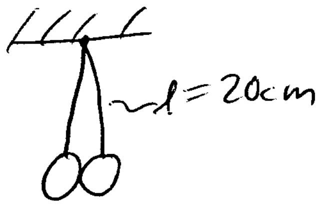
PHYS 102 – Sp 09

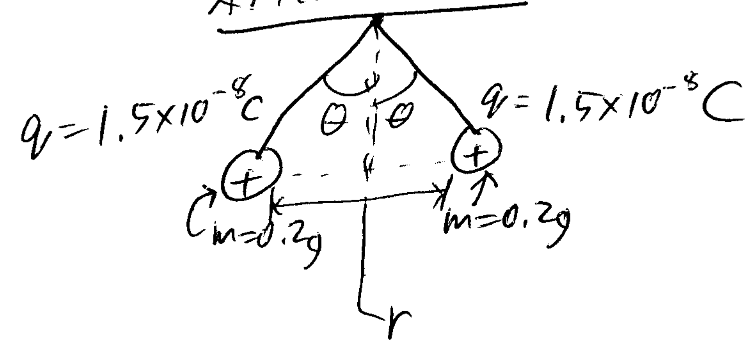
Solutions to Suggested Problem Set #1

21-10

Before Charge



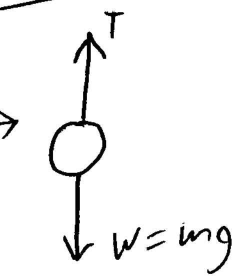
After Charge



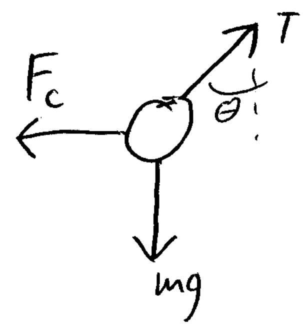
(a) Free body-diagram:

Before charging: same for each ball

assume angle is very small



After charging: Different for each:



$$F_c = \frac{kqq}{r^2}$$

(b) Before charge:

$$\sum F_y = T - mg = 0 \rightarrow T = mg$$

$$T = (0.2g)\left(\frac{1kg}{1000g}\right)(9.8 m/s^2)$$

$$\boxed{T = 1.96\times10^{-3}N}$$

After charge:

$$\sum F_y = T\sin\theta - mg = 0 \rightarrow T = \frac{mg}{\sin\theta}$$

$$\sum F_x = T\cos\theta - F_c = 0 \rightarrow T = \frac{F_c}{\cos\theta}$$

set equal to each other.

$$\frac{mg}{\sin\theta} = \frac{F_c}{\cos\theta}$$

$$\tan\theta = \frac{F_c}{mg} = \frac{kq^2}{r^2 mg}$$

$$r = 2(l\sin\theta)$$

$$\tan\theta = \frac{kq^2}{mg(2l\sin\theta)^2}$$

we'll apply the approximation $\theta \approx \sin\theta \approx \tan\theta$ for small angles. This is probably fair, given the tiny amount of charge

$\boxed{21\text{-}10}$ (cont'd)

(b) so, we have:

$$(\sin\theta)^3 \simeq \frac{kq^2}{4mgl^2}$$

$$\sin\theta = \left[\frac{(9\times10^9)(1.5\times10^{-8}C)^2}{4(0.0002kg)(9.8\,m/s^2)(0.2m)^2}\right]^{1/3}$$

$$\sin\theta \simeq 0.186 \rightarrow \theta \simeq 10.7^\circ$$

$$\therefore T = \frac{mg}{\sin\theta} = \frac{(0.0002kg)(9.8\,m/s^2)}{0.186} = \boxed{1.05\times10^{-2}N}$$

(c) $\boxed{\theta \simeq 10.7^\circ}$ (see above)

$\boxed{21\text{-}19}$

$r = 4.5\times10^{-9}m$

$F = 1.1\times10^{-11}N$

$$F = \frac{kq_1q_2}{r^2}$$ same q for each ion

$$\therefore q^2 = \frac{Fr^2}{k}$$

$$q = \left[\frac{(1.11\times10^{-11}N)(4.5\times10^{-9}m)^2}{9\times10^9}\right]^{1/2}$$

$$\boxed{q \simeq 1.58\times10^{-19}C}$$

$\boxed{q \simeq 1e} \rightarrow \boxed{Na^+}$ ion, as expected!

