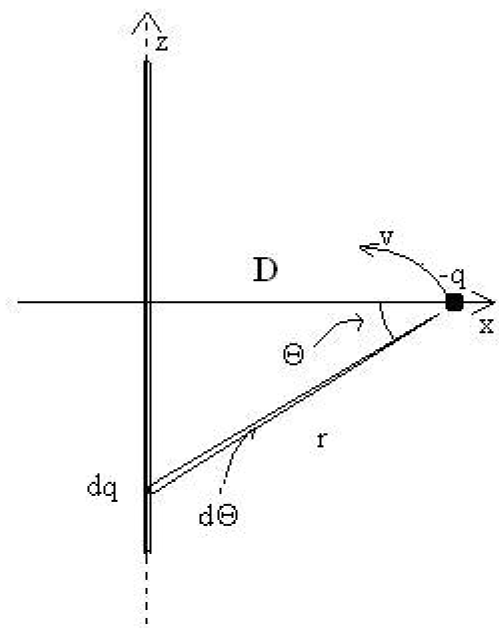
Problem Set #2 – Chapter 22 – 43, 45, 51, 55, 56, 57, 63

43. The mathematics used to find the electric field at a distance D from the wire is similar to that used to solve Problem 36, from Problem Set #1, with the limits of integration now going from $-\infty$ to $+\infty$. Because of symmetry considerations, the field a given distance from the wire on the xy plane will have a constant magnitude, and so for simplicity, we will look at the field at a point on the positive x axis. We choose a differential element of the wire, as shown in the diagram. The charge of this element is $dq = \lambda\, dx$. We find the field produced by the element, which has both x- and z-components, by integrating along the rod:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{x=-\infty}^{x=\infty} \frac{dq}{r^2}\left(\cos 6\,\hat{i} + \sin 6\,\hat{k}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{x=-\infty}^{x=\infty} \frac{dx}{r^2}\left(\cos 6\,\hat{i} + \sin 6\,\hat{k}\right).$$



Symmetry considerations again indicate that the components in the z direction will cancel, so we only need to calculate the total field attributed by all the components dq in the x direction.

To perform the integration, we must eliminate variables until we have one, for which we choose θ.
From the diagram we see that $r = D/\cos\theta$, and $z = D\tan\theta$. This gives

$dx = -D\sec^2\theta\, d\theta = -(D\, d\theta)/\cos^2\theta$.

The limits for θ are $-\pi/2$ rad to $\pi/2$ rad. When we make these substitutions, we have

$$\vec{E}(D,0) = \vec{E}(D,0)_x = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2} \frac{D\,d\Theta/\cos^2\Theta}{\left(D/\cos\Theta\right)^2}\cos\Theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2}\frac{1}{D}\cos\Theta\, d\Theta = \frac{\lambda}{4\pi\varepsilon_0 D}\left(\sin \pi/2 - \sin -\pi/2\right) = \frac{\lambda}{2\pi\varepsilon_0 D}$$

The force produced by the electric field of the wire on the negative charge is toward the wire and provides the centripetal force (where we're now referring to the distance between the wire and the negative charge as r):

$F = mv^2/r;$

$q(\lambda/2\pi\varepsilon_0 r) = mv^2/r$, which gives a speed $v =$ $\boxed{(q\lambda/2\pi\varepsilon_0 m)^{1/2}}$, which does not depend on r.

45. The force produced by the electric field of the wire on the negative charge is toward the wire and provides the centripetal force:

$F = mv^2/r$;

$q(\lambda/2\pi\varepsilon_0 r) = mv^2/r$, which gives a speed $v = (q\lambda/2\pi\varepsilon_0 m)^{1/2}$.

The period of the orbit is

$T = 2\pi r/v = [2\pi/(q\lambda/2\pi\varepsilon_0 m)^{1/2}]r$.

If the centripetal force is provided by a point charge, we have

$(1/4\pi\varepsilon_0)(qQ/r^2) = mv^2/r$, which gives

$v = (qQ/4\pi\varepsilon_0 mr)^{1/2}$.

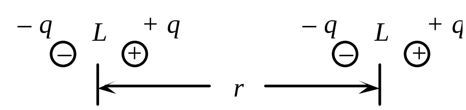
The period of the orbit is

$T_{\text{point charge}} = 2\pi r/v = 2\pi(4\pi\varepsilon_0 m/qQ)^{1/2} r^{3/2}$, which has a different r dependence.

