Problem Set #12 Solutions – Chapter 32 – 17, 21, 22, 31, 42, 48, and 79

17. Because the windings of the two solenoids are in the same direction, the induced emfs will be in the same direction. The total emf in the system is

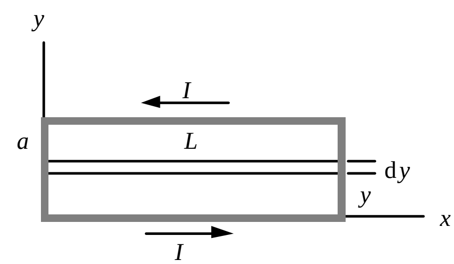
$$e = e_1 + e_2 + e_{12} + e_{21}$$

$$= -\,(L_1\,dI/dt) - (L_2\,dI/dt) - (M\,dI/dt) - (M\,dI/dt)$$

$= -L_{eq}\,dI/dt$, which gives

$$L_{eq} = L + L + M + M = 2(L + M).$$

21. Because $a << L$, we can ignore the magnetic field from the short sides. The fields from the two long sides are in the same direction, so we can double the field from one when we calculate the flux through the circuit. For the field produced by the bottom wire of radius r, we have



$B = m_0Iy/2pr^2$, $y < r$; and $B = m_0I/2py$, $y = r$.

To find the flux through the loop, we choose a strip dy a distance y from the wire:

$$\Phi_B = 2\iint \vec{B}\cdot d\vec{A} = 2\int_0^r \frac{m_0I\ell}{2\pi r^2}y\,dy + 2\int_r^a \frac{m_0I\ell}{2\pi y}\,dy = \frac{m_0I\ell}{\pi}\left[\tfrac{1}{2} + \ln\left(\tfrac{a}{r}\right)\right].$$

The self inductance is

$$L = \frac{m_0\ell}{\pi}\left[\tfrac{1}{2} + \ln\left(\tfrac{a}{r}\right)\right].$$

If $r \to 0$, $\ln(a/r) \to 8$. The radius of wire cannot be neglected.

22. We find the mutual inductance of the system by finding the mutual inductance of the loop. The magnetic field of the wire depends on the distance from the wire. To find the magnetic flux through the loop, we choose a strip a distance x from the wire with width dx:

$$\Phi_B = \iint \vec{B}\cdot d\vec{A} = \int_d^{d+a} \frac{m_0I}{2\pi x}a\,dx = \frac{m_0Ia}{2\pi}\ln\left(1 + \frac{a}{d}\right).$$

The mutual inductance is

$M = \Phi_B/I = \boxed{(m_0 a/2\mathrm{p}) \ln(1 + a/d)}$.

31. (a) The stored energies are

$U_L = ½LI^2 = ½\,(1\text{ H})(10\text{ A})^2 = 50\text{ J}$;

$U_C = ½Q^2/C = ½\,(It)^2/C = ½I^2t^2/C = ½(10\text{ A})^2(1\text{ s})^2/(1\text{ F}) = 50\text{ J}$.

The two energies are $\boxed{\text{the same}}$.

(b) The stored energies are

$U_L = ½LI^2 = ½\,(1\text{ H})(10^{-3}\text{ A})^2 = 0.50\ m\text{J}$;

$U_C = ½Q^2/C = ½\,(It)^2/C = ½I^2t^2/C = ½\,(10^{-3}\text{ A})^2(1\text{ s})^2/(1\text{ F}) = 0.50\ m\text{J}$.

The two energies are the same.

42. (a) From the cylindrical symmetry, we know that the magnetic field is circular. We apply Ampere's law to a circular path to find the magnetic field inside the wire:

$\int \vec{B} \cdot d\vec{s} = m_0 I_{\text{enclosed}}$;

$B2\mathrm{p}r = m_0(I/\mathrm{p}a^2)\mathrm{p}r^2$, which gives $B = m_0 Ir/2\mathrm{p}a^2$.

