Problem Set #6 –Chapter 25 – 54, 58, 68, 71, 73, 75, 77

54. We can consider the system to be two capacitors in series:

$$1/C = 1/C_1 + 1/C_2$$

$$= (D - d)/\varepsilon_0 A + d/\kappa\varepsilon_0 A$$

$$= (D - d + d/\kappa)]/\varepsilon_0 A, \text{ which gives}$$

$$C = \boxed{\kappa\varepsilon_0 A/[d + \kappa(D - d)]}.$$

The charge on the plates is then

$Q = CV = \kappa\varepsilon_0 AV/[d + \kappa(D - d)]$, so the electric field in the empty space is

$$E_0 = \sigma/\varepsilon_0 = Q/\varepsilon_0 A = \kappa V/[(d + \kappa(D - d)]$$

$$= (1.8)(600\text{ V})/[0.6 \times 10^{-2}\text{ m} + (1.8)(1.6 \times 10^{-2}\text{ m} - 0.6 \times 10^{-2}\text{ m})]$$

$$= \boxed{4.5 \times 10^4\text{ V/m}}.$$

The electric field in the dielectric is

$$E = E_0/\kappa = (4.5 \times 10^4\text{ V/m})/1.8 = \boxed{2.5 \times 10^4\text{ V/m}}.$$

58. Because $D << L$, we can ignore fringing fields. The potential difference must be the same on each half of the space, so we can treat the system as two capacitors in parallel:

$$C = C_1 + C_2 = \kappa_0\varepsilon_0(½L^2)/d + \kappa_1\varepsilon_0(½L^2)/d$$

$$= (\varepsilon_0(½L^2)/d)(\kappa_0 + \kappa_1) = ½(\kappa_0 + \kappa_1)(\varepsilon_0 L^2/d)$$

68. (a) Because a single potential is available, from $Q = CV$ we see that the maximum charge will be produced by the maximum capacitance. For a parallel-plate capacitor, $C = \kappa\varepsilon_0 A/d$. We need a system with maximum area and minimum separation. The minimum separation is 5 mm, and the maximum area possible is 150 cm^2. (Note that if we make a number of smaller capacitors, they will be connected in parallel to produce the maximum capacitance. This is the same as a single capacitor.) The system consists of 2 aluminum plates of area 150 cm^2, separated by 5 mm, with a 150 cm^2 piece of Bakelite between the plates. The designed capacitance is

$$C = \kappa\varepsilon_0 A/d$$

$$= (4.9)(8.85 \times 10^{-12}\text{ F/m})(150 \times 10^{-4}\text{ m}^2)/(5 \times 10^{-3}\text{ m}) = 1.30 \times 10^{-10}\text{ F}.$$

The charge on the plates is

$$Q = CV = (1.30 \times 10^{-10}\text{ F})(1200\text{ V}) = \boxed{1.56 \times 10^{-7}\text{ C}}.$$

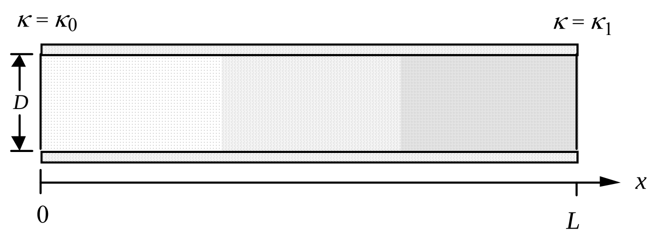
The energy stored is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(1.30 \times 10^{-10}\text{ F})(1200\text{ V})^2 = \boxed{9.37 \times 10^{-5}\text{ J}}.$$

(b) Because a single potential is available, from $E = V/d$ we see that the maximum field will be produced by the minimum separation. The Bakelite is not needed to have this electric field, so the system is the same, but with no Bakelite. The electric field is

$$E_0 = (1200\text{ V})/(5 \times 10^{-3}\text{ m}) = \boxed{2.4 \times 10^5\text{ V/m}}.$$

71.



We find the capacitance of the strip of the dielectric at x, with width dx, from

$$dC = \kappa\varepsilon_0\, dA/D = \kappa\varepsilon_0 L\, dx/D.$$

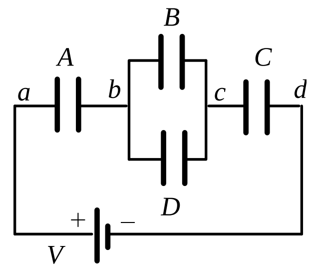
The strips that make up the capacitor are in parallel, so the equivalent capacitance is

