}(x)$ on the z-axis for x > b. Are there any points other than $+\infty$ at which the electric field is 0?
- (e) Taking the zero of the potential to be at infinity, determine the total electrostatic potential V(x) for all points on the x-axis for x > b.
- (f) Holding the spherical shell and point charge of +Q fixed in place, a point charge +q is released from rest at the point x=2b, y=2b. Determine its kinetic energy after it has escaped to infinity.



- II. (25 pts) Two very long current-carrying wires, are situated as shown below. One wire is located at x = -b, and carries current I out of the page. The other is located at x = +b and carries current I into the page.
- (a) Determine the force per unit length that the wire at x = +b exerts on the wire at x = -b.
- (b) Determine the magnetic field $\vec{B(y)}$ produced by the wires on the vertical line x = 0, that is the line midway between them. Don't forget to consider both positive and negative values for y.
- (c) A charged particle of mass m and negative charge -q is released from the point x = 0, y = b with velocity $\vec{v} = v_0 \hat{i}$. Determine its acceleration immediately after it is released.

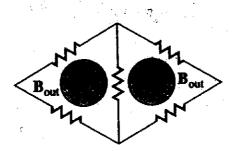


III. (25 pts) Five wires of equal length are connected to form two equilateral triangles, as shown below. Each of the five resistors has value 200. Two solenoids, each 10cm in diameter, extend a long way both into and out of the page. The magnetic fields of both solenoids point out of the page. The field strength in the left-hand solenoid is increasing at 60mT/sec, and that in the right-hand solenoid is decreasing at 25mT/sec.

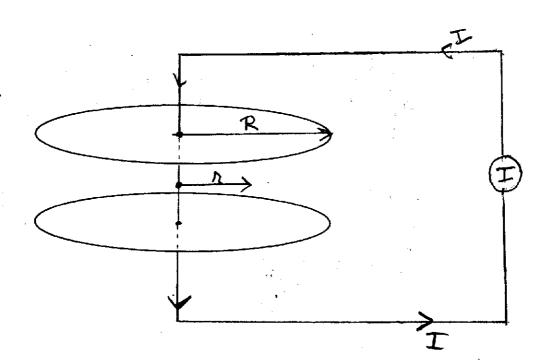
(a) Determine the emf $\mathcal{E}_{\mathcal{L}}$ induced in the left hand triangular loop.

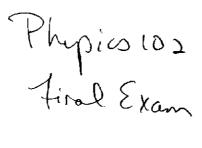
(b) Determine the emf $\mathcal{E}_{\mathcal{R}}$ induced in the right hand triangular loop.

(c) Determine the current in the central wire, shared by the two loops. Which way does the current flow?

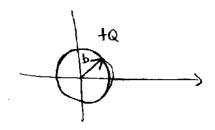


- IV. (30 pts) A very large cylindrical capacitor of radius R and plate separation d is being charged slowly with constant current I. As the capacitor charges, the electric field between the plates increases with time. Take r to be the radial distance from the axis of the capacitor, as shown in the figure below.
- (a) At a particular point in time, the surface charge density on the plates is σ . What is the electric field \vec{E} between the plates at that time?
- (b) Determine the time rate of change of the electric field between the plates, $\frac{dE}{dt}$, in terms of I, R, and other constants.
- (c) Near the center of the plates, the electric field is constant in space. Determine the magnetic field B(r) between the plates for r < R in terms of I, R and other constants. Be sure to indicate both direction and magnitude of the field.
- (d) Neglecting fringe effects around the edge of the capacitor plates, determine the magnetic field B(r) for r = R and for r > R.
- (e) Sketch B(r) for all r.





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(a) Gouss' Law gives

Take a splerical Gaussian surface with 1>6.

Because of the spherical symmetry, we know that IEI must be the same everywhere on the Gaussian surface. We also berow, from Coulomb's Low and from the spherical symmetry that E must be radially outward (the only unique direction defined by the spherical symmetry is radially outward). Hen E II d'and E. D. = Edd

SE.dA = SE,dA = E,SdA

We can take E out of the integral because it is constant over the surface. Her Gauss' Law becomes

for 12 > b, Qand = + Q and we have

$$E_{\lambda} = \frac{Q}{4\pi \epsilon_{0} \Lambda^{2}} \qquad \sigma \qquad E = \frac{Q}{4\pi \epsilon_{0} \Lambda^{2}} \qquad \Lambda > 6$$

for NCB, Quard = 0 and = 0 N>6

(b) from the definition of U,

Calculate DV from Do > 1, E. Li = E. Li = - End (1>6)

$$\Delta V = \int_{-\infty}^{\infty} E_{\Lambda} d\Lambda = \frac{Q}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{d\Lambda}{\Lambda^2} = \frac{Q}{4\pi\epsilon_0} \left(-\frac{1}{\Lambda}\right)_{\Lambda} = \frac{Q}{4\pi\epsilon_0} \Lambda$$

$$\frac{Q}{\Lambda \cos \Pi P} = (\Lambda) V$$

At N=b, En drops to Ø, so by from b >0 = 0.

