Physics 102– Pledged Problem 9

Time allowed: **2 hours at a single sitting**

Due 5PM Monday, April 16, 2007, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

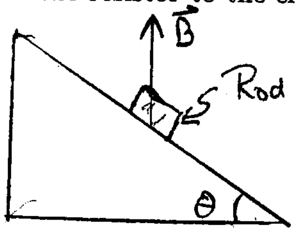
(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner. Make one vertical fold.
(c) On the outside, print your name in capital letters, your LAST NAME followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 9", followed by the due date.
(e) Write and sign the pledge, with the understanding that you may consult the materials noted above.
(f) Indicate your **start time** and **end time.**

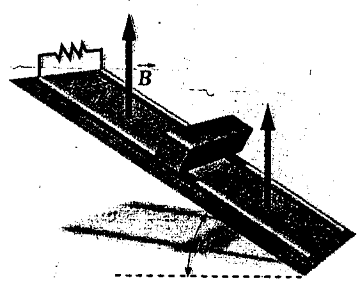
I. A conducting rod of mass m and negligible resistance slides without friction along two parallel rails, also of negligible resistance. The rails are separated by a distance l and connected together through a resistor R. The rails rest on a long inclined plane that makes and angle θ to the horizontal. There is a uniform magnetic field B directed vertically upward as shown in the figure below. The rod is released from rest and slides down the incline. Express your answers in terms of B, l, R, θ, m, and possibly other constants.

(a) When the rod has velocity v_x down the incline, determine the current induced in the circuit by the changing magnetic flux. Show that this current produces a retarding force up the incline and find the magnitude of that force.

(b) Show that there is a terminal speed v_t such that the gravitational force down the incline is balanced by the upward retarding force. Determine the value of v_t.

(c) When the rod has reached the terminal velocity v_t, what is the I^2R power dissipation in the resistor? Compare the power dissipated in the resistor to the change in gravitational potential energy of the rod.



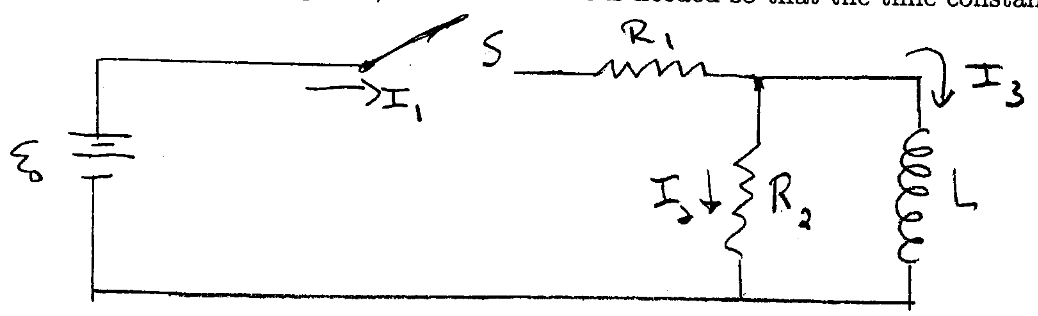


II. In the circuit shown below, the switch is initially opened, then at $t = 0$ it is closed.

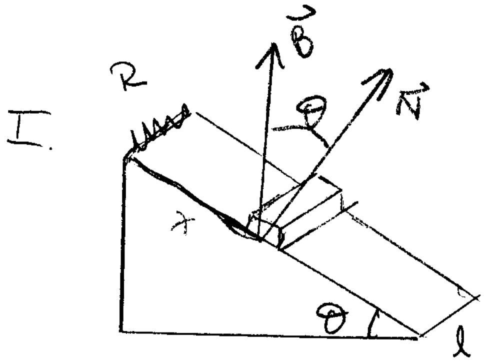
(a) Determine the currents I_1, I_2, and I_3 and the potential drop across the inductor $\mathcal{E}_L$ at $t = 0$.
(b) Determine the currents I_1, I_2, and I_3 and $\mathcal{E}_L$ at $t \to \infty$.

After the switch has been closed for a long time, it is opened.

(c) Determine the three currents immediately after the switch is opened
(d) Determine the three currents a long time after the switch is opened.
(e) Determine the $I(t)$, the current in the inductor L as a function of time, after the switch is opened.
(f) If L=1H, and $R_1 = R_2 = R$, what value of R is needed so that the time constant for discharging is 30s?



Phys102 - Pledged Problem 9

I. 

(a) $\vec{B}$ is at an angle θ to the normal vector $\vec{N}$, so the flux $\phi_B = \int \vec{B} \cdot d\vec{S} = B\cos\theta$ (area)

where area $= l\,x$

To determine the induced emf in the rod-rail loop, we need $\frac{d\phi_B}{dt}$

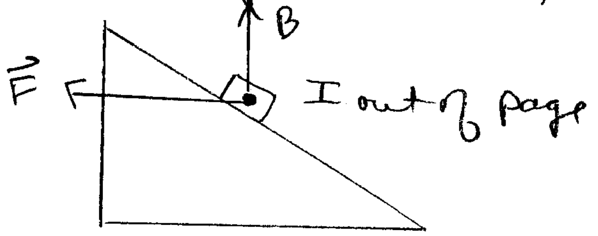
$$\frac{d\phi_B}{dt} = \frac{d}{dt}(B\cos\theta\, l x) = B\cos\theta\, l\, v_x$$

where v_x is the velocity down the incline.

