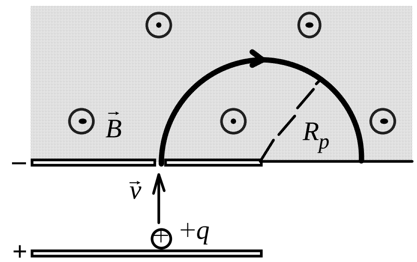
Problem Set #9 –Chapter 28 – 34, 41, 48, 49, 50, 63, 79

34. We find the speed acquired from the accelerating potential from

$v = (2qV/m)^{1/2}$.



We combine this with the expression for the radius of the path,

$R = mv/qB$, to get

$R = (2mV/q)^{1/2}/B$.

With V, q, and B the same,

$R_x/R_p = 1.4 = (m_x/m_p)^{1/2}$, which gives $\boxed{m_x/m_p = 2.0}$.

41. The force of attraction between the two charges provides the centripetal acceleration:

$F_0 = mR\omega_0^2$.

A small uniform magnetic field perpendicular to the plane of the orbit, and thus the velocity, will add an additional radial force, with a magnitude

$F_M = qvB = qR\omega B$, where $\omega = \omega_0 + \mathrm{d}\omega$.

If the direction is such that this force is toward the center, we have

$F_0 + F_M = mR\omega^2$;

$mR\omega_0^2 + qR(\omega_0 + \mathrm{d}\omega)B = mR(\omega_0 + \mathrm{d}\omega)^2$.

If we can neglect the $B\,\mathrm{d}\omega$ term, we get

$mR\omega_0^2 + qRB\omega_0 = mR(\omega_0 + \mathrm{d}\omega)^2 = mR\omega_0^2 + 2mR\omega_0\,\mathrm{d}\omega + mR\,(\mathrm{d}\omega)^2$

$= mR\omega_0^2 + 2mR\omega_0\,\mathrm{d}\omega$,

which gives $\boxed{\mathrm{d}\omega = qB/2m}$.

48. The element has length $\mathrm{d}s = R\,\mathrm{d}\theta$ and makes an angle of θ from the y-axis toward the $-x$-axis. The force on the element is

$\mathrm{d}\vec{F} = I\,\mathrm{d}\vec{s} \times \vec{B} = IR\,\mathrm{d}\theta(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \times (B\hat{i})$,

which gives

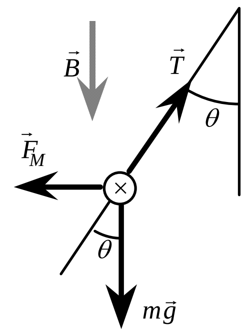
$\boxed{d\vec{F} = -IRB\cos(\mathrm{d}\theta)\,\hat{k}}$.

49. From $\vec{F} = I\vec{L} \times \vec{B}$, we see that the force on the wire produced by the magnetic field will be down, so the wire will move down. At the new equilibrium position, the magnetic force will be balanced by the increased elastic forces of the springs:

$F_B = F_{\text{elastic}}$;

$ILB = 2k\,\Delta y$, which gives $\Delta y = \boxed{ILB/2k}$.

50. Because the magnetic field is perpendicular to the wire, the magnetic force is horizontal. If we look along the support axis, we have the three forces shown in the diagram.



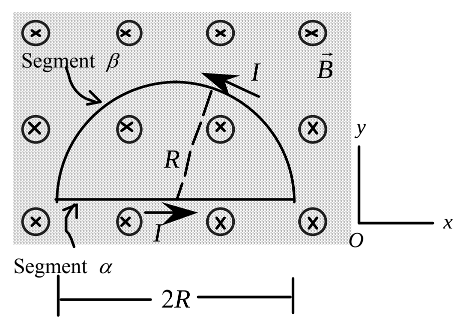
In the equilibrium condition, the resultant force is 0, so we have

$\tan\theta = F_M/mg = ILB/mg$

$= (0.55\ \text{A})(0.8\ \text{m})(0.03\ \text{T})/[(0.070\ \text{kg})(9.8\ \text{m/s}^2)] = 0.017$, so

$\theta = \boxed{1°}$.

63. (a) For the segment α, we have



$$\vec{F}_\alpha = I\vec{L} \times \vec{B} = I(2R\hat{i}) \times (-B\hat{k}) = \boxed{2IRB\hat{j}}.$$

(b) For the segment β, we choose a differential element $d\vec{s}$ at an angle θ from the x-axis. The force on every element will be directed toward the center of the arc, along the radius. By pairing elements symmetrically placed from the y-axis, we see that the resultant force will be along the $-y$-axis. The force on the element is

$$d\vec{F}_\beta = I\,d\vec{s} \times \vec{B} = I(-\sin\theta\,ds\,\hat{i} + \cos\theta\,ds\,\hat{j}) \times (-B\hat{k})$$

$$= IB\,ds\,(-\sin\theta\,\hat{j} - \cos\theta\,\hat{i}).$$

The resultant force is

$$\vec{F}_\beta = IB \int (-\sin\theta\,ds)\hat{j}.$$

We could use $ds = R\,d\theta$ and perform the integration over θ. If we recognize that $\sin\theta\,ds = dx$, we can simplify the integral:

$$\vec{F}_\beta = -IB \int dx\,\hat{j} = -IB\,\Delta x\,\hat{j} = -2IRB\hat{j}.$$

(c) For the net force, we have

$$\vec{F}_{\text{net}} = \vec{F}_{\alpha} + \vec{F}_{\beta} = 2IRB\hat{j} - 2IRB\hat{j} = \boxed{0}.$$

(d) From the analysis of part (b), which did not use the shape of the wire, we see that the net force in a uniform field will be $\boxed{\text{zero}}$ for a loop of any shape.

79. a) The angular momentum of the electrons is

$$L = \boxed{NmvR \text{ perpendicular to the orbit}}.$$

(b) The period of the orbit is $T = 2\pi R/v$. The magnetic dipole moment is

$$\mu = IA = (Ne/T)\pi R^2 = [Ne/(2\pi R/v)]\pi R^2 = \boxed{1/2NevR}.$$

(c) The ratio is

$$L/\mu = NmvR/1/2NevR = \boxed{2m/e}.$$