$\boxed{21-22}$

$F_g = 7\times10^{-7}\ N$

$r = 0.1\ m$

0.05% accuracy $\rightarrow \frac{\Delta F}{F} = 0.0005$

$\Delta F = 3.5\times10^{-10} N \rightarrow$ allowable error due to Coulomb force

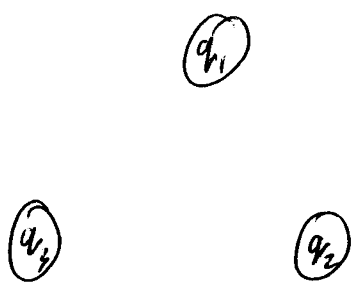
equal charges

$\Delta F = \frac{kq^2}{r^2}$

$q = \sqrt{\frac{(\Delta F) r^2}{k}} = \left[\frac{(3.5\times10^{-10})(0.1)^2}{9\times10^9}\right]^{1/2}$

$\boxed{\text{over 100 million electrons!}} \rightarrow \boxed{q = 1.97\times10^{-11}\ C}$

$\boxed{21-31}$



$F_{12} = 1.4\times10^{-2} N$ attractive at $r_{12} = 15.0$ cm

$F_{23} = 3.8\times10^{-2} N$ attractive at $r_{23} = 20.0$ cm

$F_{13} = 5.2\times10^{-2} N$ repulsive at $r_{13} = 10.0$ cm

$F_{12} = \frac{-kq_1q_2}{r_{12}^2} \rightarrow q_1q_2 = \frac{-F_{12}r_{12}^2}{k}$

$F_{23} = \frac{-kq_2q_3}{r_{23}^2} \rightarrow q_2q_3 = \frac{-F_{23}r_{23}^2}{k}$

$F_{13} = \frac{kq_1q_3}{r_{13}^2} \rightarrow q_1q_3 = \frac{F_{13}r_{13}^2}{k}$

$q_1q_2 = -3.5\times10^{-14} C^2$

$q_2q_3 = -1.7\times10^{-13} C^2$

$q_1q_3 = 5.8\times10^{-14} C^2$

$\boxed{21-31}$ cont'd

$q_1 q_2 = -3.5\times10^{-14}C^2 \rightarrow \boxed{q_2 = \frac{-3.5\times10^{-14}C^2}{q_1}}$

$q_2 q_3 = -1.7\times10^{-13}C^2 \rightarrow q_3 = \frac{-1.7\times10^{-13}C^2}{q_2} = \frac{-1.7\times10^{-13}C^2}{-3.5\times10^{-14}C^2}$

$\boxed{q_3 = 4.9\,q_1}$

$q_1 q_3 = 5.8\times10^{-14}C^2$

$\boxed{q_3 = \frac{5.8\times10^{-14}C^2}{q_1}}$

combine:

