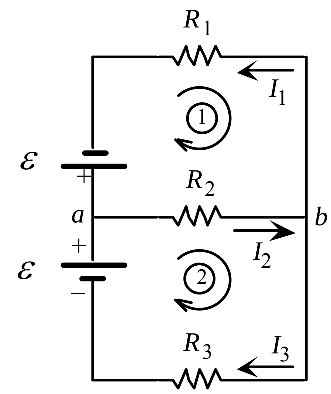
Problem Set #8 –Chapter 27 – 22, 31, 32, 33, 34, 57, 59

22. Because no two resistors have the same current and no two resistors have the same potential difference across them, we cannot combine them in series or parallel. We assume the current directions shown in the diagram. We use conservation of current at point a:

$$\sum I_{\text{in}} = 0;$$

$$I_1 - I_2 + I_3 = 0.$$



We apply the loop rule for the two loops indicated in the diagram:

loop 1: $-\mathcal{E} + I_1R_1 + I_2R_2 = 0;$

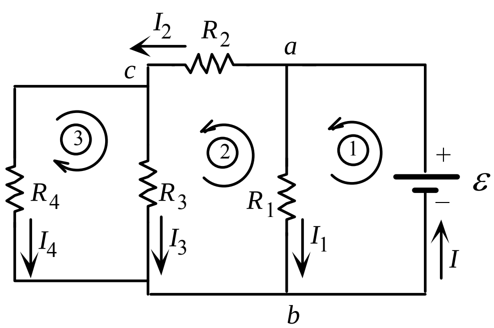
loop 2: $-I_2R_2 - I_3R_3 + \mathcal{E} = 0.$

When we combine these equations, we get

$I_1 = R_3\mathcal{E}/(R_1R_2 + R_1R_3 + R_2R_3),$

$I_2 = (R_1 + R_3)\,\mathcal{E}/(R_1R_2 + R_1R_3 + R_2R_3),$

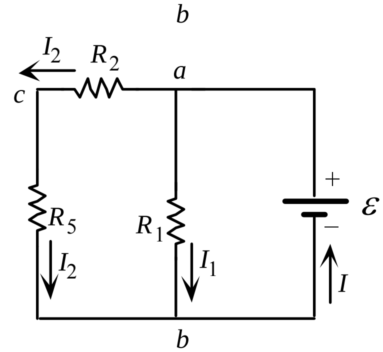
$I_3 = R_1\mathcal{E}/(R_1R_2 + R_1R_3 + R_2R_3).$



31. (a) We can reduce the circuit to a single loop by successively combining parallel and series combinations. We combine R_3 and R_4, which are in parallel:

$1/R_5 = 1/R_3 + 1/R_4 \quad = 1/(100\ \Omega) + 1/(50\ \Omega),$

which gives $R_5 = 33.3\ \Omega$.



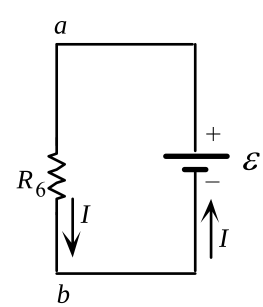
We combine R_1 and $R_2 + R_5$, which are in parallel:

$1/R_6 = 1/R_1 + 1/(R_2 + R_5) = 1/(100\ \Omega) + 1/(20\ \Omega + 33.3\ \Omega),$

which gives $R_6 = 34.8\ \Omega$.

Because $V_{ab} = 6$ V, we find I_2 from

$I_2 = V_{ab}/(R_5 + R_2) = (6\text{ V})/(33.3\ \Omega + (0\ \Omega) = 0.113$ A.