Vio constant over OLALB & equal to the value at N=b

VINCB) = 4775 b

VINCB) = 4775 b

X-components cancel

y-components add

IEI = 2

HTEND Mach change

400. E(y) = 20 GOD 1 COD = 4 d= 542+465

 $E(y) = \frac{20 \text{ y}}{47180 (y^2 + 46^2)^3 \text{ }} \hat{y} > 0$

for y 20, the magnitude is the some, but direction is -y

y co Eig) = -20/4/ 4 TES (42+462) 35 }

ž.

First consider
$$x > 4b$$
. $E = E_x$ only
$$x > 4b = \frac{Q}{4\pi\epsilon_0 x^2} + \frac{Q}{4\pi\epsilon_0 (x-4b)^2} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{x^2} + \frac{1}{(x-4b)^2} \right) \int_{a}^{a}$$

$$\vec{E}(x) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{x^2} + \frac{1}{(40+x)^2} \right) \hat{A} = (4) \vec{3}$$

For bexe4b, E from the change at x=4b changes direction

$$E(x) = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{x^2} - \frac{1}{(4b-x)^2} \right] \stackrel{\wedge}{\longrightarrow} b < x < 4b$$

X = (4P-X)=

x = 46-x

€ =0 at x=26, midway between the changes.

(12) The total potential V(x) is the scalar sum of the two contributions

$$b(x(4b)) = \frac{Q}{4\pi\epsilon_0 x} + \frac{Q}{4\pi\epsilon_0 (4b-x)}$$

These can be combined:

$$V(x) = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{|x|} + \frac{1}{|4b-x|} \right] \qquad x > 0$$

First we need to determine the potential at x=2b, y=2b V(26,26) = 20 with d= (26)2+(26)2 = 256

The change of will then have potential energy

When the charge is released, all PE will be converted to KE when greates so

T T

(9) The B field due to the wire at x=+b is given by Amperés Lew. B forms concentric loops about the wire in the ccw sense.

1B1 = hot , where is the distance from Hervis.

At x=-b, the location of the other wire, \overline{B} is vertically upward and $|B| = \frac{hoT}{2T(3b)}$.

The force on the wire at n=-bis (length L)

type of or mitself

$$\frac{E}{L} = \frac{-\lambda_0 I^2}{4\pi b} \int$$

force is repulsion.

By By By

The two contributions are

Bi &Bs as aboun.

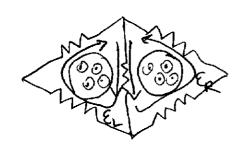
X - Components cancel

Y - components add

Direction is the same for ±y, printive y-direction.

And wo of with d = 162+ 42 $\overline{B}_{tot} = \overline{B}_1 + \overline{B}_2 =$ and $coo = \frac{1}{4}$ this expression is valid Tr (b2+43) for both ±y. Forg = -gvxB Direction will be unto the page. 1 H= q No B= q No (MoIb) = ma a = gromoth (-h)

世.



(a)
$$\varepsilon_L = -\frac{d\phi_{BL}}{dt}$$

PBL = TTR BL with 1=5cm

$$\frac{dPBL}{dt} = TTR^2 dBL with dBL = +60mT/s$$

Since tBlis increasing in the left-hand Solaroid, the direction of En will be to produce a downward field=>

En is clockwise as stown

$$E_{L} = \pi (.05 \, \text{m})^{2} (60 \, \text{xi} \, 0.47 \, \text{Ni} \, 0.47 \, \text$$

Er will therefore be in a direction to produce an upward field, counterclockwiss.

(C) The current in the central wire can be determined from Kiachoff's laws: (R=>052)

EL NI EVIZA ER

1,=1,+13

EL= i, R +i3R+i, R

EL = 21, R+13R

€ En = - 21, R + 313 R

Ee = i3R - 2i2R

=13 R-21,R+213R

En= 31312-21, R

EL+ER = 4/372

i3 = EL + ER = (4.71 x0 V) + (1.96 x10 V) = .083 x10

13 = 8.3 MA

downward in this picture

II. I constant

(a)
$$|E| = \frac{\pi}{\epsilon_0}$$
, direction is vertically downward.

(b)
$$E = \frac{I}{6} = \frac{Q}{A6} = \frac{Q}{\Pi R^3 E}$$

$$\frac{dE}{dt} = \frac{dQ/dt}{A6} = \frac{I}{\Pi R^3 E_0}$$

The displacement current is (extension of Ampereislan)

SB. LE = 168 de

Direction us Cw as riched from the top (+) plate.