$\frac{5.8\times10^{-14}C^2}{q_1} = 4.9q_1$

$q_1^2 = \frac{5.8\times10^{-14}C^2}{4.9}$

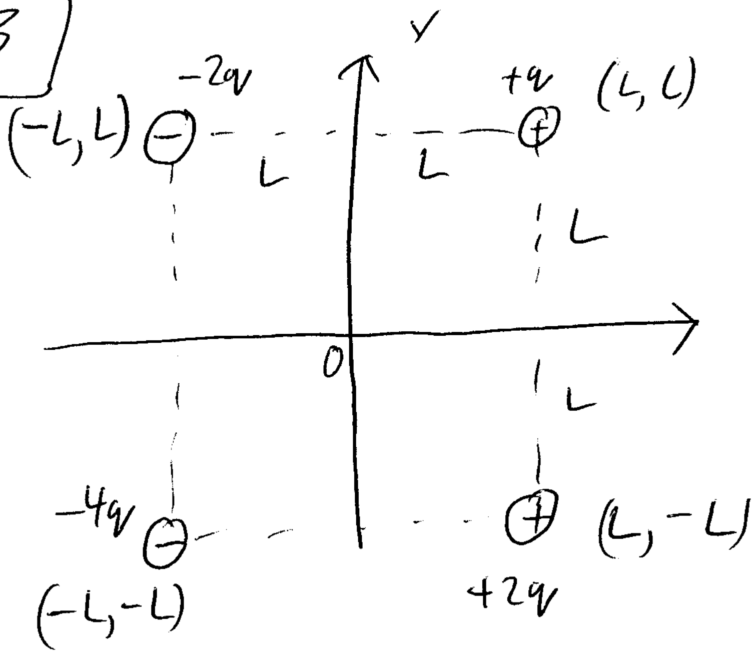
$\boxed{q_1 \approx 1.1\times10^{-7}C}$

$\frac{-3.5\times10^{-14}C^2}{q_1} = \boxed{q_2 = -3.2\times10^{-7}C}$

$4.9q_1 = \boxed{q_3 = 5.4\times10^{-7}C}$ $\rightarrow$ text book gives $5.3\times10^{-7}C$

rounding error

$\boxed{21-43}$



let $q_1 = +q$
$q_2 = +2q$
$q_3 = -2q$
$q_4 = -4q$

(a) $\underline{F_{net} \text{ on } +q:}$

$$\vec{F}_{net} = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$

$r = 2L$

unit vectors obvious

$$\vec{F}_{21} = \frac{k(2q)(q)}{(2L)^2}\hat{j} = \left(\frac{kq^2}{2L^2}\right)\hat{j}$$

$$\vec{F}_{31} = \frac{k(-2q)(q)}{(2L)^2}\hat{i} = \left(\frac{-kq^2}{2L^2}\right)\hat{i}$$

$r_{41} = 2L\sqrt{2}$

$$\hat{r}_{41} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j}) \longrightarrow \vec{F}_{41} = \frac{k(-4q)(q)}{(2L\sqrt{2})^2}\left(\frac{1}{\sqrt{2}}\right)(\hat{i} + \hat{j}) = \frac{-kq^2}{2\sqrt{2}L^2}(\hat{i} + \hat{j})$$

$$\therefore \vec{F}_{net} = \underbrace{\left(-\frac{kq^2}{2L^2}\hat{i}\right)}_{\vec{F}_{21}} + \underbrace{\left(+\frac{kq^2}{2L^2}\hat{j}\right)}_{\vec{F}_{31}} + \underbrace{\left(\frac{-kq^2}{2\sqrt{2}L^2}\hat{i}\right) + \left(-\frac{kq^2}{2\sqrt{2}L^2}\hat{j}\right)}_{\vec{F}_{41}}$$

$$\tan\theta = \frac{-0.293}{1.707}$$

$$\boxed{\theta = -9.74^\circ}$$

$$\vec{F}_{net} = \frac{kq^2}{2L^2}\left[\left(-1 - \frac{1}{\sqrt{2}}\right)\hat{i} + \left(+1 - \frac{1}{\sqrt{2}}\right)\hat{j}\right] = \boxed{\frac{kq^2}{2L^2}\left(0.293\hat{j} - 1.707\hat{i}\right)}$$

$$\boxed{\vec{F}_{net} = \frac{\sqrt{3}kq^2}{2L^2} \text{ at } \theta = 9.74 \text{ below } -x\text{-axis}}$$

$\boxed{21\text{-}43}$ (cont'd)

(b) F_{net} on $+Q$ at origin:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$= \frac{kQ}{(L\sqrt{2})^2}\left[q\hat{r}_1 + 2q\hat{r}_2 - 2q\hat{r}_3 - 4q\hat{r}_4\right]$$

$$\hat{r}_1 = \frac{1}{\sqrt{2}}(-\hat{i} - \hat{j}) \qquad \hat{r}_2 = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$$

$$\hat{r}_3 = \frac{1}{\sqrt{2}}(+\hat{i} - \hat{j}) \qquad \hat{r}_4 = \frac{1}{\sqrt{2}}(+\hat{i} + \hat{j})$$

$$\vec{F}_{net} = \frac{kqQ}{2\sqrt{2}L^2}\left[(-\hat{i} - \hat{j}) + 2(-\hat{i} + \hat{j}) - 2(+\hat{i} - \hat{j}) - 4(+\hat{i} + \hat{j})\right]$$

$$\vec{F}_{net} = \frac{kqQ}{2\sqrt{2}L^2}(-9\hat{i} - 1\hat{j}) \rightarrow \tan\theta = \frac{-1}{-9}$$