So $\mathcal{E} = Bl\cos\theta\, v_x = IR$

$$I = \frac{Bl\cos\theta\, v_x}{R}$$

The direction will be so as to oppose the change in flux, so I will be clockwise as viewed from above, or out of the page, giving a force to the left



$\vec{F} = I\vec{L} \times \vec{B}$

$\vec{F} = I\ell B\,(-\hat{i})$ — to the left.

We need the component of this force up the incline

$F_{up} = F\cos\theta = IB\ell\cos\theta$

$$\boxed{F_{up} = \frac{B^2\ell^2\cos^2\theta\, v_x}{R}}$$ — retarding force up incline

(b) As the rod's speed down the incline increases, the retarding force up the incline increases until F_{up} balances the gravitational force down.

$$\frac{B^2\ell^2\cos^2\theta\, v_t}{R} = mg\sin\theta$$

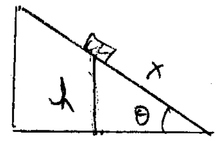
$$\boxed{v_t = \frac{Rmg\sin\theta}{B^2\ell^2\cos^2\theta}}$$

(c) At v_t, the power dissipated in the resistor is

$$P = I^2R = \frac{RB^2\ell^2\cos^2\theta}{R^2}\left[\frac{Rmg\sin\theta}{B^2\ell^2\cos^2\theta}\right]^2$$

$$\boxed{\text{Power} = \frac{R(mg\sin\theta)^2}{B^2\ell^2\cos^2\theta}}$$

Now determine the rate of change of the rod's gravitational PE



$$PE = mgh$$

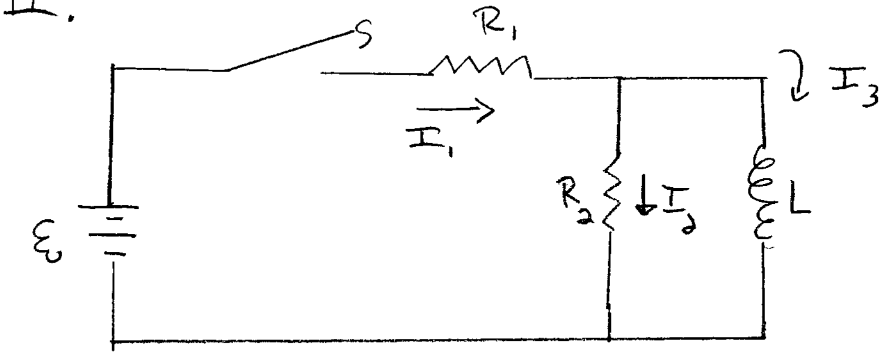
$$= mg\, x \sin\theta$$

$$\frac{d}{dt}(PE) = mg \sin\theta\, v_t \qquad \left(\frac{dx}{dt} = v_t\right)$$

$$\frac{d}{dt}(PE) = mg\sin\theta \left[\frac{R\, mg \sin\theta}{B^2 l^2 \cos^2\theta}\right]$$

$$\boxed{\frac{d}{dt}(PE) = \frac{R\,(mg\sin\theta)^2}{B^2 l^2 \cos^2\theta} = \text{Power dissipated in } R}$$

II.



(a) Immediately after S is closed, there is no current through L, since the current in an inductor cannot change abruptly. R_1 & R_2 are in series

$$\boxed{I_3 = 0 \qquad I_1 = I_2 = \frac{\mathcal{E}_0}{R_1+R_2}}$$

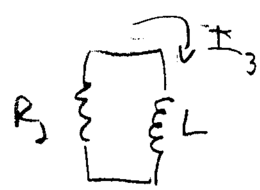
$$\boxed{\mathcal{E}_L = I_2 R_2 = \frac{\mathcal{E}_0 R_2}{R_1+R_2}}$$

(b) for $t \to \infty$, the current in L is established, and the resistance of L is negligible

$$\boxed{I_2 = 0 \qquad I_1 = I_3 = \frac{\mathcal{E}_0}{R_1}}$$

(c) Immediately after switch opened, R_1 is out of the circuit, $I_1 = 0$. The current through L can't change abruptly, so

$$\boxed{I_1 = 0 \qquad I_2 = I_3 = \frac{\mathcal{E}_0}{R_1}}$$



(d) At $t \to \infty$, all currents have decayed away

$$\boxed{I_1 = I_2 = I_3 = 0}$$

(e) The current decays with characteristic time constant $\tau = L/R$ where in this case $R = R_2$.

$I = I_0 e$ where $I_0 = \frac{\mathcal{E}_0}{R_1}$

$$\boxed{I = \frac{\mathcal{E}_0}{R_1} e^{-tR_2/L}}$$

(f) $L = 1\,H$ $\qquad \tau = L/R = 30\,\text{sec}$

$$R = \frac{L}{30\,\text{sec}} = 1/30$$

$$\boxed{R = 0.03\,\Omega}$$

Check units: L has units of $\frac{\text{flux}}{I} = \frac{B \cdot A}{I}$

B has units $\frac{N s}{C m}$ $\qquad [L] = \frac{N s}{C m} \cdot \frac{m^2 s}{C} = \frac{J s^2}{C^2}$

$(F = q v \times B)$

$$[R] = \frac{L}{\tau} = \frac{J s^2}{C^2 s} \sim \frac{J s}{C^2} = \Omega$$

Ω has units $R = \frac{V}{I} = \frac{J s}{C^2}$