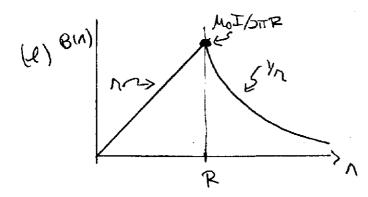
da/d

131 = MoIr direction cw longs
as viewed from above

(E ex tends only to R)

de = me de

|B| = MOI) Direction again Cwas viewed from the tops



Physics 102 Spring 2002: Final—Multiple-Choice Questions

- 1. A hollow metal sphere is electrically neutral (no excess charge). A small amount of negative charge is suddenly placed at one point P on this metal sphere. If we check on this excess charge a few seconds later we will find one of the following possibilities.
 - (a) All of the excess charge remains right around P.
 - (b) The excess charge has distributed itself evenly over the outside surface of the sphere.
 - (c) The excess charge is evenly distributed over the inside and outside surface.
 - (d) Most of the charge is still at point P, but some will have spread over the sphere.
 - (e) There will be no charge left.
- 2. A rectangular loop of wire lies in the same page as a wire carrying a constant current. The rectangular loop has a length (parallel to wire) ℓ and width w; the wire carries a current I. The mutual inductance of the system will be doubled in which of the following cases?

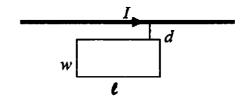
I. I is doubled

II. w is doubled

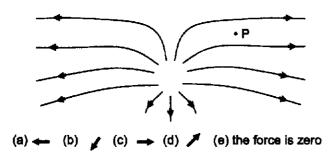
III. I is doubled

IV. d is halved

- (a) I and III
- (b) II and III
- (c) II, III and IV
- (d) I, II, III and IV
- (e) None of the options above.

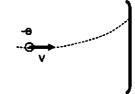


3. Consider the diagram below which depicts the electric field lines in a region of space. What is the direction of the electric force on a negative charge at point P in the diagram above.



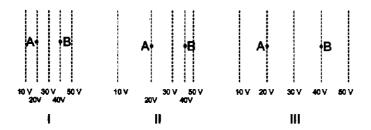
For next two questions a positive charge is placed at rest at the center of a region of space in which there is a electric field of uniform strength and direction.

- 4. When the positive charge is released from rest in the uniform electric field, what will its subsequent motion be?
 - (a) It will move at a constant speed.
 - (b) It will move at a constant velocity.
 - (c) It will move at a constant acceleration.
 - (d) It will move with a linearly changing acceleration.
 - (e) It will remain at rest in its initial position.
- 5. Which of the following statements correctly describe the electric potential energy of the positive charge after the charge is released from rest in the uniform electric field?
 - (a) It will remain constant because the electric field is uniform.
 - (b) It will remain constant because the charge remains at rest.
 - (c) It will increase because the charge will move in the direction of the electric field.
 - (d) It will decrease because the charge will move in the opposite direction of the field.
 - (e) It will decrease because the charge will move in the direction of the electric field.
- 6. An electron starts out moving horizontally toward a screen. The electron enters a region of magnetic field and follows the path depicted below. In what direction does that magnetic field point?
 - (a) toward the top of the page
 - (b) toward the bottom of the page
 - (c) into the page
 - (d) out of the page
 - (e) the magnetic field is in the direction of the path

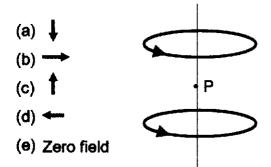


- 7. A resistor, inductor and parallel plate capacitor are connected in series across an AC emf source. The frequency of the AC source is tuned so maximum power is dissipated in the resistor. Then a dielectric slab ($\kappa > 1$) is inserted between the plates of the capacitor. Which of the following statements is (are) true?
 - I. The frequency of the AC emf source must be decreased to increase the power dissipated in the resistor.
 - II. If the frequency of the AC emf source is not changed, the current in the inductor leads the voltage of the emf source.
 - III. After the dielectric is inserted, the sum of the instantaneous voltages across the circuit elements equals the instantaneous voltage of the emf source.
 - (a) I
 - (b) I and II
 - (c) I and III
 - (d) II and III
 - (e) I, II and III

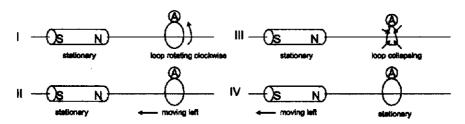
The following three questions refer to the figures below. In the figures below, the dotted lines show the equipotential lines of electric field lines. A charged object is moved directly from point A to point B. The charge on the object is $+1 \mu$ C.



