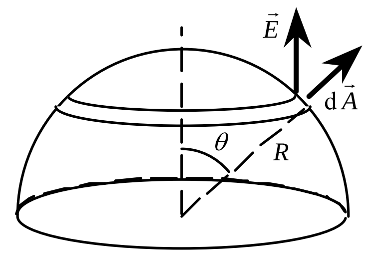
Problem Set #3 – Chapter 23 – 8, 27, 31, 36, 48, 52, 57, 59

8. Because the angle between the electric field and the area varies over the surface of the hemisphere, we find the flux by integration. We choose a strip at an angle θ with a thickness $R\ d\theta$, as shown in the diagram. The area of this strip is



$dA = (2\pi R \sin\theta)R\ d\theta = 2\pi R^2 \sin\theta\, d\theta.$

From the diagram, we see that θ is the angle between $\vec{E}$ and $d\vec{A}$,

so we have

$$\Phi = \iint \vec{E}\cdot d\vec{A} = \int_0^{\pi/2} E(\cos\theta)2\pi R^2 \sin\theta\, d\theta$$

$$= E2\pi R^2\left(\frac{\sin^2\theta}{2}\right)\Bigg|_0^{\pi/2} = E\pi R^2.$$

This is the flux of a constant field through the area of a circle of radius R.

27. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the center of the balloon, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere centered on the balloon with radius $r = 50$ cm. On this surface, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\ dA$. When we apply Gauss' law, we get

$\oint \vec{E}\cdot d\vec{A} = E4\pi r^2 = Q/\varepsilon_0$, which gives

$$E = (1/4\pi\varepsilon_0)(Q/r^2)$$

$$= (9\times 10^9\ \mathrm{N\cdot m^2/C^2})(5\times 10^{-7}\ \mathrm{C})/(0.50\ \mathrm{m})^2$$

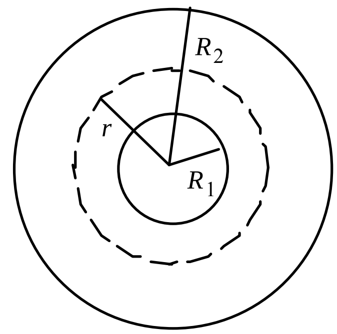
$$= 1.8\times 10^4\ \mathrm{N/C};$$

$$\vec{E} = \boxed{(1.8\times 10^4\ \mathrm{N/C})\ \hat{r}}.$$

If the balloon shrinks, the enclosed charge does not change, so the electric field will be the same:

$$\vec{E} = \boxed{(1.8\times 10^4\ \mathrm{N/C})\ \hat{r}}.$$

31. From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere of radius r. On this surface, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$. The charge density is



$\rho = Q/[4/3\pi(R_2^3 - R_1^3)]$.

For the region where $r < R_1$, we apply Gauss' law:

$\oint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0$;

$E4\pi r^2 = 0$, because there is no charge inside the Gaussian surface, which gives

$E = \boxed{0 \quad \text{for } r < R_1}$.

For the region where $R_1 < r < R_2$, we apply Gauss' law:

$\vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0$;

$E4\pi r^2 = \rho 4/3\pi\ \pi(r^3 - R_1^3)/\varepsilon_0$, which gives

$E = \rho 4/3\pi\ \pi(r^3 - R_1^3)/(4\pi\varepsilon_0 r^2) = Q(r^3 - R_1^3)/4\pi\varepsilon_0(R_2^3 - R_1^3)r^2$;

$\vec{E} = \boxed{[Q(r^3 - R_1^3)/\ 4\pi\varepsilon_0(R_2^3 - R_1^3)r^2]\hat{r} \quad \text{for } R_1 < r < R_2}$.

For the region where $R_2 < r$, we apply Gauss' law:

$\oint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0$;

$E4\pi r^2 = Q/\varepsilon_0$, which gives

$E = (Q/4\pi\varepsilon_0 r^2)$;

$\vec{E} = \boxed{(Q/\ 4\pi\varepsilon_0 r^2)\hat{r} \quad \text{for } R_2 < r}$.

36. Because the slab is infinite, we know from symmetry that the field must be perpendicular to the slab, with a constant magnitude for a constant distance from the slab. If r is positive, the field will be away from the slab. For a Gaussian surface we choose a cylinder of length $2L$ and area A, centered on the axis. On the curved side of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpendicular, so $\vec{E}\cdot d\vec{A} = 0$. On the ends, the field has a constant

magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so $\vec{E}\cdot d\vec{A} = E\, dA$.