We can now find V_{cb} from

$V_{cb} = I_2R_5 = (0.113 \text{ A})(33.3\ \Omega) = 3.75 \text{ V}.$

The current through the 50-Ω resistor is

$I_4 = V_{cb}/R_4 = (3.75 \text{ V})/(50\ \Omega) = \boxed{0.075 \text{ A}}.$

(b) For the conservation of current, we have

junction a: $I - I_1 - I_2 = 0;$

junction c: $I_2 - I_3 - I_4 = 0.$

For the three loops indicated on the diagram, we have

loop 1: $\varepsilon - I_1R_1 = 0;$

$6 \text{ V} - I_1(100\ \Omega) = 0;$

loop 2: $I_1R_1 - I_2R_2 - I_3R_3 = 0;$

$I_1(100\ \Omega) - I_2(20\ \Omega) - I_3(100\ \Omega) = 0;$

loop 3: $-I_3R_3 + I_4R_4 = 0;$

$-I_3(100\ \Omega) + I_4(50\ \Omega) = 0.$

When we solve these equations, we get

$I_1 = 0.060 \text{ A}, I_2 = 0.113 \text{ A}, I_3 = 0.038 \text{ A}, I_4 = 0.075 \text{ A}.$

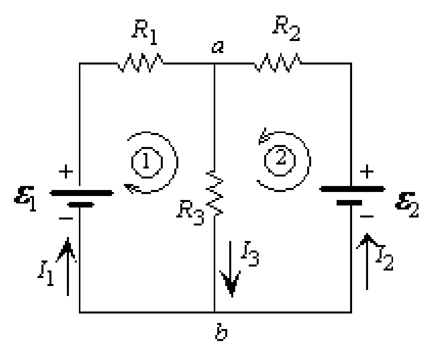
Thus, the current through the 50-Ω resistor is $\boxed{0.075 \text{ A}}$.

32. For the conservation of current at point a, we have

$\sum I_{\text{in}} = 0;$

$I_1 + I_2 - I_3 = 0;$

$I_1 + I_2 - 0.1 \text{ A} = 0.$



For the two loops indicated on the diagram, we have

loop 1: $\varepsilon_1 - I_1R_1 - I_3R_3 = 0;$

$+3 \text{ V} - I_1(5\ \Omega) - (0.1 \text{ A})R_3 = 0;$

loop 2: $\varepsilon_2 - I_2R_2 - I_3R_3 = 0;$

$$+6\text{ V} - I_2(20\ \Omega) - (0.1\text{ A})R_3 = 0.$$

When we solve these equations, we get

$$I_1 = -0.04\text{ A},\ I_2 = 0.14\text{ A, and } R_3 = \boxed{32\ \Omega}.$$

If $I_3 = -0.1$ A, the equations become

$$I_1 = -I_2 - 0.1\text{ A};$$

$$+3\text{ V} - I_1(5\ \Omega) - (-0.1\text{ A})R_3 = 0;$$

$$+6\text{ V} - I_2(20\ \Omega) - (-0.1\text{ A})R_3 = 0.$$

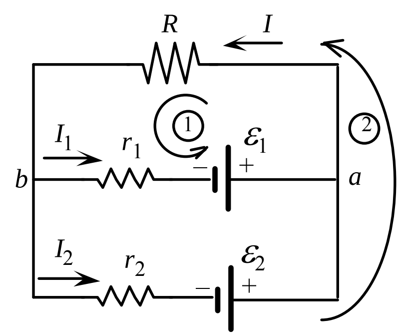
When we solve these equations, we get $I_1 = -0.2$ A, $I_2 = 0.1$ A, and $R_3 = -40\Omega$.

Because we cannot have a negative resistance, it is $\boxed{\text{not possible}}$ to have $I_3 = -0.1$ A.

33. For the conservation of current at point b, we have

$$\sum I_{\text{in}} = 0;$$

$$I - I_1 - I_2 = 0;$$



For the two loops indicated on the diagram, we have

loop 1: $\mathcal{E}_1 - I_1 r_1 - IR = 0;$

$$+12\text{ V} - I_1(0.1\ \Omega) - I(5\ \Omega) = 0;$$

loop 2: $\mathcal{E}_2 + I_2 r_2 - IR = 0;$

$$+10\text{ V} - I_2(10\ \Omega) - I(5\ \Omega) = 0.$$

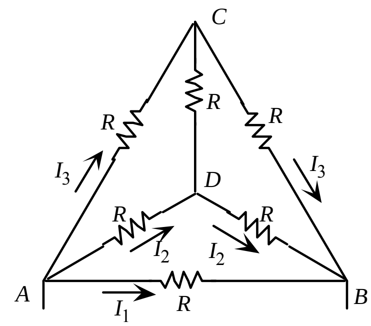
When we solve these equations, we get

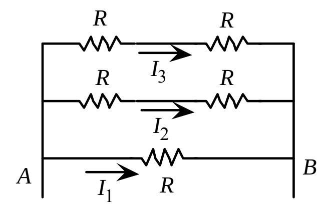
$$I_1 = 2.52\text{ A},\ I_2 = -0.17\text{ A, and } I = 2.35\text{ A}.$$

The current through the load resistor is $\boxed{2.35\text{ A}}$.