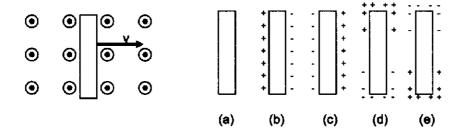
- 6. How does the amount of work needed to move this charge compare for the above cases?
 - (a) The least work required in I.
 - (b) The least work required in II.
 - (c) The least work required in III.
 - (d) I and II require the same amount of work, which is less than III.
 - (e) All three would require the same amount of work.
- 7. Rank the magnitudes of the electric fields at point B in these three cases.
 - (a) I>II>III
 - (b) III>I>II
 - (c) III>II>I
 - (d) II>I>III
 - (e) **I=II=III**
- 8. For case III, what is the direction of the electric force exerted by the field on the +1 μ C charged object when at A and when at B?
 - (a) left at A and left at B
 - (b) right at A and left at B
 - (c) left at A and right at B
 - (d) right at A and left at B
 - (e) no electric force at either
- 9. Two identical loops of wire carry identical currents *I*. The loops are located as shown in the diagram. Which arrow represents the direction of the magnetic field at the point P midway between the two loops?



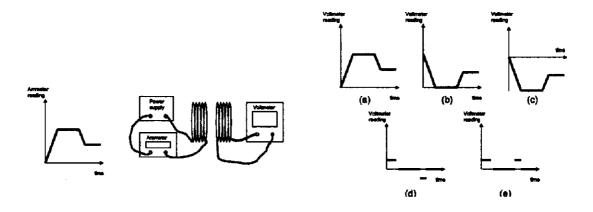
10. The four separate figures below involve a cylindrical magnet and a sensitive ammeter connected to the ends of a loop of copper wire. The plane of the wire is perpendicular to the reference axis. The states of motion of the magnet and the loop of wire are indicated in the diagram. In which of the figures will the ammeter detect current?



- (a) I, II, IV
- (b) II
- (c) II, III, IV
- (d) II, IV
- (e) all four cases
- 11. A neutral metal bar is moving at constant velocity to the right through a region where there is a uniform magnetic field pointing out of the page. Which of the following diagrams best describes the charge distribution on the surface of the metal bar?



12. A variable power supply is connected to a coil and an ammeter and the time dependence of the ammeter is shown. A nearby coil is connected to a voltmeter. Which of the following graphs correctly shows the time dependence of the voltmeter reading?

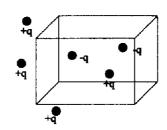


13. What is the total flux through the rectangular prism surface due to charge configuration in the figure below?

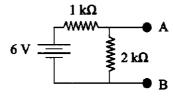


- (b) 0
- (c) q/ϵ_0
- (d) $2q/\epsilon_0$

(e) Not enough symmetry to easily calculate.



The following two questions refer to the figure to the right in which a capacitor is charged to 12 V and then connected between points A and B in the figure below, with its positive plate to A.



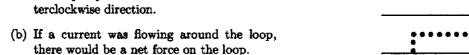
16. What is the current through the 2 k Ω resistor immediately after the capacitor is connected?

- (a) 0 mA
- (b) 1 mA
- (c) 2 mA
- (d) 3 mA
- (e) 6 mA

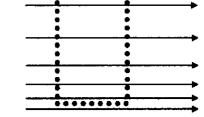
17. What is the current through the 2 $k\Omega$ resistor a long time after the capacitor is connected?

- (a) 0 mA
- (b) 1 mA
- (c) 2 mA
- (d) 3 mA
- (e) 6 mA

18. Two point sources of electromagnetic radiation (otherwise identical) are such that the power of source 1 (P₁) is three times the power of source 2 (P₂). If an observer is 7 m from source 1, and measures the peak electric fields from each source to have the same magnitude, how far is that observer from source 2?
(a) 0.78 m
(b) 2.33 m
(c) 3.50 m
(d) 4.04 m
(e) 4.95 m
19. For the magnetic field and loop depicted to the right, which (if any) of statements (a)-(d) is NOT true?
(a) The line integral of the magnetic field around the loop is positive when taken in the coun-



- (c) If a current was flowing around the loop, there would be a net torque on the loop.
- (d) There must be some current encircled by the loop going out of the page to produce such a magnetic field.



- (e) None of the above statements are false.
- 20. When a switch is closed, a solenoid with inductance L and resistance R is connected to a battery with voltage V_0 . Which of the following statements is (are) true?
 - I. After the switch is closed, the current through the solenoid decreases exponentially to zero with a time constant $\tau = L/R$.
 - II. After the switch is closed, the voltage drop across the solenoid decreases exponentially to zero with a time constant $\tau = L/R$.
 - III. After the switch is closed, an electric field that circles the axis of the solenoid is induced with a magnitude that decreases exponentially to zero with a time constant $\tau = L/R$.
 - (a) I
 - (b) II
 - (c) III
 - (d) I and II
 - (e) I, II and III