Problem Set #7 –Chapter 26 – 34, 40, 55, 57, 58, 77, 86

34. We find the length of the equivalent single wire from

$R = V/I = \rho L/A$;

$(1.5\ \mathrm{V})/(0.14\ \mathrm{A}) = (1.7 \times 10^{-8}\ \Omega\cdot\mathrm{m})L/[1/2\pi(0.24 \times 10^{-3}\ \mathrm{m})^2]$, which gives $L = 28.5$ m.

The distance to the short is $d = L/2 = \boxed{14.3\ \mathrm{m}}$.

40. We express the voltage in terms of the current and radius:

$V = IR = I\rho L/A = I\rho L/\pi r^2$.

If we use this for the two wires and take the ratio, we have

$V_1/V_2 = (I_1\rho L/I_2\rho L)(\pi r_2^2/\pi r_1^2) = (I_1/I_2)(r_2/r_1)^2$.

We see that r_2/r_1 will be minimum when V_2/V_1 has its maximum value of 1.5:

$1/1.8 = (1/2)(r_2/r_1)_{\mathrm{min}}^2$, which gives $(r_2/r_1)_{\mathrm{min}} = \boxed{1.05}$.

55. The current in the 24-Ω resistor is

$I_1 = V_{AB}/(24\ \Omega) = (16\ \mathrm{V})/(24\ \Omega) = \boxed{0.67\ \mathrm{A}}$.

Since the equivalent resistance of the upper branch of the circuit is
$R = 8\ \Omega + (12\ \Omega)(6\ \Omega)/(12\ \Omega + 6\ \Omega) = 12\ \Omega$, the current in the 8-Ω resistor is
$I_2 = V_{AB}/(12\ \Omega) = (16\ \mathrm{V})/(12\ \Omega) = \boxed{1.33\ \mathrm{A}}$.

This current is split between the two remaining resistors, with
$I_3 = [6\ \Omega/(6\ \Omega + 12\ \Omega)]\ I_2 = 1/3(1.33\ \mathrm{A}) = \boxed{0.44\ \mathrm{A}}$ in the 12-Ω resistor and
$I_4 = I_2 - I_3 = 1.33\ \mathrm{A} - 0.44\ \mathrm{A} = \boxed{0.89\ \mathrm{A}}$ in the 6-Ω resistor.

57. Call the lower branch with the single 12-Ω resistor branch 1 and the other one (which contains all the other resistors) branch 2. First, we find R_2, the equivalent resistance of branch 2. Combine the 2-Ω and 4-Ω resistors in parallel to obtain $(2\ \Omega)(4\ \Omega)/(2\ \Omega + 4\ \Omega) = 1.33\ \Omega$, which we add to the 8-Ω resistor and put the resultant 9.33-Ω resistor in parallel with the 24-Ω resistor: $(9.33\ \Omega)(24\ \Omega)/(9.33\ \Omega + 24\ \Omega) = 6.72\ \Omega$. Add this to the 6-Ω resistor to obtain $R_2 = 12.72\ \Omega$.
We now combine R_1 (= 12 Ω) and R_2 in parallel to obtain the equivalent resistance between A and B:

$R_{\mathrm{eq}} = R_1 R_2/(R_1 + R_2) = (12.72\ \Omega)(12\ \Omega)/(12.72\ \Omega + 12\ \Omega) = 6.18\ \Omega$.

The voltage across AB, i.e., that across the 12-Ω resistor, is then

$V_{12\,\Omega} = I_{AB}R_{eq} = (20\text{ A})(6.18\ \Omega) = \boxed{124\text{ V}}$.

The current in the 12-Ω resistor is

$I_{12\,\Omega} = V_{AB}/(12\ \Omega) = (124\text{ V})/(12\ \Omega) = \boxed{10.3\text{ A}}$,

And the current through the 6-Ω resistor is then

$I_{6\,\Omega} = 20\text{ A} - 10.3\text{ A} = \boxed{9.7\text{ A}}$, which requires a voltage of

$V_{6\,\Omega} = I_{6\,\Omega}(6\ \Omega) = (9.7\text{ A})(6\ \Omega) = \boxed{58\text{ V}}$.

