

Problem Set #13 Solutions - Chapter 33 - 9, 42, 44, 54, 80, 84

9. If we call the secondary windings N_2 and N_3 , we have $\mathbf{e}_2/\mathbf{e}_1 = N_2/N_1$ and $\mathbf{e}_3/\mathbf{e}_1 = N_3/N_1$, which combines to $\mathbf{e}_3/\mathbf{e}_2 = N_3/N_2$;

$$(11 \text{ V})/(220 \text{ V}) = N_3/(1000 \text{ turns}), \text{ which gives } N_3 = \boxed{50 \text{ turns}}.$$

42. For the maximum voltages to be equal, we have $I_{\max}R = I_{\max}X_C = I_{\max}X_L$, or $R = X_C = X_L$, which gives

$$1/\omega C = R, \quad \text{or} \quad C = \boxed{1/2\text{pf}R};$$

$$\omega L = R, \quad \text{or} \quad L = \boxed{R/2\text{pf}}.$$

44. We find the equivalent values for each of the components:

$$\text{two resistors in series: } R_{\text{eq}} = R_1 + R_2;$$

$$\text{two capacitors in series: } 1/C_{\text{eq}} = 1/C_1 + 1/C_2;$$

$$\text{two inductors in series: } L_{\text{eq}} = L_1 + L_2.$$

For the reactances, we have

$$X_{C_{\text{eq}}} = 1/\omega C_{\text{eq}} = 1/\omega C_1 + 1/\omega C_2 = X_{C1} + X_{C2};$$

$$X_{L_{\text{eq}}} = \omega L_{\text{eq}} = \omega L_1 + \omega L_2 = X_{L1} + X_{L2}.$$

The total impedance is

$$Z_{\text{total}} = [(X_{L_{\text{eq}}} - X_{C_{\text{eq}}})^2 + R_{\text{eq}}^2]^{1/2} = \{[(X_{L1} + X_{L2}) - (X_{C1} + X_{C2})]^2 + (R_1 + R_2)^2\}^{1/2}$$

54. The impedance is

$$Z = \mathbf{e}_{\max}/I_{\max} = 170 \text{ V}/0.4 \text{ A} = 425 \Omega = \boxed{0.4 \text{ k}\Omega}.$$

From $P = I_{\text{rms}}^2 R = (I_{\max}/\sqrt{2})^2 R$ we get

$$R = 2P/I_{\max}^2 = 2(18 \text{ W})/(0.4 \text{ A})^2 = 225 \Omega = \boxed{0.2 \text{ k}\Omega}.$$

Finally, from $Z = [R^2 + (\omega L)^2]^{1/2}$ we solve for L :

$$L = (Z^2 - R^2)^{1/2}/\omega = [(425 \Omega)^2 - (225 \Omega)^2]^{1/2}/[2\pi(60 \text{ Hz})] = 0.956 \text{ H} \sim \boxed{1 \text{ H}}.$$

80. The equivalent capacitance of the two capacitors in series is

$$C = C_1 C_2 / (C_1 + C_2)$$

$$= (4 \text{ nF})(9 \text{ nF}) / (4 \text{ nF} + 9 \text{ nF}) = 2.8 \text{ nF}.$$

The reactances are

$$X_C = 1/\omega C = 1/[2\pi(600 \text{ Hz})(2.8 \times 10^{-6} \text{ F})] = 95 \Omega;$$

$$X_L = \omega L = 2\pi(600 \text{ Hz})(70 \times 10^{-6} \text{ H}) = 0.26 \Omega.$$

The impedance of the circuit is

$$Z = |X_L - X_C| = |(95 \Omega) - (0.26 \Omega)| = 95 \Omega.$$

(a) We find the maximum current from

$$V_0 = I_0 Z;$$

$$4\text{V} = I_0(95 \Omega), \text{ which gives } I_0 = \boxed{0.042\text{A}}.$$

(b) We have

$$\omega_0 = (1/LC)^{1/2} = [1/(70 \times 10^{-6} \text{ H})(2.8 \times 10^{-6} \text{ F})]^{1/2} = 7.1 \times 10^4 \text{ rad/s},$$

so the resonant frequency is

$$f_0 = \omega_0/2\pi = (7.1 \times 10^4 \text{ rad/s})/2\pi = \boxed{1.1 \times 10^4 \text{ Hz}}.$$

84. The equivalent capacitance of the two capacitors in parallel is

$$C = C_1 + C_2 = 20 \text{ nF} + 30 \text{ nF} = 50 \text{ nF}.$$

The reactances are

$$X_{C1} = 1/\omega C_1 = 1/[2\pi(400 \text{ Hz})(20 \times 10^{-6} \text{ F})] = 20 \Omega;$$

$$X_{C2} = 1/\omega C_2 = 1/[2\pi(400 \text{ Hz})(30 \times 10^{-6} \text{ F})] = 13 \Omega;$$

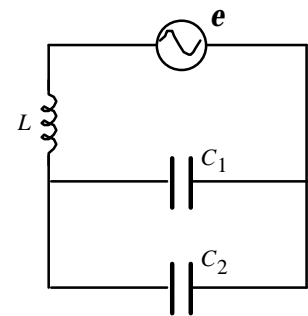
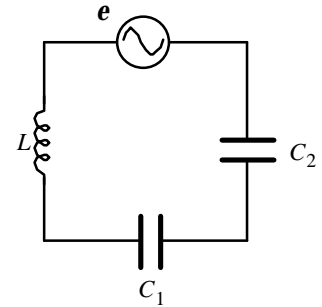
$$X_C = 1/\omega C = 1/[2\pi(400 \text{ Hz})(50 \times 10^{-6} \text{ F})] = 8.0 \Omega;$$

$$X_L = \omega L = 2\pi(400 \text{ Hz})(10 \times 10^{-3} \text{ H}) = 25 \Omega.$$

The impedance of the circuit is

$$Z = [(X_L - X_C)^2]^{1/2} = |25 \Omega - 8.0 \Omega| = 17 \Omega.$$

(a) The maximum current in the circuit, which is the maximum current



in the inductor, is

$$I_0 = I_L = V_0/Z = (12 \text{ V})/(17 \ \Omega) = \boxed{0.71 \text{ A}}.$$

The voltage across the equivalent capacitance is the voltage across each capacitor:

$$I_0 X_C = I_{C1} X_{C1} = I_{C2} X_{C2};$$

$$(0.71 \text{ A})(8.0 \ \Omega) = I_{C1}(20 \ \Omega) = I_{C2}(13 \ \Omega), \text{ which gives } I_{C1} = \boxed{0.28 \text{ A}}, \text{ and } I_{C2} = \boxed{0.44 \text{ A}}.$$

(b) The resonant frequency is

$$f_0 = \omega_0/2\pi = (1/2\pi)/(LC)^{1/2} = (1/2\pi)/[(10 \times 10^{-3} \text{ H})(50 \times 10^{-6} \text{ F})]^{1/2} = \boxed{225 \text{ Hz}}.$$