Physics 102– Pledged Problem 9

Time allowed: **2 hours at a single sitting**

Due 5PM Monday, April 3, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.

(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.

(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.

(d) Below your name, print the phrase "Pledged Problem 9", followed by the due date.

(e) Also indicate **start time** and **end time.**

(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

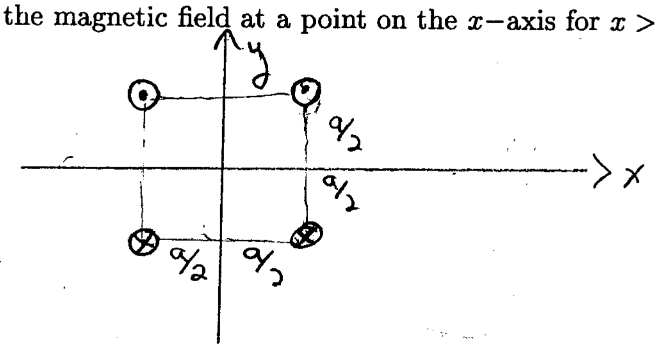
I. Four long, parallel, straight wires pass through the corners of a square whose side has length a. Take the center of the square to be the origin, with the x and y axes as shown. The top two wires carry current I out of the page, and the bottom two wires carry current I into the page.

(a) Determine the magnetic field $\vec{B}$ at the location of the top left wire due to the other three wires.

(b) Determine the force per unit length on the top left wire due to the other three wires.

(c) Determine the magnetic field $\vec{B}$ at the center of square.

(d) Determine the *direction* of the magnetic field at a point on the x–axis for $x > a$.



II. A very long wire carrying current I is bent to form a semicircle of radius R as shown below. After the semicircle is formed, the wire makes a sharp bend and continues to the right. The point C is at the center of the semicircle.

(a) Determine the magnetic field at the point C due to the curved (semicircular) part of the wire.

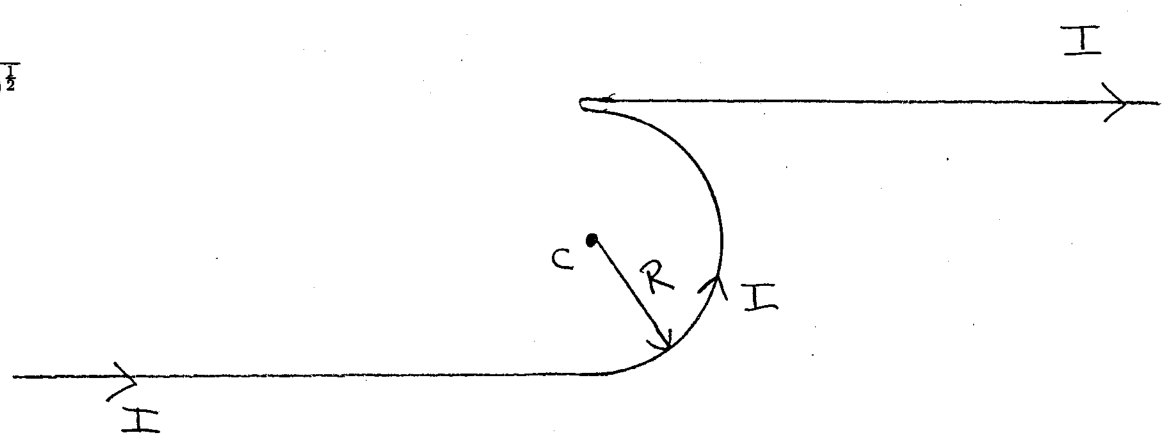
(b) Determine the magnetic field at C due to the long straight section of wire to the left of the semicircle.

(c) Determine the magnetic field at C cue to the long straight section of wire to the right of the semicircle.

(d) Compare your results in (b) and (c) to the magnetic field due to a single infinitely long wire.