The voltage difference across the 24-Ω resistor is now

$V_{24\,\Omega} = 123.6\text{ V} - 58.2\text{ V} = \boxed{66\text{ V}}$, which drives a current of

$I_{24\,\Omega} = (65.4\text{ V})/(24\ \Omega) = \boxed{2.7\text{ A}}$, leaving the current in the 8-Ω resistor as

$I_{8\,\Omega} = 9.7\text{ A} - 2.7\text{ A} = \boxed{7.0\text{ A}}$, which requires a voltage of

$V_{8\,\Omega} = I_{8\,\Omega}(8\ \Omega) = (7.0\text{ A})(8\ \Omega) = \boxed{56\text{ V}}$.

The voltage applied on the two remaining resistors (4-Ω and 2-Ω) is then

$V_{4\,\Omega} = V_{2\,\Omega} = 123.5\text{ V} - 58.2\text{ V} - 56.0\text{ V} = \boxed{9.3\text{ V}}$, which drives a current of

$I_{4\,\Omega} = (9.3\text{ V})/(4\ \Omega) = \boxed{2.3\text{ A}}$ in the 4-Ω resistor and

$I_{2\,\Omega} = (9.3\text{ V})/(2\ \Omega) = \boxed{4.7\text{ A}}$ in the 2-Ω resistor.

58. The two parallel branches has an equivalent resistance of $R(R + x)/[R + (R + x)]$, so for the equivalent resistance of the entire load we have

$R_{eq} = 3R + R(R + x)/(2R + x) = x;$

$x^2 - 2Rx - 7R^2 = 0;$

$x = \boxed{(1 + \sqrt{8})R \approx 3.83\,R}$.

77. The energy taken out of the battery is

$U = Pt = IVt = (50 \times 10^{-3}\text{ A})(6\text{ V})(18\text{ h})(3600\text{ s/h}) = \boxed{1.94 \times 10^4\text{ J}\ \ (5.4\text{ kWh})}$.

86. (a) When the bulbs are connected in series, the equivalent resistance is

$R_{series} = \sum R_i = 10R_{bulb}$.

The power consumption is

$P = V_{ab}^2/R_{eq}$;

$50\text{ W} = (120\text{ V})^2/(10R_{bulb})$, which gives $R_{bulb} = \boxed{28.8\ \Omega}$.

(b) In part (a), the power consumption of each bulb is 5 W, which is the maximum power rating, so the voltage across each bulb, 12 V, must be the maximum allowed. With the limiting resistor connected in series with the parallel bulb combination, we find the equivalent resistance of the ten bulbs from

$1/R_{\text{parallel}} = \sum(1/R_i) = 10/R_{\text{bulb}} = 10/(28.8\ \Omega)$, which gives $R_{\text{parallel}} = 2.88\ \Omega$.

For the maximum consumption, the voltage across each bulb, and thus R_{parallel}, is 12 V, so we have

$V_{\text{parallel}} = I_{\text{total}}R_{\text{parallel}}$;

$12\ \text{V} = I_{\text{total}}(2.88\ \Omega)$, which gives $I_{\text{total}} = 4.17\ \text{A}$.

With the series resistor, we have

$V_{ab} = I_{\text{total}}(R_{\text{parallel}} + R_2)$;

$120\ \text{V} = (4.17\ \text{A})(2.88\ \Omega + R_2)$, which gives $R_2 = \boxed{25.9\ \Omega \text{ in series}}$.

The power loss in the added resistor is

$P_2 = I_{\text{total}}{}^2R = (4.17\ \text{A})^2(25.9\ \Omega) = \boxed{4.5 \times 10^2\ \text{W}}$.