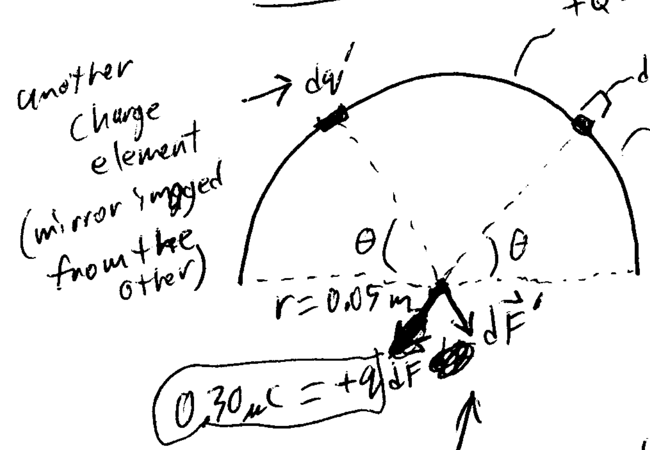
$$\boxed{\theta = 6.34°}$$ below $-x$-axis

$$\boxed{\vec{F}_{net} \approx \frac{3.2\,kqQ}{L^2} \text{ at } \theta = 6.34° \text{ below } -x\text{-axis}}$$

$\boxed{21-55}$

Find $\vec{F}_{net}$ on $+q$:

small charge element along wire

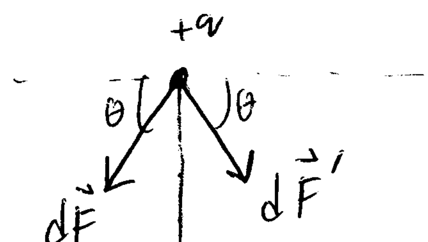


$\lambda = \dfrac{+Q}{\pi r}$

$dq = \lambda \underbrace{ds}_{\text{arc length}} = \lambda r \, d\theta$

$dq = \left(\dfrac{Q}{\pi r}\right) r \, d\theta = \dfrac{Q}{\pi} d\theta$

let's look closely here:



→ horizontal components equal and opposite!

$\vec{F}_y = \vec{F}_{net} = (-2 \, dF \sin\theta)\hat{j}$

net is sum of vertical components

recall definition of Coulomb force from a continuous object:

$$\vec{F}_{net} = \int d\vec{F} \rightarrow \vec{F}_y = -\int (dF \sin\theta)\hat{j} = -\hat{j}\int \frac{kq\,dq}{r^2} \sin\theta$$

$$= -\left(\frac{kq}{r^2}\right)\hat{j}\int dq \sin\theta = \left(-\frac{kq}{r^2}\right)\hat{j}\int_{\theta=0}^{\pi} \left(\frac{Q}{\pi}\right)\sin\theta \, d\theta$$

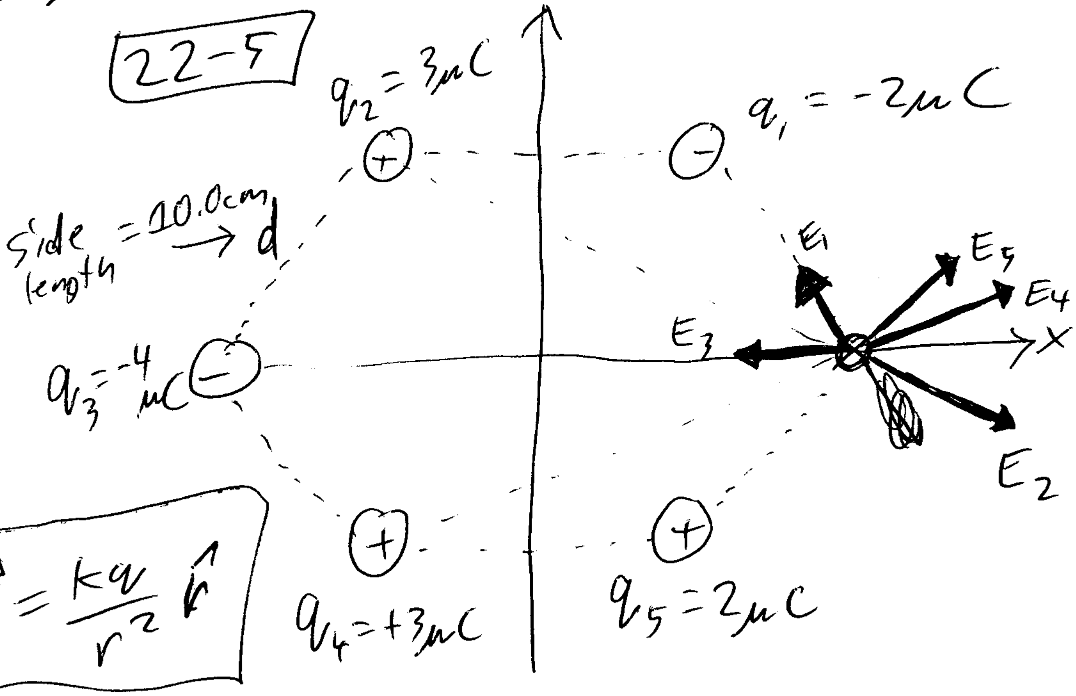
Use symmetry to change limits:

$$\vec{F}_{net} = \left(-\frac{2kqQ}{\pi r^2}\right)\hat{j}\int_{\theta=0}^{\pi/2} \sin\theta \, d\theta = \hat{j}\,\frac{(-2)(9\times10^9)(0.30\,\mu C)(0.75\,\mu C)}{\pi(0.05)^2} \cancel{\int \sin\theta}$$

$$\vec{F}_{net} = -0.516\,\hat{j}\left[-\cos\theta\right]_0^{\pi/2} = \boxed{(-0.52\,N)\,\hat{j}}$$

~~[illegible]~~

$\boxed{22-5}$



$$\boxed{\vec{E} = \frac{kq}{r^2}\hat{r}}$$

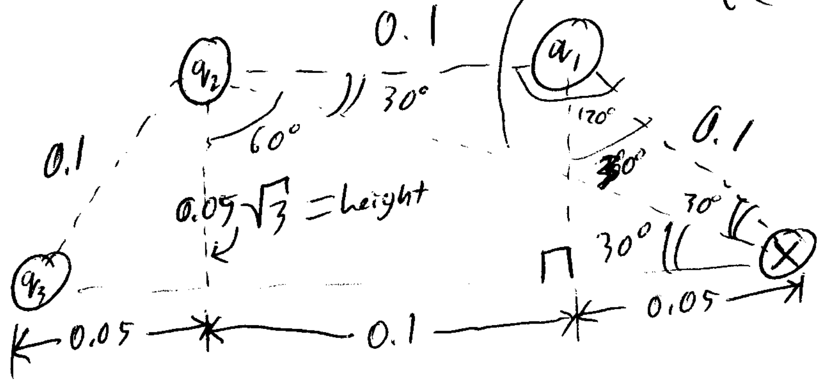
$\boxed{\text{Find } \vec{E} \text{ at empty vertex}}$

↖ we find these directions for E ~~vectors~~ thinking qualitatively

Let's examine the geometry closer by taking upper half only:

$s^2 = (0.05\sqrt{3})^2 + (0.15)^2$ (lengths in meters)

$\boxed{s = 0.173\,m}$



Note also: E_2 and E_4 should be mirror images, so we'll expect only their horizontal components to count.

Now, to find unit vectors from each q_i to the point of interest...

$\hat{r}_1 = (\cos 60°)\hat{i} - (\sin 60°)\hat{j}$

$\hat{r}_2 = (\cos 30°)\hat{i} - (\sin 30°)\hat{j}$

$\hat{r}_3 = \hat{i}$

$\hat{r}_4 = (\cos 30°)\hat{i} + (\sin 30°)\hat{j}$

$\hat{r}_5 = (\cos 60°)\hat{i} + (\sin 60°)\hat{j}$

Now, we'll combine these and calculate magnitudes...

