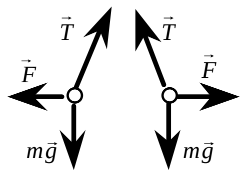
Problem Set #1 – Chapter 21 – 10, 22, 24, 43, 47, 63; Chapter 22 – 7, 10, 36

Chapter 21 Problems

10. (a)



(b) Before the charge is added, the cork balls are hanging vertically, so the tension is

$$T_1 = mg = (0.2\text{x}10^{-3}\text{ kg})(9.8\text{ m/s}^2) = \boxed{2.0\text{x}10^{-3}\text{ N}}.$$

After the charge is added, the charge will be shared equally by the two cork balls, and there is a horizontal Coulomb force. From the force diagram, we apply $\sum \vec{F} = 0$:

horizontal: $T \sin \theta = F = kqq/r^2$;

vertical: $T \cos \theta = mg$.

where θ is the angle of displacement from the horizontal. If we divide the two equations, we get

$$\tan \theta = F/mg = kq^2/r^2mg = kq^2/(2L \sin \theta)^2mg$$

$$= (9\text{x}10^9\text{ N} \cdot \text{m}^2/\text{C}^2)(1\text{x}10^{-7}\text{ C})^2/[2(0.20\text{ m}) \sin \theta]^2(0.2\text{x}10^{-3}\text{ kg})(9.8\text{ m/s}^2) = 0.0065/(\sin^2 \theta).$$

This equation has only one unknown, θ, but the presence of trigonometric functions makes the algebra a little messy. We can solve by calculating both sides for a range of angles, or even simpler, note that for small angles $\tan \theta \approx \sin \theta$, substitute $\sin \theta$ for $\tan \theta$ on the left hand side and solve for $\sin \theta$. Using either of these approaches, we get

$$\sin \theta = 0.19, \quad \theta = 11°.$$

Comparing sin 11° = 0.191 to tan 11° = 0.194, we confirm our substitution is within the degree of uncertainty associated with the numbers being used (the mass of the ball is given as 0.2g, indicating we have uncertainties past one significant figure).

The tension is

$$T_2 = mg/(\cos\,\theta) = (0.2\text{x}10^{-3}\text{ kg})(9.8\text{ m/s}^2)/(\cos 11°) = \boxed{2.0\text{x}10^{-3}\text{ N}}$$

(Not a great amount of change – $T_2 = T_1/(\cos 11°) = 1.02{*}T_1$).

(c) From the analysis in part (b), we have $\theta = \boxed{11°}$.

22. For the Coulomb force to be 0.05% of the measured force, we have

$F = kq_1q_2/r^2$;

$(0.05\text{x}10^{-2})(7\text{x}10^{-7}\text{ N}) = (9\text{x}10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2/(0.10\text{ m})^2$,

which gives

$q = \boxed{2.0\text{x}10^{-11}\text{ C}}$.

24. (a) The attractive Coulomb force provides the centripetal acceleration:

$F = mv^2/r = mr\omega^2$;

$ke^2/r^2 = mr\omega^2$, which we write as $ke^2 = mr^3(2\pi/T)^2$;

$(9\text{x}10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\text{x}10^{-19}\text{ C})^2 = (9.11\text{x}10^{-31}\text{ kg})r^3[2\pi/(24\text{ h})(3600\text{ s/h})]^2$,

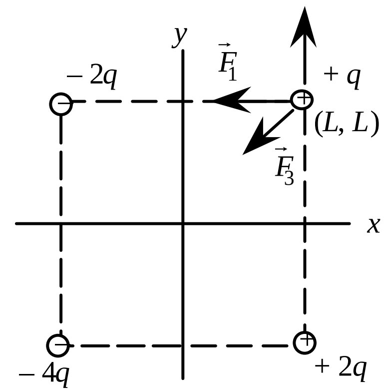
which gives $r = \boxed{3.6\text{x}10^3\text{ m}}$.

(b) For the hydrogen orbit, we have

$(9\text{x}10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\text{x}10^{-19}\text{ C})^2 = (9.11\text{x}10^{-31}\text{ kg})r^3[2\pi/(4\text{x}10^{-16}\text{ s})]^2$,

which gives $r = \boxed{1.0\text{x}10^{-10}\text{ m}}$.