You *may* find the following indefinite integral useful:

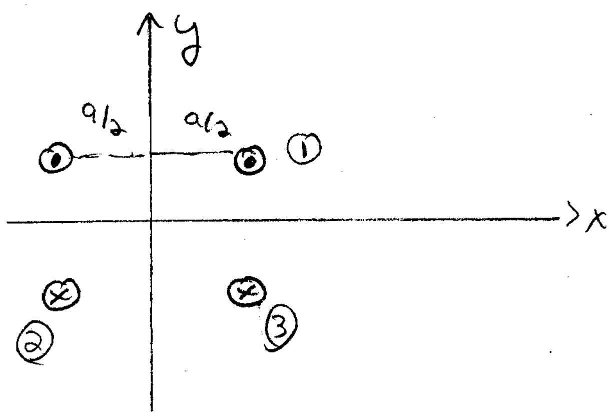
$\int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2(x^2+a^2)^{\frac{1}{2}}}$



Phys 102

Pledged Problem 9

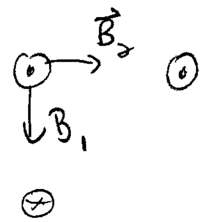
I.



(a) Find $\vec{B}$ at the location of the top left wire.

Consider each wire separately!

$\vec{B}_1$ (due to wire labeled 1 above)



From Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

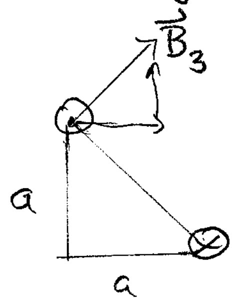
$$2\pi r B = \mu_0 I$$

$$|B| = \frac{\mu_0 I}{2\pi r}$$ for 1 & 2 $r = a$

for 3 $r = \sqrt{2}\, a$

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi a}(-\hat{j}) \quad \vec{B}_2 = \frac{\mu_0 I}{2\pi a}(\hat{i})$$

For the third wire, we need to be careful of the angles!



$$\vec{B}_3 = \frac{\mu_0 I}{2\sqrt{2}\pi a}\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$$

Total $\vec{B}$ is the sum:

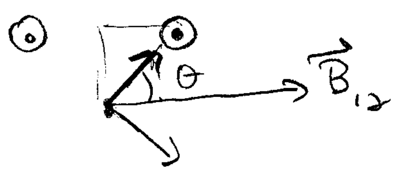
$$\boxed{\vec{B}_{TOT} = \frac{\mu_0 I}{2\pi a}\left[\frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right]}$$

(b) $\vec{F} = I\vec{L}\times\vec{B}$ Current in this case is in $+\hat{k}$ direction

$$\vec{F} = IL\left[\frac{\mu_0 I}{2\pi a}\left(\frac{3}{2}\underbrace{\hat{k}\times\hat{\imath}}_{+\hat{\jmath}}\right) - \frac{1}{2}\left(\underbrace{\hat{k}\times\hat{\jmath}}_{-\hat{\imath}}\right)\right]$$

$$\boxed{\frac{\vec{F}}{L} = \frac{\mu_0 I^2}{2\pi a}\left[\frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath}\right]}$$

(c) $\vec{B}$ at the center of the square - use the symmetry to simplify.



First consider the top two wires. The net field is in the $+x$ direction, since the vertical components cancel.

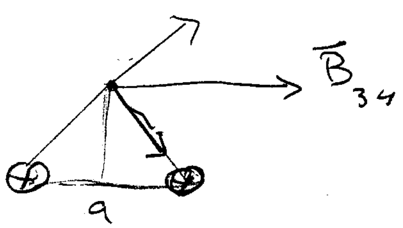
$\theta = 45°$

$$\vec{B}_{12} = \frac{\mu_0 I}{2\pi(a/\sqrt{2})} (2 \cos\theta)\, \hat{i}$$

$$\vec{B}_{12} = \frac{\cancel{\sqrt{2}}\, \mu_0 I}{2\pi a} \left(\frac{2}{\cancel{\sqrt{2}}}\right) \hat{i}$$

$$\vec{B}_{12} = \frac{\mu_0 I}{\pi a}\, \hat{i}$$

Similarily for the lower two wires!