$\boxed{22-5}$ (cont'd)

$$r_1 = d \rightarrow \vec{E}_1 = \frac{kq_1}{r_1^2}\hat{r}_1 = \frac{(9\times10^9)(-2\times10^{-6}C)}{(0.1)^2}\left(\cos 60^\circ\,\hat{\imath} - \sin 60^\circ\,\hat{\jmath}\right)$$

$$r_2 = s \rightarrow \vec{E}_2 = \frac{kq_2}{r_2^2}\hat{r}_2 = \frac{(9\times10^9)(3\times10^{-6}C)}{(0.173)^2}\left(\cos 30^\circ\,\hat{\imath} - \sin 30^\circ\,\hat{\jmath}\right)$$

$$r_3 = 0.2 \rightarrow \vec{E}_3 = \frac{kq_3}{r_3^2}\hat{r}_3 = \frac{(9\times10^9)(-4\times10^{-6}C)}{(0.2)^2}\left(\hat{\imath}\right)$$

$$r_4 = s \rightarrow \vec{E}_4 = \frac{kq_4}{r_4^2}\hat{r}_4 = \frac{(9\times10^9)(3\times10^{-6}C)}{(0.173)^2}\left(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}\right)$$

$$r_5 = d \rightarrow \vec{E}_5 = \frac{kq_5}{r_5^2}\hat{r}_5 = \frac{(9\times10^9)(2\times10^{-6}C)}{(0.1)^2}\left(\cos 60^\circ\,\hat{\imath} + \sin 60^\circ\,\hat{\jmath}\right)$$

$$\vec{E}_1 = (-9\times10^5)\hat{\imath} + (1.6\times10^6)\hat{\jmath}$$

$$\vec{E}_2 = (7.8\times10^5)\hat{\imath} + (-4.5\times10^5)\hat{\jmath}$$

$$\vec{E}_3 = (-9\times10^5)\hat{\imath}$$

$$\vec{E}_4 = (7.8\times10^5)\hat{\imath} + (4.5\times10^5)\hat{\jmath}$$

Sum these up $\rightarrow$

$$\vec{E}_5 = (9\times10^5)\hat{\imath} + (1.6\times10^6)\hat{\jmath}$$

$$\vec{E}_{net} = (2.46\times10^5)\hat{\imath} + (3.2\times10^6)\hat{\jmath}$$

$$\tan\theta = \frac{3.2\times10^6}{2.46\times10^5}$$

$$\boxed{|\vec{E}_{net}| \simeq 3.21\times10^6\ N/C \text{ at } \theta = 85.6^\circ \text{ above x-axis}}$$

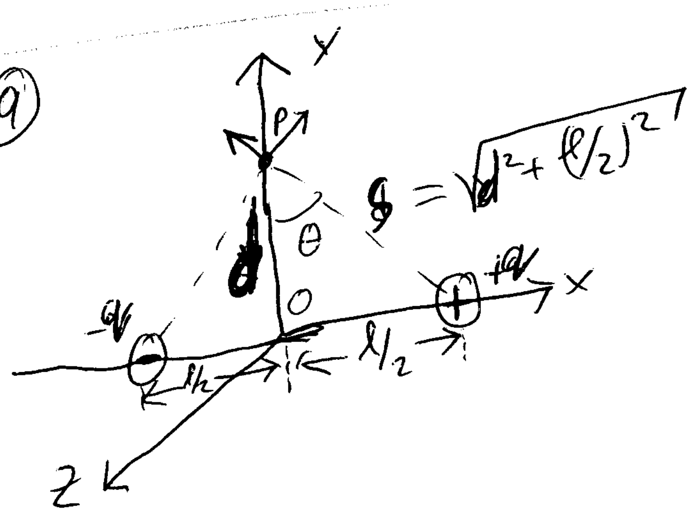
rounding error

$\boxed{22-7}$ there's a typo in this problem, given the figure provided.

part (c) should read:

"For part (b), what would the electric field be in the entire xz plane specified by $y = 0$?"

(a)



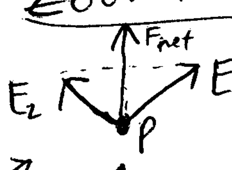
$$s = \sqrt{d^2 + (l/2)^2}$$

$$\vec{E}_{(0,0,0)} = \frac{-kq}{(l/2)^2}\hat{i} + \frac{kq}{(l/2)^2}(-\hat{i}) = \boxed{-\frac{8kq}{l^2}\hat{i}}$$

(b) let $-q \rightarrow +q$:

$$\vec{E}_{(0,0,0)} = \frac{kq}{(l/2)^2}\hat{i} + \frac{kq}{(l/2)^2}(-\hat{i}) = \boxed{0}$$

(c) Zoom in on point P:



→ ~~horizontal~~ out-of-plane components cancel!