43. (a) The three forces acting on q are shown in the figure.



Their magnitudes are

$F_1 = F_2 = k2qq/(2L)^2 = \frac{1}{2}\, kq^2/L^2$;

$F_3 = k4qq/(2L\sqrt{2})^2 = \frac{1}{2}\, kq^2/L^2$.

The net force acting on q is

$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-\frac{1}{2}\, kq^2/L^2)\,\hat{i} + (\frac{1}{2}\, kq^2/L^2)\,\hat{j}$ –

$\{[(\frac{1}{2}\, kq^2/L^2) \cos 45°]\,\hat{i} + [(\frac{1}{2}\, kq^2/L^2) \sin 45°]\,\hat{j}\}$

$= (\frac{1}{2}\, kq^2/L^2)\{[-(2 + \sqrt{2})/2]\,\hat{i} + [(2 - \sqrt{2})/2]\,\hat{j}\}$

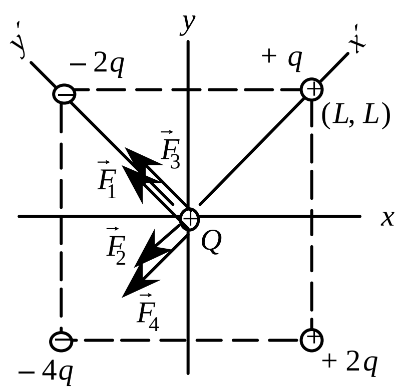
$= \boxed{(\sqrt{3})kq^2/2L^2,\ 9.7° \text{ above the } -x\text{-axis}}$.

(b) The four forces acting on Q are shown in the figure. Their magnitudes are

$F_1 = F_3 = k2qQ/(L\sqrt{2})^2 = kqQ/L^2$;

$F_2 = kqQ/(L\sqrt{2})^2 = kqQ/2L^2$;

$F_4 = k4qQ/(L\sqrt{2})^2 = 2kqQ/L^2$.



To find the net force, we use a rotated $x'y'$-coordinate system, as shown on the diagram. Thus

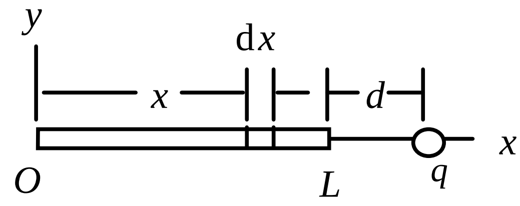
$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= (kqQ/L^2)\,\hat{j}\,' - (kqQ/2L^2)\,\hat{i}\,' + (kqQ/L^2)\,\hat{j}\,' - (2kqQ/L^2)\,\hat{i}\,'$

$= (kqQ/L^2)[-2.5\,\hat{i}\,' + 2\,\hat{j}\,']$

$= 3.2kqQ/L^2$, 38.7° above the $-x'$-axis, or $\boxed{3.2kqQ/L^2,\ 6.3° \text{ below the original } x\text{-axis}}$.

$\boxed{47}$. We align the rod along the x-axis with one end at the origin, as shown in the figure. The linear charge density is $\lambda = Q/L$, so the charge on the element dx is d$Q = (Q/L)$ dx. All elements of the rod produce a force in the + x-direction. The total force is

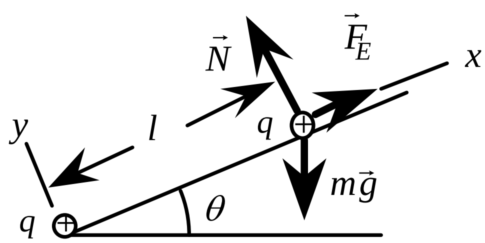


$$\vec{F} = \int \hat{i}\, \mathrm{d}F_x = \int_0^L \frac{kql}{r^2}\hat{i}\, \mathrm{d}x = \frac{kqQ}{L}\hat{i}\int_0^L \frac{\mathrm{d}x}{(L - x + d)^2}$$

$$= \frac{kqQ}{L}\hat{i}\left(\frac{1}{L - x + d}\right)\Big|_0^L = \frac{kqQ}{L}\hat{i}\left(\frac{1}{d} - \frac{1}{L + d}\right) = \frac{kqQ}{d(L + d)}\hat{i}.$$

The force on q is $\boxed{kqQ/d(L + d) \text{ away from the rod}}$.