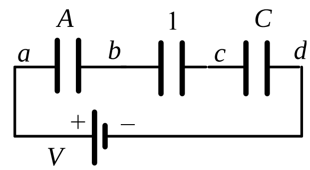
$$C = \int dC = \int_0^L \frac{\kappa\varepsilon_0 L}{D}\, dx = \varepsilon_0 \int_0^L \left[\kappa_0 + \frac{(\kappa_1 - \kappa_0)x}{L}\right]\frac{L}{D}\, dx$$

$$= \frac{\varepsilon_0 L}{D}\left[\kappa_0 x + \frac{(\kappa_1 - \kappa_0)x^2}{2L}\right]\Bigg|_0^L = \frac{\varepsilon_0 L}{D}\left[\kappa_0 L + \frac{(\kappa_1 - \kappa_0)L^2}{2L}\right], \text{ which reduces to}$$

$$C = \tfrac{1}{2}\boxed{(\kappa_0 + \kappa_1)(\varepsilon_0 L^2/D)}.$$

73. We find the equivalent capacitance of the circuit.





B and D are in parallel:

$$C_1 = C_B + C_D = 4.3\ \mu\text{F} + 2.1\ \mu\text{F} = 6.4\ \mu\text{F}.$$

We now have three capacitors in series:

$$1/C_{\text{equ}} = 1/C_A + 1/C_1 + 1/C_C$$

$$= 1/(5.4\ \mu\text{F}) + 1/(6.4\ \mu\text{F}) + 1/(3.2\ \mu\text{F}),$$

which gives $C_{\text{equ}} = 1.53\ \mu\text{F}$.

We find the charge on the equivalent capacitor, which is

also the charge on each capacitor in series, from

$$Q_{\text{equ}} = Q_A = Q_1 = Q_C = C_{\text{equ}}V_{ab}$$

$$= (1.53\ \mu\text{F})(3000\ \text{V}) = 4.6 \times 10^3\ \mu\text{C}.$$

We find the potential differences from

$$V_A = V_{ac} = Q_A/C_A = (4.6 \times 10^3\ \mu\text{C})/(5.4\ \mu\text{F}) = \boxed{8.5 \times 10^2\ \text{V}};$$

$$V_B = V_D = V_{cd} = Q_1/C_1 = (4.6 \times 10^3\ \mu\text{C})/(6.4\ \mu\text{F}) = \boxed{7.2 \times 10^2\ \text{V}};$$

$$V_C = V_{db} = Q_C/C_C = (4.6 \times 10^3\ \mu\text{C})/(3.2\ \mu\text{F}) = \boxed{1.43 \times 10^3\ \text{V}}.$$

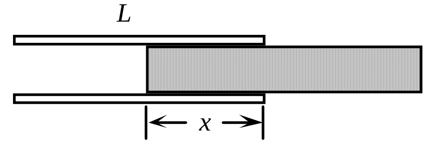
75. The energy stored in the capacitor is

$$U_0 = ½C_0V_0^2 = ½\,(3.0 \times 10^{-6}\ \text{F})(1500\ \text{V})^2 = \boxed{3.4\ \text{J}}.$$

Because the source is disconnected, the charge on the capacitor does not change, and we have

$$C = \kappa C_0;\ V = V_0/\kappa.$$

The energy stored after the dielectric is inserted is



$$U = ½CV^2 = ½\kappa C_0(V_0/\kappa)^2 = (1/\kappa)(½C_0V_0^2) = (1/\kappa)U_0.$$

We find the work required to insert the dielectric from

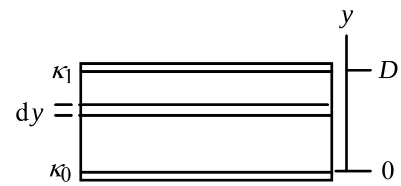
$$W = \Delta U = (1/\kappa - 1)U_0$$

$$= (1/2.8 - 1)(3.4\ \text{J}) = \boxed{-2.2\ \text{J}}.$$

The negative value means that the dielectric is drawn into the region between the plates.

77. We take a strip of the dielectric perpendicular to the y-axis, with thickness Δy, as a capacitor. The capacitance of this strip is

$$C_y = \kappa\varepsilon_0 L^2/\Delta y.$$



All of the strips from $y = 0$ to $y = D$ are in series, so we find the total capacitance from

$$1/C = \sum(1/C_i) = \sum(\Delta y/\kappa\varepsilon_0 A).$$

In the limit $\Delta y \to 0$, this sum becomes an integral:

$$\frac{1}{C} = \int_0^D \frac{dy}{\kappa\varepsilon_0 L^2} = \frac{1}{\varepsilon_0 L^2}\int_0^D \frac{dy}{\kappa_0 + \left[\left(\kappa_1 - \kappa_0\right)y/D\right]}$$

$$= \frac{D}{\varepsilon_0 L^2\left(\kappa_1 - \kappa_0\right)} \ln\left\{\kappa_0 + \left[\left(\kappa_1 - \kappa_0\right)y/D\right]\right\}\Bigg|_0^D = \frac{D}{\varepsilon_0 L^2\left(\kappa_1 - \kappa_0\right)} \ln\left[\frac{\kappa_0 + \left(\kappa_1 - \kappa_0\right)}{\kappa_0}\right]$$

$$= \frac{D}{\varepsilon_0 L^2\left(\kappa_1 - \kappa_0\right)} \ln\left(\frac{\kappa_1}{\kappa_0}\right).$$

The capacitance is

$$C = \boxed{(\kappa_1 - \kappa_0)\varepsilon_0 L^2/[D\ln(\kappa_1/\kappa_0)]}.$$