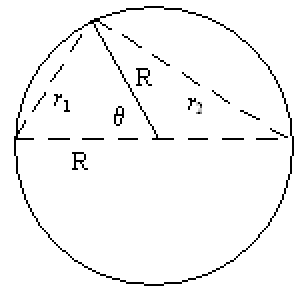
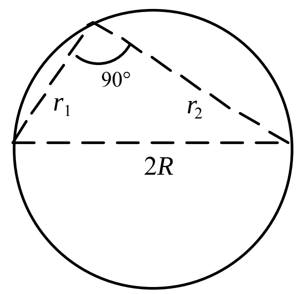
Problem Set #4 – Chapter 24 – 19, 28, 36, 38, 52, 59, 77, 85



19. (a) The distance from the point in question needs to be determined for both point sources. r_1 is given. Using R = a = b and r_1 = c, one can apply the Law of Cosines ($c^2 = a^2 + b^2 - 2ab\cos\theta$) to find the angle θ, and therefore, the angle (180° - θ), and then solve for r_2, again using the Law of Cosines. Alternatively, one might recall that the diameter of a circle subtends an angle of 90° at any point on the circle. Thus the distance from the negative charge to the point is

$$r_2 = [(2R)^2 - r_1^{\,2}]^{1/2} = [(50\text{ cm})^2 - (30\text{ cm})^2]^{1/2} = 40\text{ cm}.$$



The potential at the point is

$$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2)$$

$$= (9 \times 10^9\text{ C}^2/\text{N}\cdot\text{m}^2)[(24 \times 10^{-8}\text{ C})/(0.30\text{ m}) + (-10 \times 10^{-8}\text{ C})/(0.40\text{ m})]$$

$$= \boxed{+5.0 \times 10^3\text{ V}}.$$

(b) The work required is

$$W = q\,\Delta V = (-0.2 \times 10^{-6}\text{ C})(5.0 \times 10^3\text{ V} - 0) = \boxed{-1.0 \times 10^{-3}\text{ J}}.$$

The negative value indicates that the negative charge wants to “fall” to the higher potential.

28. We need to find the potential energy stored in the charge Q distributed uniformly over the spherical shell. We do this by successively bringing a differential charge in from infinity. The total potential energy is the sum (integral) of the differential work done. As we bring in a differential charge, the charge q already on the shell appears to be a point charge, so the work required to bring in the next differential charge is

$$dW = (1/4\pi\varepsilon_0)(q/r)\,dq.$$

The potential energy is the total work required:

$$U_1 = W_1 = \frac{1}{4\pi\varepsilon_0}\int_0^Q \frac{q}{R}\,dq = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}\right).$$

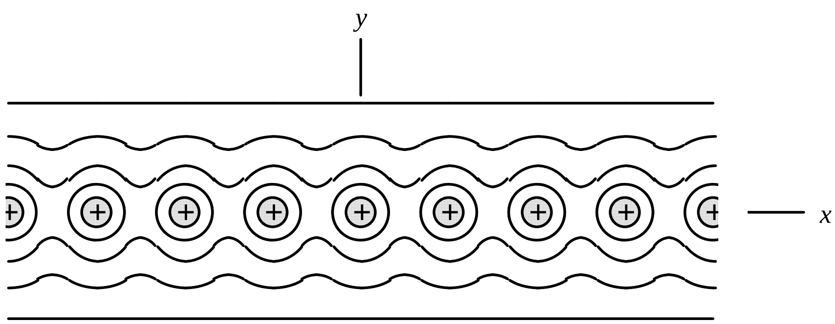
For a shell with half the radius, we have

$$U_2 = \tfrac{1}{2}Q^2/4\pi\varepsilon_0(R/2) = 2U_1.$$

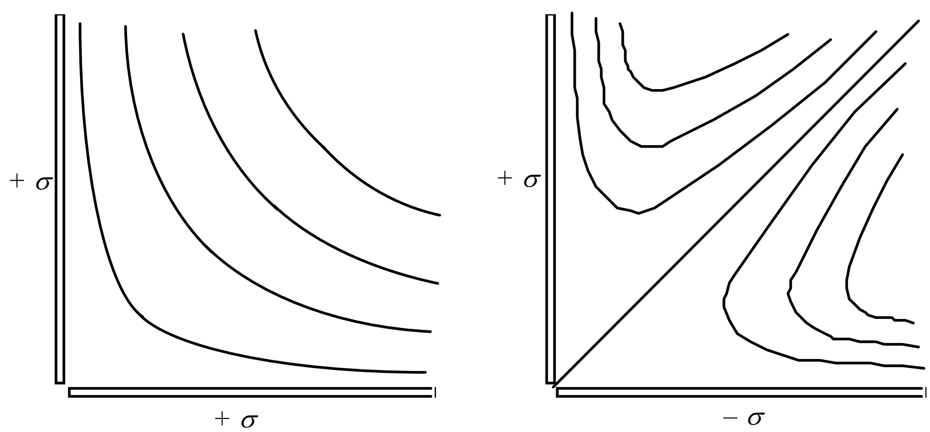
The work required to move the charges is

$W = \Delta U = 2U_1 - U_1 = U_1 = \boxed{\tfrac{1}{2}\ Q^2/4\pi\varepsilon_0 R}$.