63. In the equilibrium position, the net force is zero. From the diagram,



$\sum F_x = F_E - mg \sin\theta = 0;$

$kqq/\ell^2 = mg \sin\theta;$

$(9\text{x}10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(2\text{x}10^{-8}\ \text{C})^2/(0.08\ \text{m})^2 = (0.5\text{x}10^{-3}\ \text{kg})(9.8\ \text{m/s}^2) \sin\theta$, which gives

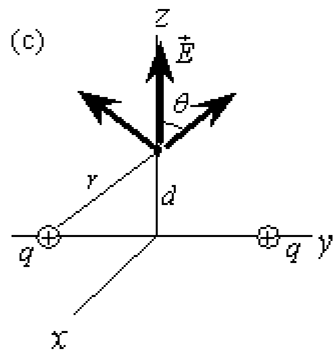
$\sin\theta = 0.115, \quad \theta = \boxed{6.6°}.$

Chapter 22 Problems

7. (a) With the charges on the x-axis, the electric fields produced by the charges will have the same magnitude and point in the – x-direction. The resultant field will be

$\vec{E} = 2(1/4\pi\varepsilon_0)q/(\ell/2)^2(-\hat{i}) = -(1/4\pi\varepsilon_0)(8q/\ell^2)\hat{i}.$

(b) The fields produced by the charges will have the same magnitude and point in opposite directions. The resultant field will be $\vec{E} = \boxed{0.}$



(c) We take a representative point on the z-axis (note – this is the corrected figure – Figure 22-28 in the textbook has the charges incorrectly located on the x axis). From the diagram, we see that the electric fields produced by the charges will have the same magnitude, and the resultant field will point away from the origin. If we call the distance from the origin d, we have

$$E = 2(1/4\pi\varepsilon_0)(q/r^2)\cos\theta = (1/4\pi\varepsilon_0)(2q/r^2)(d/r)$$

$$= (1/4\pi\varepsilon_0)\{2qd/[d^2 + (\ell/2)^2]^{3/2}\}.$$

Note, this is the answer given in the back of the book but we can do a little more work and express the answer in terms of x and z. We begin by noting that the magnitude of the field will remain the same at all points on the xz plane the same distance d from the origin, and the direction of the field will point away from the origin. So, along a circle of radius d, the field will be

$$\vec{\mathrm{E}} = (1/4\pi\varepsilon_0)\{2\mathrm{qd}/[\mathrm{d}^2 + (\ell/2)^2]^{3/2}\}(\sin\varphi\,\hat{\mathrm{i}} + \cos\varphi\,\hat{\mathrm{k}})$$

where φ is the angle from the z axis in the xz plane and $\sqrt{x^2 + z^2} = \mathrm{d}$. Since

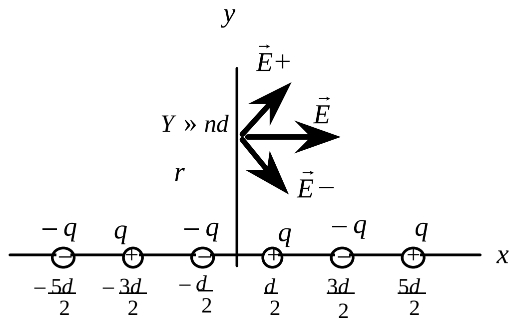
$\cos\varphi = \mathrm{x}/\sqrt{x^2 + z^2}$ and $\sin\varphi = \mathrm{z}/\sqrt{x^2 + z^2}$, the field can finally be expressed as

$$\vec{\mathrm{E}} = (1/4\pi\varepsilon_0)\{2\mathrm{q}/[\mathrm{x}^2 + \mathrm{z}^2 + (\ell/2)^2]^{3/2}\}(\mathrm{x}\,\hat{\mathrm{i}} + \mathrm{z}\,\hat{\mathrm{k}})$$

10. We treat the line of charges as n pairs symmetrically placed about the y-axis. From the diagram, we see that a pair of charges produces an electric field parallel to the x-axis. For a pair with $r^2 = Y^2 + x^2$, we add the x-components to get the

magnitude of the field:

$E = 2(1/4\pi\varepsilon_0)(q/r^2)(x/r) = 2qx/4\pi\varepsilon_0(Y^2 + x^2)^{3/2}$.