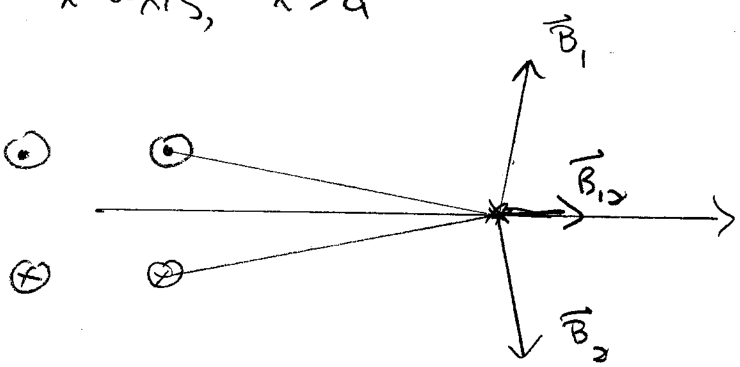


$\vec{B}_{34}$ will be the <u>same</u> as $\vec{B}_{12}$!

$$\vec{B}_{34} = \frac{\mu_0 I}{2\pi(a/\sqrt{2})} \left(\frac{2}{\sqrt{2}}\, \hat{i}\right) \hat{i}$$

$$\boxed{\vec{B}_{TOT} = \vec{B}_{12} + \vec{B}_{34} = \frac{2\mu_0 I}{\pi a}\, \hat{i}}$$

(d) Determine the <u>direction</u> of the magnetic field on the x-axis, $x > a$

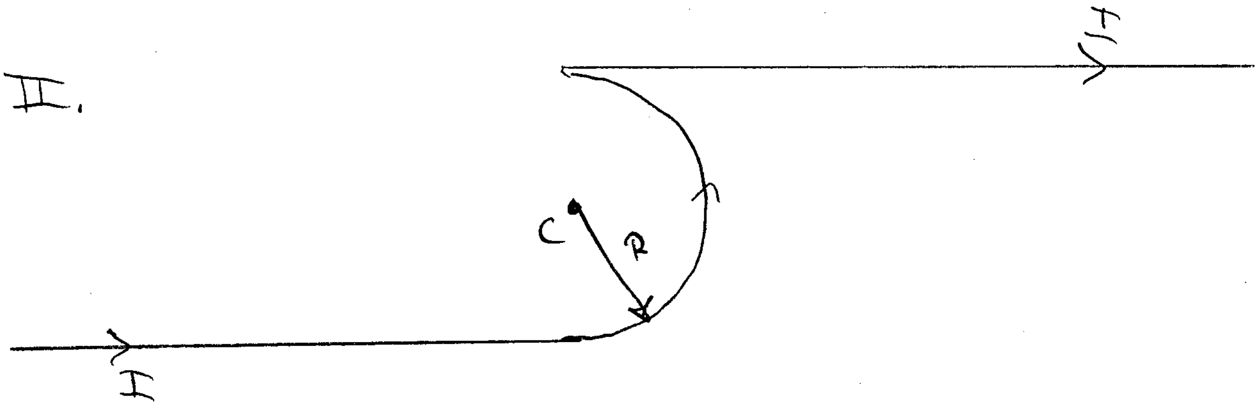


First consider the two wires closest to P. Vertical components cancel, horizontal components add.. $\vec{B}_{12}$ is in $+x$ direction

Same arguements apply to the other two wires

$$\boxed{\Rightarrow \vec{B} \text{ on x-axis is in the } \underline{\underline{+x}} \text{ direction}}$$

II.



(a) Determine $\vec{B}$ at C due to semicircular part of wire.

Use Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

Direction of $d\vec{l} \times \hat{r}$ is up out of page.

$r^2 \longrightarrow R^2$, same for all current elements $d\vec{l}$

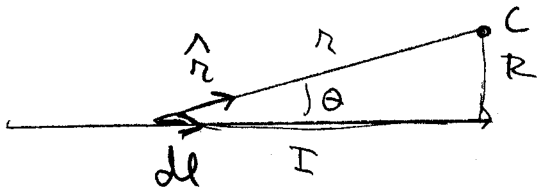
$$|dB| = \frac{\mu_0 I}{4\pi} \frac{dl}{R^2} \qquad \int dl = \pi R$$

$$|\vec{B}| = \frac{\mu_0 I}{4\pi}\left(\frac{\cancel{\pi} \cancel{R}}{R^{\cancel{2}}}\right) = \frac{\mu_0 I}{4R}$$

(due to semicircle)

$$\boxed{\vec{B}_{(\text{semic.})} = \frac{\mu_0 I}{4R}\,\vec{k} \quad (\text{up out of page})}$$

(b) Now determine $\vec{B}$ due to the left straight section.