The energy density of the magnetic field is

$u_B = ½\,B^2/m_0 = \boxed{m_0 I^2 r^2/8\mathrm{p}^2 a^4}$.

(b) Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell centered on the wire, with radius $r < a$, thickness dr, and length L. The energy per unit length is

$$\frac{U_B}{L} = \frac{1}{L}\int_0^a \frac{m_0 I^2 r^2}{8\pi^2 a^4} L2\pi r\,\mathrm{d}r = \frac{m_0 I^2}{4\pi a^4}\int_0^a r^3\,\mathrm{d}r = \frac{m_0 I^2}{16\pi}.$$

48. Before the switch is open there is a stable current in the inductor. After the switch is opened at $t = 0$ we have an RL circuit, with $R = 15\ \Omega + 20\ \Omega = 35\ \Omega$ and $L = 0.5$ H. The current I as a function of time is

$I = I_0\, e^{-Rt/L}$, and so

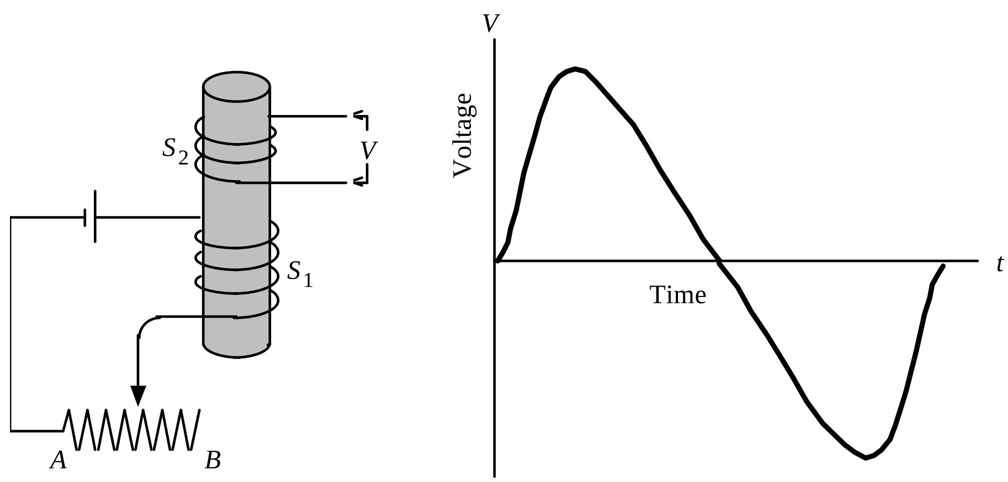
$t = (L/R) \ln(I_0/I)$

$= [(0.5\text{ H})/(35\ \Omega)] \ln(I_0/0.25I_0) = [(0.5\text{ H})/(35\ \Omega)] \ln 4.0 = 0.020\text{ s} = \boxed{20\text{ ms}}$.

Since $U_L = \frac{1}{2}LI^2$ is proportional to I^2, it decrease to 25% of its initial value as I^2 drops to 0.25 I_0^2, i.e., as I drops to $(0.25)^{1/2}\, I_0 = 0.50\, I_0$. The time it takes is

$t = (L/R) \ln(I_0/I) = (0.5\text{ H}/35\ \Omega) \ln(I_0/0.50I_0) = \boxed{10\text{ ms}}$.

79.



While the slider moves from A to B, the current in solenoid S_1 decreases, causing a decrease in the magnetic flux in the core. This decrease in flux generates an induced emf in solenoid S_2, which opposes the decrease and depends on the mutual inductance: $V = -M\, dI/dt$. The time dependence of dI/dt is determined by the specific motion of the slider and the self-inductance of solenoid S_1. We assume a smooth motion, with starting and stopping regions that will give a maximum rate of change of the current a short time after starting. Note that, as the resistance increases, the rate of the fractional change in the resistance will decrease. When the motion of the slider turns around to return to A, the current and the flux will go through a minimum; the induced emf will be zero. Then the flux will start to increase, and the sign of the voltage will change. Assuming the same type of motion, the voltage will be the reverse of the first stage.