For all pairs, we have $Y >> x$, so we get

$$E \cong 2qx/4\pi\varepsilon_0 Y^3.$$

Because the pairs alternate in sign, the direction of E will alternate. The electric field of the ith pair is

$$\vec{E}_i = [(-1)^i 2qx_i/4\pi\varepsilon_0 Y^3]\hat{i}, \text{ with } i = 1, 2, 3, \ldots, n.$$

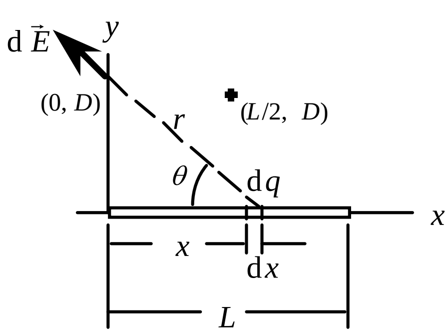
The values of x_i are $d/2$, $3d/2$, $5d/2$, …, so when we sum the n pairs, we get

$$\vec{E} = \sum \vec{E}_i = \sum[(-1)^i 2qx_i/4\pi\varepsilon_0 Y^3]\hat{i} = (2q/4\pi\varepsilon_0 Y^3)(d/2)(-1 + 3 - 5 + 7 - \ldots)\hat{i}.$$

For the first few terms, the result of the summation is – 1, + 2, – 3, + 4, … . Thus the general result of the summation is $(-1)^n n$. The resultant electric field is

$$\vec{E} = (2q/4\pi\varepsilon_0 Y^3)(d/2)(-1)^n n\hat{i} = (-1)^n(qnd/4\pi\varepsilon_0 Y^3)\hat{i}.$$

36.



To find the electric field at the point $(0, D)$, we choose a differential element of the rod, as shown in the diagram. The charge of this element is $dq = (Q/L)\,dx$. We find the field produced by the element, which has both x- and y-components, by integrating along the rod:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{x=0}^{x=L} \frac{\mathrm{d}q}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_{x=0}^{x=L} \frac{\mathrm{d}x}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right).$$

To perform the integration, we must eliminate variables until we have one, for which we choose θ. From the diagram we see that $r = D/\sin\theta$, and $x = D\cot\theta$. This gives

$$\mathrm{d}x = -D\csc^2\theta\,\mathrm{d}\theta = -(D\,\mathrm{d}\theta)/\sin^2\theta.$$

The limits for θ are $\pi/2$ rad to $\theta_0 = \cos^{-1}[L/(D^2 + L^2)]$. When we make these substitutions, we have

$$\vec{E}(0, D) = \frac{Q}{4\pi\varepsilon_0 L}\int_{\pi/2}^{\theta_0} \frac{(-\mathrm{d}\theta)/\sin^2\theta}{(D/\sin\theta)^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\int_{\pi/2}^{\theta_0} \mathrm{d}\theta\left(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\left(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)\Big|_{\pi/2}^{\theta_0}$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\left[(\sin\theta_0 - 1)\,\hat{i} + (\cos\theta_0 - 0)\,\hat{j}\right];$$

$$\vec{E}(0, D) = \frac{Q}{4\pi\varepsilon_0 LD}\left[\left(\frac{D}{\sqrt{D^2 + L^2}} - 1\right)\hat{i} + \left(\frac{L}{\sqrt{D^2 + L^2}}\right)\hat{j}\right].$$

Because the point $(L/2, D)$ is opposite the midpoint of the rod, we know that the field there will have only a y-component. Instead of doing another integration, we use the result from the text (which Prof. Dunning also went over in class):

$$\vec{E}(L/2, D) = \frac{2\lambda}{4\pi\varepsilon_0 D}\left[\frac{L/2}{\sqrt{D^2 + (L/2)^2}}\right]\hat{j} = \frac{Q}{4\pi\varepsilon_0 D}\left(\frac{2}{\sqrt{4D^2 + L^2}}\right)\hat{j}.$$