36.



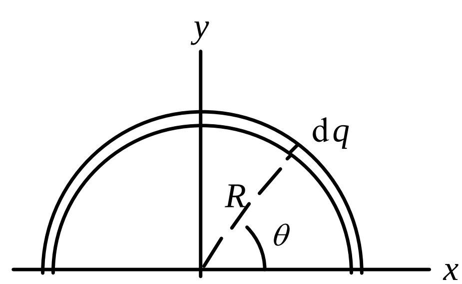
38.



52. The potential at the origin from a differential element of the charge is

$$dV = (1/4\pi\varepsilon_0)(dq/R).$$

To find the potential at the origin, we add (integrate) the contributions from all elements:



$$V = \int (1/4\pi\varepsilon_0)(dq/R)$$

$$= (1/4\pi\varepsilon_0 R)\int dq = q/4\pi\varepsilon_0 R = \lambda\pi R/4\pi\varepsilon_0 R$$

$$= \boxed{\lambda/4\varepsilon_0}.$$

59. As discussed in class and in the book, before the two spheres are connected together, their potentials will depend on the amount of deposited charge and their spherical radius:

$$V_2 = (1/4\pi\varepsilon_0)(q_2/r_2)$$

$$V_1 = (1/4\pi\varepsilon_0)(q_1/r_1)$$

After connection, the two spheres must have the same potential:

$$V = (1/4\pi\varepsilon_0)(q_1'/r_1) = (1/4\pi\varepsilon_0)(q_2'/r_2), \text{ or } \quad q_1' = (r_1/r_2)q_2'.$$

Because charge is conserved we have

$$q_1 + q_2 = q_1' + q_2'.$$

When we combine these two equations, we get

$$q_2' = (q_1 + q_2)[r_2/(r_1 + r_2)].$$

The amount of charge that moves between the two spheres is

$$\Delta q_2 = q_2' - q_2 = \boxed{(q_1 r_2 - q_2 r_1)/(r_1 + r_2)}.$$

77. Because the electric field is purely radial (from symmetry considerations), we will chose a radial path from ∞ to a radial position r within the sphere and set $V = 0$ at $r = \infty$. Using Gauss' Law, we can derive the electric fields in the different regions as:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}, \quad r \geq R$$

$$\vec{E} = \frac{Qr}{4\pi\varepsilon_0 R^3}, \quad r < R$$

For r inside the sphere, we have

$$\begin{aligned} V(r) &= -\int_{\infty}^{R} E_{outside}\,dr - \int_{R}^{r} E_{inside}\,dr \\ &= -\int_{\infty}^{R} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr - \int_{R}^{r} \frac{Qr}{4\pi\varepsilon_0 R^3}\,dr \\ &= -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{R} \frac{1}{r^2}\,dr - \frac{Q}{4\pi\varepsilon_0 R^3}\int_{R}^{r} r\,dr \\ &= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}\right) + \frac{Q}{8\pi\varepsilon_0 R}\left(1 - \frac{r^2}{R^2}\right) \\ &= \frac{Q}{8\pi\varepsilon_0 R^3}\left(3 - \frac{r^2}{R^2}\right) \end{aligned}$$

The potential energy of a charge $-q$ is

$$U = -qV = \boxed{-(qQ/8\pi\varepsilon_0 R)(3 - r^2/R^2)}.$$

The variable part of the potential energy has the form of the elastic potential energy of a spring:

$$U = \tfrac{1}{2}kr^2,$$

so the motion can be an oscillation, like the mass on a spring.

Comparing the coefficients, we have

$$k = \boxed{qQ/4\pi\varepsilon_0 R^3}.$$

85. Assume that $r_1 < r_2$. Place the origin of the coordinate system at the center of both shells. For $r > r_2$ the electric field is identical to that of a point charge, $Q = q_1 + q_2$, at the origin. So

$$V = (1/4\pi\varepsilon_0)Q/r = \boxed{(1/4\pi\varepsilon_0)(q_1 + q_2)/r \quad (r_2 < r)}.$$

Between r_1 and r_2, the E-field produced by q_2 is zero, so the potential due to q_2 remains the same as its value at r_2, i.e., $(1/4\pi\varepsilon_0)q_2/r_2$. For q_1, the E-field is still equivalent to that of a point charge q_1 at the origin, so the contribution to V due to q_1 is $(1/4\pi\varepsilon_0)q_1/r$. Add both contributions up to obtain

$$V = \boxed{(1/4\pi\varepsilon_0)(q_2/r_2 + q_1/r) \quad (r_1 < r < r_2)}.$$

Once $r < r_1$, there is no electric field, so the potential no longer changes once it reaches its value at r_1. Thus

$$V = \boxed{(1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2) \quad (r < r_1)}.$$