$\mathcal{E}_1$ supplies $\boxed{2.52\text{ A}}$; $\mathcal{E}_2$ supplies $\boxed{\text{no current}}$, it is being charged by $\mathcal{E}_1$.

34. On the diagram, we show the potential difference applied between points A and B. Because all of the resistors are the same, symmetry means that the three currents leaving point A must be the same three currents entering point B. This means that there is no current in the resistor between points C and D, which can be removed without changing the currents. When we redraw the circuit, we see that we have three parallel branches between points A and B. The currents are





$$I_1 = V_{AB}/R = (4 \text{ V})/(1 \ \Omega) = 4 \text{ A};$$

$$I_2 = I_3 = V_{AB}/(R + R) = (4 \text{ V})/(1 \ \Omega + 1 \ \Omega) = 2 \text{ A}.$$

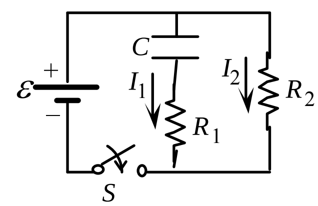
The power dissipated in each of the resistors is

$$P_{AB} = I_1^2 R = (4 \text{ A})^2(1 \ \Omega) = \boxed{16 \text{ W}};$$

$$P_{CD} = \boxed{0};$$

$$P_{\text{all others}} = I_2^2 R = (2 \text{ A})^2(1 \ \Omega) = \boxed{4 \text{ W}}.$$

57. Because there is no internal resistance in the battery, the potential difference across R_2 and across the capacitor branch is $\mathcal{E}$. The current in R_2 is constant:



$$I_2 = \mathcal{E}/R_2.$$

The charging current in the capacitor branch is

$$I_1 = (\mathcal{E}/R_1)e^{-t/R_1C}.$$

From the junction, the current in the battery is

$$\boxed{I_{\text{battery}} = I_1 + I_2 = (\mathcal{E}/R_1)e^{-t/R_1C} + (\mathcal{E}/R_2).}$$

59. The possible capacitance values that we have are

$$C_1 = C_2 = 5 \ \mu\text{F};$$

$$C_{\text{parallel}} = C_1 + C_2 = 5 \ \mu\text{F} + 5 \ \mu\text{F} = 10 \ \mu\text{F};$$

$$C_{\text{series}} = C_1C_2/(C_1 + C_2) = (5 \ \mu\text{F})(5 \ \mu\text{F})/[(5 \ \mu\text{F}) + (5 \ \mu\text{F})] = 2.5 \ \mu\text{F}.$$

We need to combine the resistors to produce one of the following resistance values:

$$R_a = RC/C_1 = (1 \times 10^{-3} \text{ s})/(5 \times 10^{-6} \text{ F}) = 200 \ \Omega;$$

$R_b = RC/C_{\text{parallel}} = (1 \times 10^{-3}\ \text{s})/(10 \times 10^{-6}\ \text{F}) = 100\ \Omega;$

$R_c = RC/C_{\text{series}} = (1 \times 10^{-3}\ \text{s})/(2.5 \times 10^{-6}\ \text{F}) = 400\ \Omega;$

If we connect the 300-Ω resistors in parallel, we get

$$R_3 = R_2R_2/(R_2 + R_2)$$

$$= (300\ \Omega)(300\ \Omega)/[(300\ \Omega) + (300\ \Omega)] = 150\ \Omega.$$

We see that we can produce R_c by putting this combination in series with the 250-Ω resistor:

$$R_c = R_1 + R_3 = 250\ \Omega + 150\ \Omega = 400\ \Omega.$$

Thus we connect the 300-Ω resistors in parallel with each other and in series with the 250-Ω resistor and the two capacitors.