To find the field outside the slab, we use the fact that the field will be away from the slab. If we place our Gaussian cylinder so that one end is at $z = L$ and the other end is at $z = -L$, the fields at each end will be directed out of the surface and have the same magnitude. The charge contained within the Gaussian surface will be ρtA (where A is the cross-sectional area of the cylinder). When we apply Gauss' law, we have

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{top}} \vec{E}\cdot d\vec{A} + \iint_{\text{bottom}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A}$$

$$= EA + EA + 0 = Q/\varepsilon_0; \quad \text{which gives}$$

$E = \boxed{\rho t/2\varepsilon_0 \text{ away from the slab, where } z > t/2 \text{ or } z < -t/2}$. The field is uniform outside the slab.

To find the field inside the slab, we use the same Gaussian surface, with the ends of the cylinder at $\pm z$, where $z < t/2$. The enclosed charge is only that part of the slab inside the cylinder and will equal ρ2z A. Applying Gauss' law:

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{top}} \vec{E}\cdot d\vec{A} + \iint_{\text{bottom}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A}$$

$$= EA + EA + 0 = Q/\varepsilon_0; \text{ which gives}$$

$$\vec{E} = \boxed{(\rho z/\varepsilon_0)\hat{k} \quad \text{where} \quad -t/2 < z < t/2}.$$

48. If we choose a Gaussian surface around the earth just outside the earth's surface, so that the radius of the Gaussian surface is $\approx R_{earth}$, we have

$\oint \vec{E}\cdot d\vec{A} = -EA = Q/\varepsilon_0$, so we have

$Q = -\varepsilon_0 4\pi R^2 E$

$= -[1/(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)](6.37\times10^6\ \text{m})^2(100\ \text{N/C}) = \boxed{-4.5\times10^5\ \text{C, on the surface}}$.

The surface charge density is

$\sigma = Q/A = \varepsilon_0 E = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-100\ \text{N/C}) = \boxed{-8.9\times10^{-10}\ \text{C/m}^2}$.

52. From Example 23–7, we know that the electric field inside the sphere is

$E = (Q/4\pi\varepsilon_0)(r/R^3)$ radial.

Because the charges have opposite signs, the force on the point charge is toward the center of the sphere, with magnitude

$F = qQr/4\pi\varepsilon_0 R^3$,

and is a restoring force proportional to the displacement from the center, as in simple harmonic motion, with an effective force constant of

$$k = qQ/4\pi\varepsilon_0 R^3.$$

The resulting motion, with $r = R$ at $t = 0$, is

$$\boxed{r = R\cos(\omega t)}, \qquad \text{with } \omega = (k/m)^{1/2}; \text{ the motion is simple harmonic.}$$

The period of the motion is

$$\tau = 2\pi/\omega = 2\pi(m/k)^{1/2} = \boxed{2\pi(4\pi\varepsilon_0 mR^3/qQ)^{1/2}}.$$

The total energy is the initial potential energy:

$$E = U = \tfrac{1}{2}kR^2 = \tfrac{1}{2}(qQ/4\pi\varepsilon_0 R^3)R^2 = \boxed{qQ/8\pi\varepsilon_0 R}.$$

57. Because there is no charge enclosed by the tetrahedron, the net flux through all sides is 0:

$$\Phi_{\text{net}} = \Phi_{\text{upper sides}} + \Phi_{\text{bottom}}.$$

Thus we find the flux through the three upper sides from

$$\Phi_{\text{upper sides}} = -\,\Phi_{\text{bottom}} = -\,(E\hat{k})\cdot A(-\hat{k}) = +\,E(\tfrac{1}{2}L)(L\sin 60°) = \boxed{0.433EL^2}.$$

59. From the symmetry of the field we construct a Gaussian surface which is a cylinder of length L and radius a, with its axis along the axis of the field. Because the field is parallel to the ends of the cylinder, we have

$$\oint \vec{E}\cdot d\vec{A} = E2\pi aL = Q_{\text{enclosed}}/\varepsilon_0 .$$

If there is a charge distribution $\rho(r)$ within the cylinder, we have

$$Q_{\text{enclosed}} = \int_0^a 2\pi rL\rho(r)\,dr. \quad \text{Thus}$$

$$\varepsilon_0 E2\pi aL = \int_0^a 2\pi rL\rho(r)\,dr, \quad \text{or} \quad \varepsilon_0 Ea = \int_0^a r\rho(r)\,dr.$$

We can write the left-hand side as an integral to get

$$\varepsilon_0 E\int_0^a dr = \int_0^a r\rho(r)\,dr.$$

Comparing the two integrands, we see that

$$\rho(r) = \boxed{\varepsilon_0 E/r}.$$

Note that this function diverges when $r \to 0$. The required field can be set up only beginning at some distance r_0 from the axis. Within r_0 only the total charge has to correspond to the required field:

$q/L = \varepsilon_0 E 2\pi r_0$, which is finite.