Again, from the Biot - Savart law,

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \qquad d\vec{l} \times \hat{r} = dl \sin\theta \ \hat{k} \text{ (upward)}$$

$$\sin\theta = \frac{R}{r} \qquad dl \to dx$$

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi} \frac{\sin\theta \, dx}{x^2 + R^2} \qquad r = \sqrt{x^2 + R^2}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{R \, dx}{(x^2+R^2)^{3/2}}$$

We need to integrate from $x = -\infty$ to 0

$$B = \frac{\mu_0 I R}{4\pi} \int_{-\infty}^{0} \frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 I R}{4\pi} \frac{x}{R^2 (x^2+R^2)^{1/2}} \Bigg|_{-\infty}^{0}$$

$$|\vec{B}| = \frac{\mu_0 I}{4\pi R} \left(0 - (-1) \right) = \frac{\mu_0 I}{4\pi R}$$

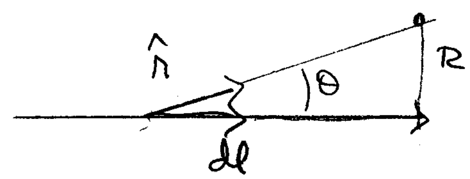
Note! $\displaystyle \lim_{x \to -\infty} \frac{x}{(x^2+R^2)^{1/2}} = \lim_{x \to -\infty} \frac{x}{|x|} = -1$

$$\boxed{\underset{\text{(left section)}}{\vec{B}} = \frac{\mu_0 I}{4\pi R} \hat{k} \text{ (up out of page)}}$$

- equal to ½ of an infinite wire!

$4\pi R$

Alternative way to do the integral — change to an integral over angles!



$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{R^2 + x^2}$$

$$\tan\theta = \frac{R}{x}$$

$$x = \frac{R\cos\theta}{\sin\theta}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{\sin\theta \, R\, d\theta}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{R^2}$$

$$dx = R\left(-\frac{\sin\theta}{\sin\theta} - \frac{\cos^2\theta}{\sin^2\theta}\right) d\theta$$

$$= -R\left(\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}\right) d\theta = -\frac{R}{\sin^2\theta} d\theta$$

$$|dx| = \frac{R\, d\theta}{\sin^2\theta}$$

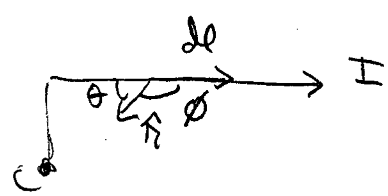
$$B = \frac{\mu_0 I}{4\pi R} \int_0^{\pi/2} \sin\theta \, d\theta$$

$$\sin\theta = \frac{R}{\sqrt{x^2 + R^2}}$$

$$B = \frac{\mu_0 I}{4\pi R} \underbrace{-\cos\theta \Big|_0^{\pi/2}}_{= 1}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi R} \hat{k} \quad \text{as before!}$$

(c) Determine $\vec{B}$ due to the right straight section!



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0 I \, dl \sin\phi}{4\pi (R^2 + x^2)} (-\hat{k})$$

(direction is down).

Use trig identity

$$\sin(\phi) = \sin(\pi - \theta) = \sin\pi\cos\theta - \cos\pi\sin\theta = +\sin\theta$$

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{R\,dx}{(x^2+R^2)^{3/2}}$$ $\sin\theta = \frac{R}{(x^2+R^2)^{1/2}}$

Same as in (b)

$$B = \frac{\mu_0 I R}{4\pi} \underbrace{\int_0^{\infty} \frac{dx}{(x^2+R^2)^{3/2}}}_{1/R^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi R}(-\hat{k}) \quad \text{(right section)}}$$

Direction is down, into page

Note that the contributions from the two long straight sections cancel!

(d) A single long wire has field

$$|\vec{B}| = \frac{\mu_0 I}{2\pi R}$$ — each half-infinite wire has half that value at the point C.