51. The torque on the dipole is

$$\vec{\tau} = \vec{p} \times \vec{E} = qL(\cos 45°\,\hat{i} + \sin 45°\,\hat{j}) \times E\hat{i}$$
$$= -\,qLE \sin 45°\ \hat{k}$$
$$= (2 \times 10^{-6}\ \text{C})(0.10\ \text{m})(10\ \text{N/C}) \sin 45°\ \hat{k}$$
$$= \boxed{-(1.41 \times 10^{-6}\ \text{N}\cdot\text{m})\,\hat{k}}.$$

55. Each dipole is a pair of charges q separated by a distance L, such that $p = qL$. To find the force on the dipole on the right, we find the electric field at each of the charges produced by the charges of the other dipole. The field at the positive charge is



$$E_+ = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r^2} - \frac{q}{(r+L)^2}\right] = \frac{q}{4\pi\varepsilon_0 r^2}\left\{1 - \frac{1}{[1+(L/r)]^2}\right\}\ \text{to the right}.$$

The field at the negative charge is

$$E_- = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-L)^2} - \frac{q}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r^2}\left\{\frac{1}{[1-(L/r)]^2} - 1\right\}\ \text{to the right}.$$

The force on the dipole is

$$F = qE_+ + (-q)E_- = \frac{q^2}{4\pi\varepsilon_0 r^2}\left\{1 - \frac{1}{[1+(L/r)]^2} - \frac{1}{[1-(L/r)]^2} + 1\right\}\ \text{to the right.}$$

Because $L \ll r$, we make use of the approximation $(1 \pm x)^{-2} \approx 1 - 2x + 3x^2 - \ldots$ and expand the terms:

$$F \sim \frac{q^2}{4\pi\varepsilon_0 r^2}\left\{1 - \left[1 - 2\frac{L}{r} + 3\left(\frac{L}{r}\right)^2\right] - \left[1 + 2\frac{L}{r} + 3\left(\frac{L}{r}\right)^2\right] + 1\right\} = \frac{q^2}{4\pi\varepsilon_0 r^2}\left[-6\left(\frac{L}{r}\right)^2\right] = -\frac{6q^2L^2}{4\pi\varepsilon_0 r^4} = -\frac{6p^2}{4\pi\varepsilon_0 r^4}\text{(attraction).}$$

Note, for possible future reference, that the approximation used, $(1 + x)^{-2} \cong 1 - 2x + 3x^2$, can be generalized, so that

$(1 + x)^n \cong 1 + nx + (n(n - 1)/2)\,x^2$.

Setting n = -2, of course, produces the first statement.

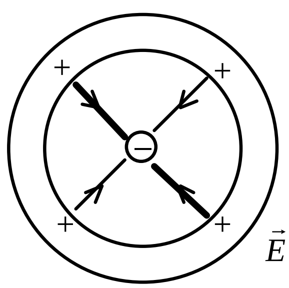
56. The potential energy of a dipole in an electric field is $U = -\vec{p}\cdot\vec{E}$. The maximum energy occurs when $\vec{p}$ and $\vec{E}$ are in opposite directions, and the minimum energy occurs when $\vec{p}$ and $\vec{E}$ are parallel:

$U_{max} = pE,\ U_{min} = -pE,$ and

$\Delta U = 2pE;$

$4.4 \times 10^{-25}\text{ J} = 2p(10^4\text{ N/C})$, which gives $p = \boxed{2.2 \times 10^{-29}\text{ C}\cdot\text{m}}$.

57.



$\vec{E} = 0$ outside

63. Each infinite plate produces a constant field perpendicular to the plate. The total electric field is

$\vec{E} = (\sigma_1/2\varepsilon_0)\hat{j} + (\sigma_2/2\varepsilon_0)\hat{i}$.

The force produced by this field on the particle causes a constant acceleration:

$\vec{a} = q\vec{E}/m = (q/2\varepsilon_0 m)(\sigma_1\hat{j} + \sigma_2\hat{i})$.

If the particle starts from rest, its position is

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + {}^1\!/_2\,\vec{a}t^2 = (1\text{ m})\hat{i} + (1\text{ m})\hat{j} + 0 + (q/4\varepsilon_0 m)(\sigma_2\hat{i} + \sigma_1\hat{j})t^2$

$= \{1 + [(1 \times 10^{-7}\text{ C})/4(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1 \times 10^{-3}\text{ kg})](+3 \times 10^{-6}\text{ C/m}^2)t^2\}\hat{i}$

$+\ \{1 + [(1 \times 10^{-7}\text{ C})/4(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1 \times 10^{-3}\text{ kg})](-5 \times 10^{-6}\text{ C/m}^2)t^2\}\hat{j}$

$= \boxed{[(1 + 8.5t^2)\hat{i} + (1 - 14t^2)\hat{j}]\text{ m, with } t \text{ in s}}$.