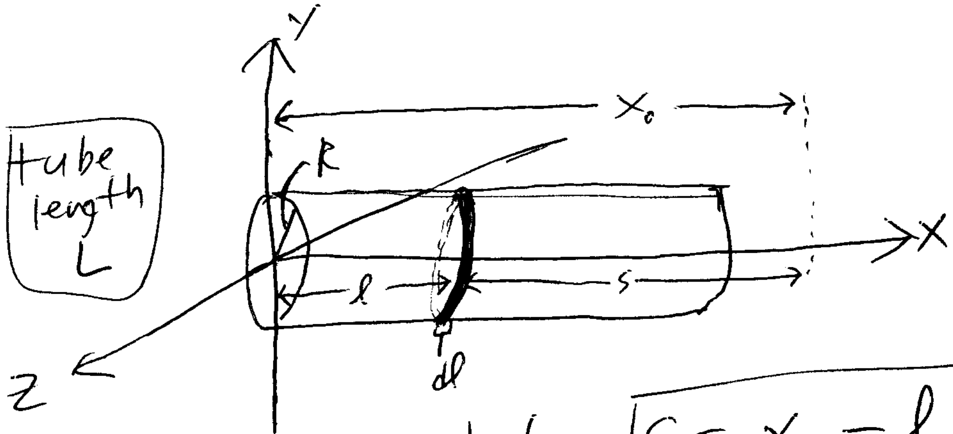
→ only in-plane field:

yz plane runs perpendicular to page

$$E_{Net} = 2E_{1x} = \frac{2kq}{s^2}\cos\theta = \frac{2kq}{(d^2 + (l/2)^2)}\left(\frac{d}{(d^2 + (l/2)^2)^{1/2}}\right)$$

$$\boxed{E_{Net} = \frac{2kqd}{(d^2 + (l/2)^2)^{3/2}} \text{ away from origin!}}$$

22-30 Finding $\vec{E}$ along axis of hollow tube:



define: we are looking for $\vec{E}_{x_0}$ for arbitray x_0

let $\boxed{s = x_0 - l}$

Also: let $\sigma = \frac{Q_{total}}{\text{surface area}}$ and $\sigma\, dl = \lambda$ for the ring

use a ring of thickness dl along x-axis as element → we'll integrate a stack of these to represent the tube.

Recall: $dE_x = \frac{k\,dQ\, s}{(R^2+s^2)^{3/2}}$ for the field from a ring at some point s along its central axis

$$\vec{E} = \int dE_x\, \hat{\imath} \rightarrow E = \int \frac{k\,dQ\, s}{(R^2+s^2)^{3/2}}$$

$dQ = \lambda(2\pi R)\,dl$ → charge on the ring

$$\vec{E} = \hat{\imath}\int \frac{k\lambda(2\pi R)(dl)(s)}{(R^2+s^2)^{3/2}} = \hat{\imath}\, k\lambda(2\pi R)\int \frac{s\, dl}{(R^2+s^2)^{3/2}}$$

$s = x_0 - l \rightarrow ds = -dl$, Integrate from $l=0$ to $l=L$

$$\vec{E} = \hat{\imath}\, k\lambda(2\pi R)\int_0^L \frac{s\, ds}{(R^2+s^2)^{3/2}} \rightarrow \begin{array}{l}\text{let } u = R^2+s^2\\ du = 2ds\end{array} \rightarrow = \hat{\imath}\,\frac{-2\pi k\lambda R}{2}\int_{l=0}^{L}\frac{du}{u^{3/2}}$$

$$\vec{E} = \hat{\imath}\,{-\pi k\lambda R}\left[\frac{1}{u^{1/2}}\right]_{l=0}^{L} = -\pi k\lambda R\left[\frac{1}{\left(R^2+(x_0-l)^2\right)^{1/2}}\right]_0^L \hat{\imath}$$

continued →

$\boxed{22\text{-}30}$ (cont'd)

$$\boxed{\vec{E} = -(\pi k \lambda R)\hat{i}\left[\frac{1}{(R^2+(x_0-L)^2)^{1/2}} - \frac{1}{(R^2+x_0^2)^{1/2}}\right]}$$